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Modal Testing and Finite Element Analysis of Crack Effects on Turbine Blades

K.R.P. Babu [1, 2 *], B. R. Kumar [2], K. M. Rao [3]

[1]*Research Scholar, Department of Mechanical Engineering, JNTUK Kakinada, India*
[2]*Department of Mechanical Engineering, KLEF University, Guntur, India*
[3]*Department of Mechanical Engineering, JNTUK Kakinada, India*

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ABSTRACT

The study of vibration response of a turbine blade helps to detect the crack presence in the blade which alters its dynamic characteristics. The change is characterized by changes in the modal parameters associated with natural frequencies. In this paper, study of vibration response is made for turbine blade in the presence of a crack like defect. Turbine blade is initially assumed as a cantilever beam. Modal testing has been carried out for both the beams with different crack depth and crack location ratios using FFT spectrum analyzer and ANSYS software. From the analysis, it has been observed that the crack depth and its location have noticeable effect on the natural frequencies. Later the same cantilever beam was twisted with different angle of twists to validate the cantilever beam model to turbine blade.

Keywords : Vibration response; Finite element analysis; Twisted cantilever beam; Turbine blade; FFT analyzer.

1 INTRODUCTION

SOME machine failures are catastrophic events in which the machine condition deteriorates from good to damage in a very short time. But the greater number of machine failures; however is the culmination of a long slow deterioration during which the machine gets progressively worse over a period of week or even months. It has been noticed that about 80% of failures of rotating machinery leads to a significant changes in vibration. The machine can talk, but it speaks in its own languages, it communicates in the form of vibration imbedded in the vibration signal are all the internal defects of the machine. It is a time where we needed a translator that is called as FFT spectrum analyzer. It translates the complex vibration signal coming from machine into something that we human can understand. In machine diagnosis, we are playing the role of machine's doctor. It is superior to look some instrument to machine and let the machine tell us what its need of repair is. So, vibration signal is the major source of information available from the machinery itself for fault detection and diagnosis.

Cracks may propagate due to fatigue in rotating components of machine and can have an adverse effect on the reliability of the machine. Machines like turbines are more expensive and require proper condition monitoring of their blades. Early crack detection and repair enhances the reliability as well as the durability of the machines. Vibration monitoring is one of the promising non-destructive testing methods which can be carried out on the components of the machine when the defect location is not accessible.

[*]Corresponding author. Tel.: +91 944 026 2702.
E-mail address: krpb_me@kluniversity.in (K.R. P. Babu).

In modal testing method, changes in system modal characteristics, such as changes in system natural frequencies response to specially applied excitation due to the presence of a crack, are employed for crack detection. Most modal testing methods need the rotor to be stationary while some require the rotor to be running at a fraction of its operating speed. The failure of turbine blades is a common experience in power plants and is predominantly vibration related. So vibration studies have acquired considerable importance and a good deal of work on the determination of natural frequencies and mode shapes has been contributed by several researchers.

Dimarogonas [1,2] noticed that for small crack depths the reduction in natural frequency is proportional to the square of the crack depth ratio. Since only monitoring cracks with small depths are of engineering interest, monitoring the change of natural frequency (critical speed in the case of turbine rotors) was not important and other methods ought to be developed. For a stepped rotor the authors used a transfer matrix technique to compute the change in critical speed of a shaft due to crack. The results confirmed their findings. Wendtland [3] reported in his dissertation on an experimental program for the computation of the change of natural frequencies of beams of different geometries and boundary conditions using machined slots to simulate cracks. He computed the cracked section flexibility from a crude simple beam analysis of the reduced section. Although he clearly stated that his results were not applicable to real cracks, only to saw cuts, there was some confusion later on because several authors compared finite element analysis results, modeling the crack with dense grid rather than a special cracked element, and reported agreement with him, naturally, because their model was also a notched beam. Adams et al [4] treated a free-free bar with localized damage, such as crack. The damage was modeled by a linear spring of infinitesimal length separating two sections of the bar. Natural frequencies of longitudinal vibration were determined analytically and experimentally.

