Using Fuzzy Intervals Ranking to Rank the Decision Making Units in Date Envelopment Analysis

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Abstract

Ranking Decision Making Units (DMUs) is one of the most important subjects in industrial, economic, education and so on. There are several methods for ranking DMUs. This paper, by technical efficiency in combination with other sources of available performance information e.g expert opinions.

KEYWORDS: Date Envelopment Analysis. Technical Efficiency, Ranking, Fuzzy Number and interval. Membership Function.

1. Introduction

In 1978 Charnes, Cooper and Rhodes (CCR) described a mathematical programming formulation for the empirical evaluation of a Decision Making Unit (DMU) on the basis of the observed quantities of inputs and outputs for a group of similar DMUs. They termed this approach Date Envelopment Analysis (DEA). In 1984, Banker, Charnes and cooper (BCC) extended this technique.

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DEA does not require a priori a priori weights on inputs and outputs and is value- free which is strength and a weakness as well. This strength is sufficient to delineate the multiple –output analysis, without any need for a parametric specification. However, this value-freest is weakness, because no expert opinions are Introduced into the measurement problem. Provisions for introducing expert opinions in measurement are done by controlling factor weights in DEA. In Charnes et al.(1989), Jahanshahloo et al.(1997), Roll et al. (1991,1993) and Thompson et al. (1986,1990) some frameworks for locating appropriate bounds is suggested.

There are several methods for ranking the efficient units in DEA. The first method was developed in Andersen et al. (AP model) (1993).The main difficulty about this method is that the method compares the efficient DMUs with the inefficient one's. The other difficulties about AP model are discussed in detail in et al.(1999),Thompson et al.(1993) and Zerafat el al.(2000). In Mehrabian et al. (MAJ model)(1999), a different ranking method was developed. However, ranking by AP and JAM models break down in case of units with at least one zero input, and these methods do not introduce performance information's in measurement. A different method is suggested by Saati et al (SZMJ) (2001).This method which is a simple but an important modification of MAJ method, ranks DMUs in both input and output orientation, simultaneously.

Another approach is suggested in Hougaard (1999). This approach is based on ranking of fuzzy intervals. Determining the components of presented utility function in Hougaard, are some of difficulties about his method.

2. Preliminary Definitions

Since terms link *fuzzy sets*, membership functions and *fuzzy* intervals from *fuzzy* set theory will be used several times in the sequel, we shall consider a few necessary definitions.

Definition 1. If X is a collection of objects denoted generically by χ , then a fuzzy set $\tilde{A}$ in x is a set of ordered pairs :

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) \mid x \in X\}$$

$\mu_{\tilde{A}}(x)$ Is called the membership function which associates with each $x \in X$ a number in [0, 1] indicating to what degree x is a member of $\tilde{A}$.

In this paper, we shall make extensive use of a particular kind of fuzzy subset of **R** called a fuzzy interval. Fuzzy intervals can be seen as generalization of usual (crisp) intervals. To be more precise a fuzzy interval is defined as follows.

Definition 2. A real fuzzy interval, is a fuzzy subset of R, such that its membership function $\mu_{\tilde{A}}(x)$ is :

1. a continuous mapping from R to the closed interval $[0,\omega], 0 < \omega \le 1$,
2. constant on $(-\infty,\alpha]; \mu_{\tilde{A}}(x) = 0$ for $-\infty < x \le \alpha$,
3. strictly increasing on $[\alpha,\gamma]$,
4. constant on $[\gamma,\delta]; \mu_{\tilde{A}}(x) = 1$ for $\gamma \le x \le \delta$,
5. strictly decreasing on $[\delta,\beta]$
6. constant on $[\beta,\infty); \mu_{\tilde{A}}(x) = 0$ for $\beta \le x \le \infty$,

where α,β,γ and δ are real number and $\alpha \le \gamma \le \delta \le \beta$.

In order to ranking fuzzy intervals methods are suggested. In these methods, a utility function is defined which represents the preference of intervals. One of these methods recently is presented in Hougaard (1999). There is some difficulty whit it, e.g. in determining the probability distribution and the parameter of utility function.