Cawley and Adams [5] employed sensitivity analysis to deduce the location of damage of a flat plate based on finite element technique. The results of the analysis were found to agree well with the experimental findings. The cracked Euler-Bernoulli beam vibration theory and extensive analytical and experimental results on the natural frequencies and vibration modes of simple continuous and lumped mass beams with surface cracks were introduced by Chondros and Dimarogonas [6]. They studied the effect of a crack in a welded joint on the dynamic behavior of beams with various boundary conditions. Crack depth was estimated from knowledge of changes in the system natural frequencies. A torsional elastic stiffness was used to model the effect of the crack. The torsional stiffness was determined experimentally from measurements of change in natural frequencies for various crack depth values. Dimarogonas and Massouros [7] introduced the influence of a peripheral crack upon the torsional dynamic behavior of a shaft. They used the linear spring model to find that the introduction of crack results in lower torsional natural frequencies, because of the added flexibility. Experimental results were in close agreement with their analysis. The results showed that the changes in dynamic response due to the crack were high enough to allow the detection of crack and estimation of its location or magnitude.

Gudmundson [8] considered a bar with free ends and a transverse crack at the center. Using a perturbation method, he developed an equation for the changes in the natural frequencies. The results compared well with values obtained by the experiments and the finite element method for small cracks. Dimarogonas and Papadopoulos [9] computed the local flexibility of a cracked shaft and verified it by experiment. They reported that sub harmonic resonance is a primary source of information for the identification of the presence of a crack. They developed also a stability chart for varying crack depths. Anifantis et al. [10] developed the spectral method for identification of earthquake induced defects in reinforced concrete frames by analysis of the changes in the vibration frequency spectrum. They also showed that any localized damage, such as a crack, would affect each vibration mode differently, for various structures, depending on the particular location, orientation and magnitude of the crack. Dentsoras and Dimarogonas [11] analyzed a cantilever beam with a crack at the fixed end. A longitudinal harmonic force was applied at the free end, and the crack was modeled as a linear spring. Lowest three natural frequencies were determined. Very little use was made of the changes in the mode shapes in detecting a crack or damage in a structure.

Sato [12] showed that natural frequency changes do increase with increasing slot width by the introduction of slots in beams. However, his results were on slots much deeper than the cracks necessary to detect in many nondestructive testing applications, so he carried out an investigation of the relatively small cracks and slots on the natural frequencies of a component. Nahvi and Jabbari [13] established analytical as well as experimental approach to the crack detection in cantilever beams. An experimental set up was designed in which a cracked cantilever beam is excited by a hammer and the response is obtained using an accelerometer attached to the beam. Contours of the normalized frequency in terms of the normalized crack depth and location were plotted. The intersection of contours with the constant modal natural frequency planes was used to relate the crack location and depth. Lele et al. [14] extended frequency based methods of crack detection to short beams by taking into account the effects of rotational inertia and shear deformation to estimate the crack location. Methods for solving both the forward and inverse problems were also presented. Babu and Prasad [15] used differences in curvature mode shapes to detect a crack in beams.

The main objective of this paper is to study the vibration response of the blade with incorporated crack with the help of experimental simulator. A crack in a turbine blade introduces local flexibility such that it would affect its vibration response. This is because of the decrease in the stiffness at that portion of the blade. Modeling of turbine blade can be treated as cantilever beam so that the effect of the crack on dynamic response of the cantilever beam can be extended to develop fault diagnosis in turbine blades.

Hence the present work focuses on vibration response cantilever beam using vibration response. This is performed in two steps. First, an experimental set up has been designed in which a cracked cantilever beam is excited by an impact hammer and the response is obtained using accelerometer attached to the beam. Secondly, the finite element model of the cracked cantilever beam is established with the help of ANSYS Software. The beam is discretised into number of elements. Crack is assumed to be in different locations. For each location crack depth is varied. Natural frequencies have been evaluated for different crack depth and its location. Experimental and theoretical results have been found to be close enough. Later same analysis has been carried out for the twisted cantilever beams to validate cantilever beam model to turbine blade. The results of this scenario will be analyzed in this paper and comparison of the vibration analysis of cantilever beam and twisted cantilever beam with V-notch cracks will be made using the measured change in natural frequencies.

2 PROBLEM FORMULATIONS

Determination of modal parameters (natural frequency & mode shape) in a cantilever beam is an eigen value problem. ANSYS Software is used for theoretical modal analysis of the beam.

2.1 Governing equation

The governing equation for general eigen value problem is

$$[M]\{\ddot{X}\}+[K]\{X\}=0 \quad (1)$$

Disposing the brackets without ambiguity Eq. (1) is rewritten as follows:

$$M\ddot{X}+KX=0 \quad (2)$$

Pre-multiplying both sides of Eq. (2) by M^{-1}:

$$M^{-1}M\ddot{X}+M^{-1}KX=M^{-1}0$$

Now, $M^{-1}M\ddot{X}=I\ddot{X}$ and $M^{-1}KX=AX$ (say)

$$I\ddot{X}+AX=0 \quad (3)$$

where, $A=M^{-1}K$ = system matrix.

Assuming harmonic motion $\ddot{X}=-\lambda X$; where, $\lambda=\omega^2$ Eq. (3) becomes

$$\left[A-\lambda I\right]X=0 \quad (4)$$

The characteristic of motion is then:

$$\Delta=\left|A-\lambda I\right|=0 \quad (5)$$

The n-roots λ_i , where $i = 1,2,3,.....n$ of the characteristic Eq. (5) are called eigen values. The natural frequencies are found as:

$$\omega_i = \sqrt{\lambda_i}, i = 1,2,3,.....n \tag{6}$$

Substitution of the eigen values ($\lambda_i s$) in Eq. (4) gives the mode shapes X_i corresponding to λ_i . These are eigen vectors.

2.2 Geometry

Cantilever beam with a *V*-notch crack and its zoomed finite element model at notch position has been shown in Figs. 1 and 2 respectively. To find out eigen values and eigen vectors finite element method (FEM) has been used. The analysis has been carried out by ANSYS Software. This code is a general purpose Software on finite element analysis. It contains a library of different types of elements and different types of analysis. To solve the present problem, 4-node shell element (shell 63) has been used.

Fig.1
Cantilever beam with *V*-notch crack.

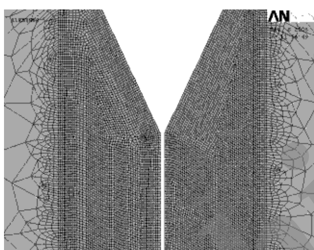

Fig.2
The discretised model of cantilever beam with V-notch and pre-crack.

2.3 Boundary conditions

For shell 63 (4 node shell element) every node is having 6 degrees of freedom, 3 translations Ux, Uy, Uz & 3 rotations Rotx, Roty, Rotz. Now, all degrees of freedom of the nodes, which are at fixed support, are restrained to simulate the condition of cantilever beam.

3 RESULTS AND DISCUSSION

In this analysis, it is assumed that crack is of *V*-notch shape. The depth (a) and locations (c) of these notches are normalized to the height and length of the cantilever beam respectively.

3.1 Experimental results

Experiments are conducted with the mild steel cantilever beam. Twelve specimens are made for the test, where three samples of each four categories are made by varying the crack depth. The crack depths are maintained as 5*mm*, 10*mm*, 15*mm* and no crack.

The length, width and thickness of the beam are 240*mm*, 20*mm* and 20*mm* respectively. All samples are machined and made grinding finish. Non-destructive test of the specimens are conducted to ensure that no surface or subsurface defect is present before pre-cracking of the sample. Pre-cracking of the beam specimens is made by fatigue cracking by Electro Magnetically controlled high frequency fatigue machine (AMSLER-HFP 5100) using three point bending method. The machine can generate high frequency up to 300 *Hz* by applying the loading on the three points on the specimen where the total span length (*S*) between the extreme two bending point is maintained by the relation $S = 4W$, where *W* is the depth of the specimen. The machine is automatically controlled and set to the desired crack depth.

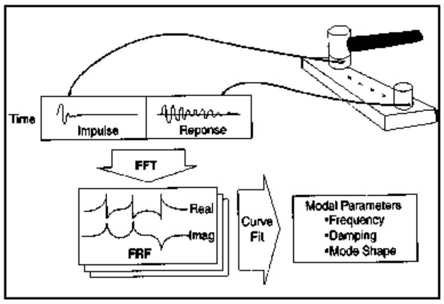

Fig.3
Natural frequency setup.

To assess the natural frequency of the cantilever beam, the fixture is used to hold the specimen. The holding device is a square block of heavy mass consisting of a T-type top plate and a channel groove in the bottom part. The sample is placed inside the groove and tightened with the top plate by bolting arrangement. The specimen is held rigid by tightly pressing the plate. Inserted length and the free length of the test specimen have been varied by properly placing the sample inside the groove to see the effect of crack location from the fixed end as shown in Fig. 3. Experiments are conducted with three different free lengths (fl) 200*mm* and corresponding inserted lengths (il) 40*mm* for all the crack depths to study the effect of crack location on the natural frequency. Excitation is provided by the impact hammer. A charge amplifier has been used for amplifying the force response. A miniature accelerometer is used to get the response of the vibrating specimen. To ensure the repeatability of the result the position of the accelerometer is varied over the free length. A FFT analyzer of OROS-25 is used to analyze the vibration signature. Some of the experimental results and their corresponding FRF diagram are given in Tables 1. to 2 and Fig. 4 respectively.

Table 1
Correlation of FE analysis with experimental data of 1st mode for beam specimen (fl = 200*mm*, il = 40*mm*, c = 80*mm*).

Depth of crack (*mm*)	Experimental				ANSYS	Percentage Error
	Set 1	Set 2	Set 3	Average		
0.0	410	418	417	415.00	440	5.7
5.0	404	394	391	396.33	430	7.9
10.0	350	345	362	352.33	400	12
15.0	250	264	245	253.00	298	15

Table 2
Correlation of FE analysis with experimental data of 2nd mode for beam specimen (fl = 200*mm*, il = 40*mm*, c = 80*mm*).

Depth of crack (*mm*)	Experimental				ANSYS	Percentage Error
	Set 1	Set 2	Set 3	Average		
0.0	2375	2382	2385	2380.67	2602	8.5
5.0	2142	2156	2152	2150.00	2427	11.4
10.0	1912	1930	1922	1921.33	2234	14.0
15.0	1670	1650	1678	1666.00	2000	17.0

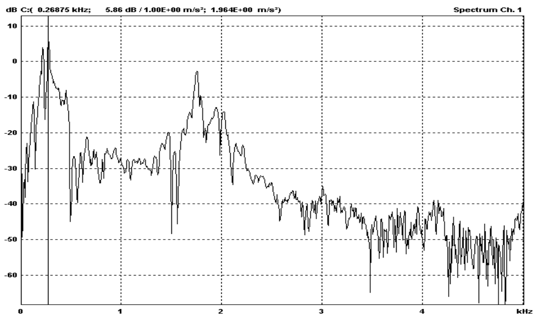

Fig.4
FRF diagram of thick beam (a = 15*mm*, c = 80*mm*, il = 40*mm*, fl = 200*mm*).

3.2 ANSYS results

The first two mode shapes for the beam were calculated using ANSYS software and were shown below for different crack depths and crack location ratios. Studies have been carried out for cantilever beam having length (l) = 200*mm*, width (w) = 20*mm* and thickness (h) = 20*mm*, parametric studies have been carried out. Crack depth (a) has been varied from 2.5*mm* to 15*mm* in steps of 2.5*mm*. Crack location (c) has been varied from 20*mm* to 100*mm* in steps of 20*mm*.

In case of cantilever beam it is observed from Tables 1. and 2 that for uncracked beam the error vary from 5.7% for 1st mode and 8.5% for 2nd mode. For the beam with 5*mm* crack the corresponding values are 7.9 and 11.4. For 10*mm* crack the corresponding values are 12 and 14. For 15*mm* crack the corresponding values are 15 and 17. So, it can be inferred that as the crack depth is increasing, the error is also increasing. The probable reason is that as crack grows deeper the body acts as two different bodies touching at some point and the stress intensity factor becomes considerably high, which was not considered while evaluating the natural frequencies by ANSYS. When the location of crack from the fixed end is 80*mm* i.e., the errors are 5.7 to 15% (Table 1) for 1st mode and 8.5 to 17% (Table 2) for 2nd mode.

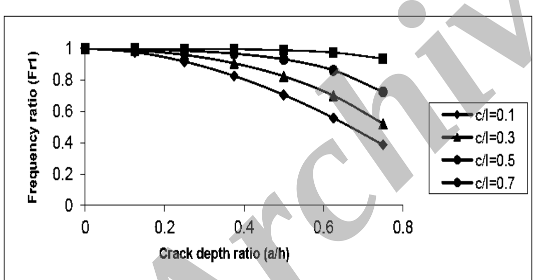

Fig.5
Variation of frequency ratio (1st mode) with crack depth ratio for different crack location ratios.

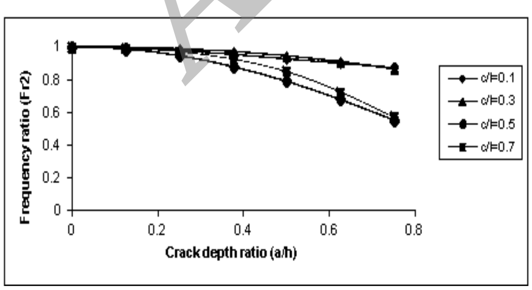

Fig.6
Variation of frequency ratio (2nd mode) with crack depth ratio for different crack location ratios.

Variation of natural frequency with crack depth has been shown in Figs. 5 and 6. It is also found that closer is the crack from the fixed end, more is the effect on the fundamental frequency one as seen in Fig. 5. From Fig. 6 for 2nd mode the effect is not significant when the crack is closer to the fixed end (c/l = 0.1 & c/l = 0.3). It is also seen that effect of crack is more at peak/through position and reduces as crack is closer to the node (the position of node is c/l = 0.783 for 2nd mode). The minimum value of normalized frequency for the 1st mode is 0.4 and that for 2nd mode is 0.6. This means natural frequencies of 1st and 2nd modes are decreased by 60% and 40% for a/h=0.75 with that of uncracked beam. So it can be inferred that effect of crack is maximum for 1st mode.

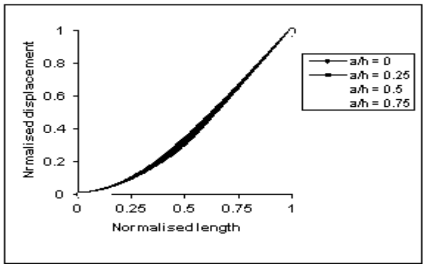

Fig.7
Mode shape (1st mode) of uncracked and cracked cantilever beam for c/l=0.5.

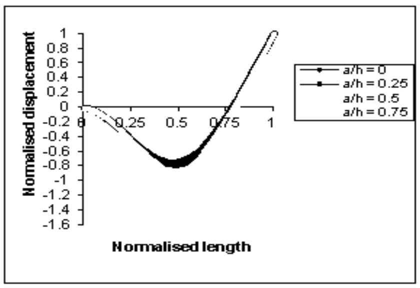

Fig.8
Mode shape (2nd mode) of uncracked and cracked cantilever beam for c/l=0.5.

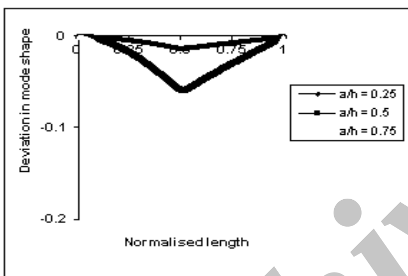

Fig.9
Deviation in mode shape (1st mode) of cracked beam w.r.t uncracked beam for c/l=0.5.

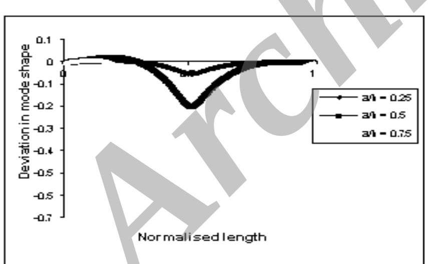

Fig.10
Deviation in mode shape (2nd mode) of cracked beam w.r.t uncracked beam for c/l=0.5.

Also mode shapes are studied for crack location (c/l=0.5). Mode shapes are shown in Fig. 7 for 1st mode and Fig. 8 for 2nd mode. The deviation in mode shape is also plotted against normalised length. It is observed in Figs. 9 and 10 that deviation in mode shape is 0.2 for 1st mode and 0.6 for 2nd mode for c/l=0.5 for thick beam when compared to that of 0.1 for 1st mode and 0.35 for 2nd mode of thin beam for the same crack depth and location ratios.

3.3 Twisted beam results

Finite element modeling of the cracked twisted cantilever beams are shown in Fig. 11. Cantilever beams are twisted for the angles 0^o, 15^o, 30^o, 45^o and 60^o and natural frequencies have been evaluated for different crack locations and depths. Some of the comparisons of natural frequencies of cantilever beam with different angles of twist are shown in Tables 3. to 6. Variation of natural frequency with crack depth has been shown in Figs. 12, 13, 16 and 17. It is also found that closer is the crack from the fixed end, more is the effect on the fundamental frequency one as seen in Figs. 12 and 13 for twisted beams with angle of twist 15^o and 60^o. From Figs. 16 and 17 for 2nd mode the effect is not significant when the crack is closer to the fixed end (c/l = 0.1 & c/l = 0.3). It is also seen that effect of crack is more at peak/through position and reduces as crack is closer to the node (the position of node is c/l = 0.783 for 2nd mode). The minimum value of normalized frequency for the 1st mode is 0.4 and that for 2nd mode is 0.6. This means natural frequencies of 1st and 2nd modes are decreased by 60% and 40% for a/h=0.75 with that of uncracked beam. So it can be inferred that effect of crack is maximum for 1st mode.

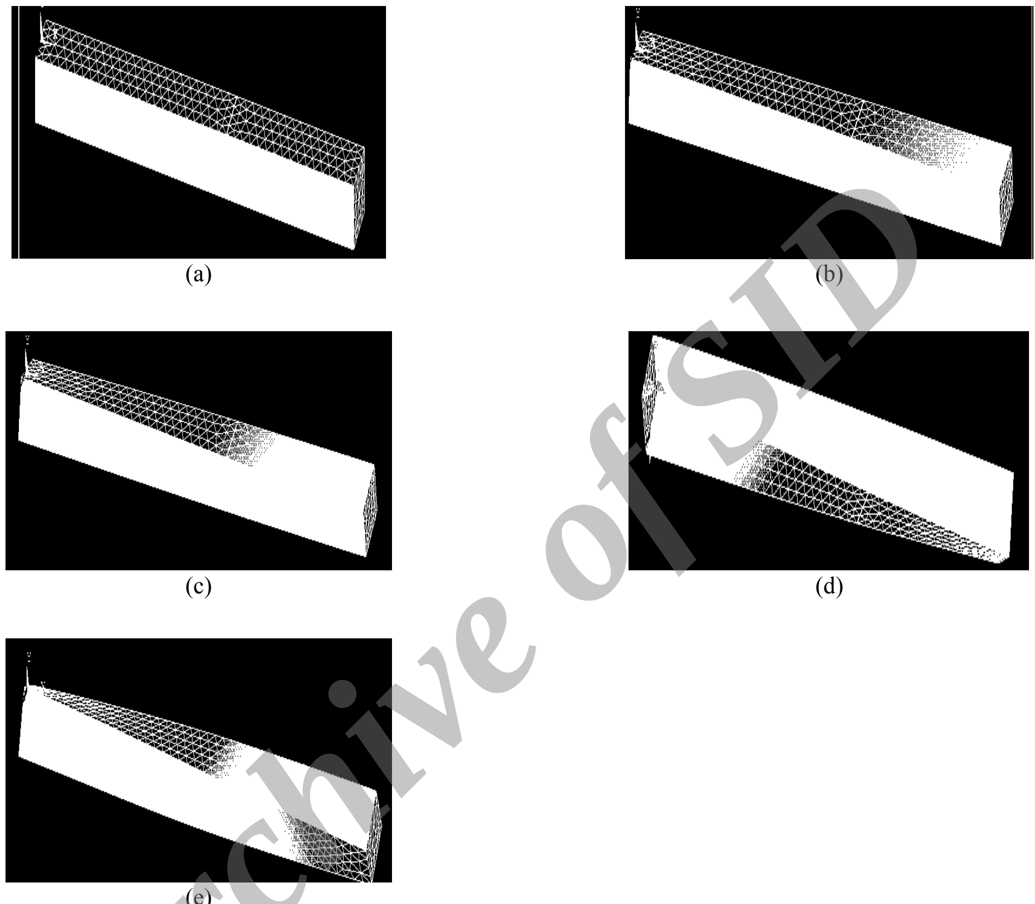

Fig.11
Finite element model of twisted cantilever beam with *V*-notch (a) 0^o (b) 15^o (c) 30^o (d) 45^o (e) 60^o.

Table 3
Comparison of natural frequencies of cantilever beam with different angles of twist (for uncracked beam).

Angle of Twist	Frequency 1	Frequency 2	Frequency 3
0^o	440.00	2602.0	6439.2
15^o	437.53	2543.1	6434.0
30^o	437.72	2542.0	6433.6
45^o	437.26	2541.9	6430.4
60^o	437.10	2540.7	6427.4

Table 4
Comparison of natural frequencies of cantilever beam with different angles of twist (c=80*mm* and a=5*mm*).

Angle of Twist	Frequency 1	Frequency 2	Frequency 3
0^o	427.63	2534.5	6763.6
15^o	417.48	2478.4	6635.9
30^o	417.52	2476.1	6635.9
45^o	415.94	2465.2	6608.0
60^o	418.52	2483.1	6639.2

Table 5
Comparison of natural frequencies of cantilever beam with different angles of twist (c=80*mm* and a=10*mm*).

Angle of Twist	Frequency 1	Frequency 2	Frequency 3
0^o	397.77	2315.2	6693.5
15^o	386.67	2260.0	6588.6
30^o	387.10	2262.8	6580.2
45^o	388.77	2270.4	6586.3
60^o	387.21	2256.8	6575.7

Table 6
Comparison of natural frequencies of cantilever beam with different angles of twist (c=80*mm* and a=15*mm*).

Angle of Twist	Frequency 1	Frequency 2	Frequency 3
0^o	298.00	1933.8	6536.3
15^o	303.05	1878.0	6460.0
30^o	303.45	1887.5	6445.1
45^o	305.07	1895.3	6445.6
60^o	305.93	1887.3	6442.5

While comparing cantilever beam results with twisted cantilever beam results, it is found that the deviation is much less as we see from Figs. 12, 13, 16 and 17. The natural frequencies of 1st and 2nd modes are decreased nearly by 60% and 40% for a/h=0.75 with that of uncracked beam for cantilever beam with different angle of twists.

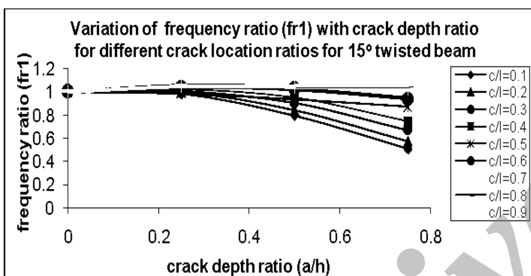

Fig.12
Variation of frequency ratio (1st mode) with crack depth ratio for different crack location ratios for 15^o twisted beam.

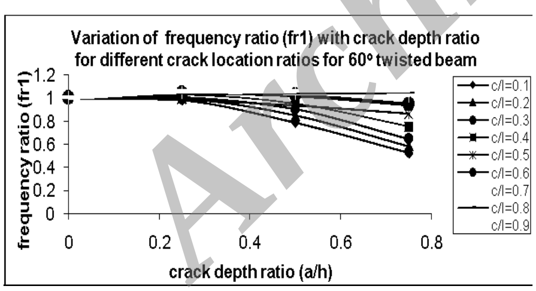

Fig.13
Variation of frequency ratio (1st mode) with crack depth ratio for different crack location ratios for 60^o twisted beam.

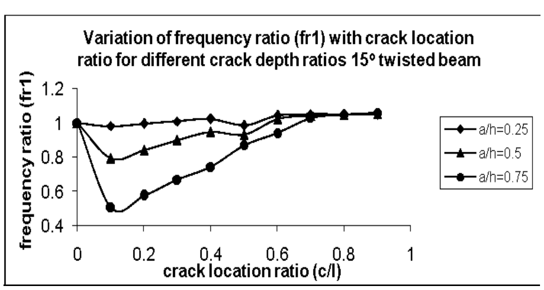

Fig.14
Variation of frequency ratio (1st mode) with crack location ratio for different crack depth ratios for 15^o twisted beam.

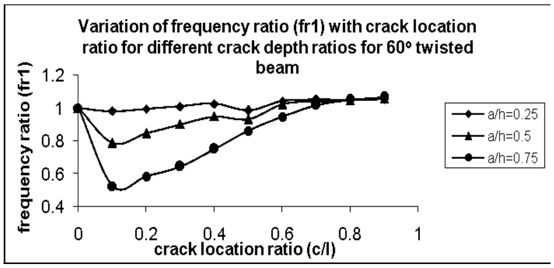

Fig.15
Variation of frequency ratio (1st mode) with crack location ratio for different crack depth ratios for 60^o twisted beam.

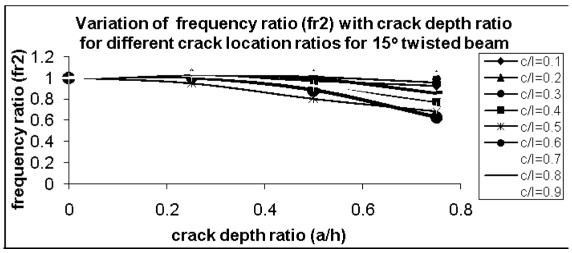

Fig.16
Variation of frequency ratio (2nd mode) with crack depth ratio for different crack location ratios for 15^o twisted beam.

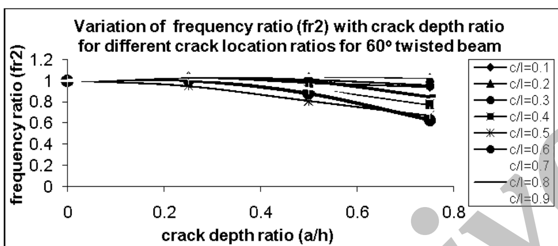

Fig.17
Variation of frequency ratio (2nd mode) with crack depth ratio for different crack location ratios for 60^o twisted beam.

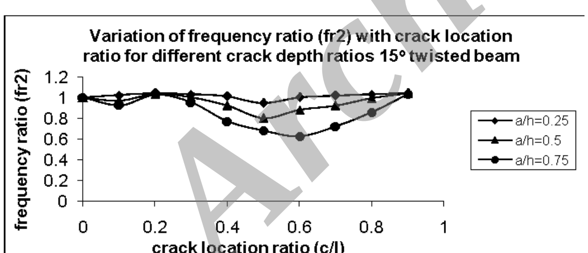

Fig.18
Variation of frequency ratio (2nd mode) with crack location ratio for different crack depth ratios for 15^o twisted beam.

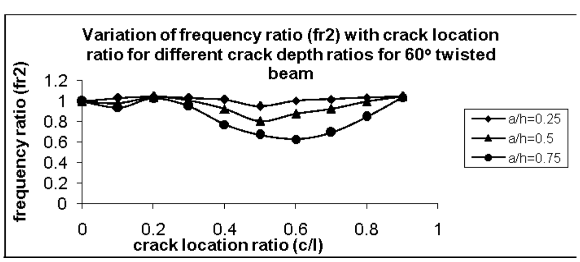

Fig.19
Variation of frequency ratio (2nd mode) with crack location ratio for different crack depth ratios for 60^o twisted beam.

4 CONCLUSIONS

A comparison for vibration analysis of cantilever beam and twisted cantilever beam with crack has been attempted in this paper. Steam turbines have number of stages and employs number of blades ranging from few centimeters in length in first stage to almost one meter long blades in the last stage. Here cantilever beam considered of l/h ratio equal to10. Assuming turbine blade as a cantilever beam parametric studies have been carried out using ANSYS Software to evaluate modal parameters (natural frequencies and mode shapes) for different crack parameters. From the analysis it has been observed that both crack location and crack depth have noticeable effects on the modal parameters of the cracked cantilever beam. Also the natural frequencies cantilever beams are high compared to twisted beams. It is also observed, for the same crack parameters the eigen value changes are same for all angle of twists. It means that the crack in a twisted beam changes the dynamic behavior similar to that of cantilever beam. So, turbine blade can be modeled as a cantilever beam with the required angle of twist for its analysis using vibration response.

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