Another approach is proposed in Memariani and Dadkhah (2000) which is more efficient than Hougaard's method. They consider three case for two intervals $I_1 = [a_1, b_1]$ and $I_2 = [a_2, b_2]$ as follows:

1. $b_1 < a_2$. In this case I_2 is proffered to I_1 .
2. $a_1 < a_2 < b_1 < b_2$ In this case, I_2 is preferred to I_1 .

3. $a_1 < a_2$ And $b_2 < b_1$ In this case, the preference of intervals is determined by considering a feasible point as:

$$\lambda = \frac{b_1 a_2 - b_2 a_1}{b_1 - a_1 + a_2 - b_2}$$

And, hence, the preference of intervals is determined as:

$$R_1 = \frac{b_1 - \lambda}{b_1 - a_1} \qquad\qquad R_2 = \frac{\lambda - a_1}{b_1 - a_1}$$

Where R_1 and R_2 are preferences of I_1 and I_2, respectively.

If $R_1 > R_2$ then I_1 is preferred to I_2, and if there were no feasible point, then I_2 is preferred to I_1.

3. Ranking by an Alternative Method

The standard DEA methods assign an efficiency score less then one to inefficient DMUs. From which a ranking can be derived. However, efficient DMUs all have an efficient DMUs were proposed by Andersen and Peteren (1993), Mehrabian et al. (1999) and Saati et al. (2001) These models, like other standard DEA models, do not consider the expert opinions in evaluation. In this section, an alternative method based on CCR and BCC models and expert's opinion is suggested.

Suppose that, one wishes to rank n similar DUMs. Toward this end, he / she evaluates technical efficiency of each DMU by CCR and BCC models as follows:

BCC model

$$B_p = \max UY_p + u_o$$

s.t : $VX_p = 1$,

$$UY_j - VX_j + u_o \le 0, \qquad j = 1,\dots,n,$$
$$U \ge 0, \qquad V \ge 0,$$

CCR model

$$C_p = \max UY_p$$

s.t : $VX_p = 1$,

$$UY_j - VX_j + u_o \le 0, \qquad j = 1,\dots,n$$
$$U \ge 0, \qquad V \ge 0,$$

Let C_j and B_j be efficiency scores of DMU$_j$ which have been obtained by CCR and BCC models, respectively. It is clear that $C_j \le B_j$.A less formal performance judgements, on the other hand, including qualitative aspects, for example, in the form of various expert evaluations is given.

For each DMU, let $\tau_j = [C_j, B_j]$ be the interval of technical efficiency, and $\xi_j = [a_j, b_j]$ be the subjective efficiency interval as judged by the expert.

Now, based on the information represented by the intervals τ_j and ξ_j, construct the interval $[L_j, U_j]$, where:

$$L_j = \min\{C_j, a_j\} \qquad U_j = \max\{B_j, b_j\}$$

Therefore for each DUM, there was an efficiency interval. By ranking these intervals by introduced method in the previous section DMUs will be ranked.

4. A Numerical Example

As an example considers 15 DMUs as table 1, which each DMU consumes 2 inputs to produces 2 outputs.

DMU	I_1	I_2	O_1	O_2
S01	6.63	7.25	121	1.11
S02	6.63	7.75	294	2.04
S03	9.06	10.75	338	1.35
S04	17.56	9.50	503	2.08
S05	16.71	15.50	215	2.60
S06	9.29	12.50	337	2.84
S07	8.89	6.25	173	0.77
S08	5.89	5.50	134	2.40
S09	10.18	7.50	322	1.72
S10	8.07	8.25	281	2.19
S11	6.80	7.75	331	2.55
S12	17.98	11.50	167	1.59
S13	10.63	7.25	405	1.60
S14	6.00	6.25	166	1.38
S15	6.71	5.25	94	1.39

Table 1: Data for numerical example

Table 2 represents the results of proposed method for ranking DUMs , CCR and BCC models, expert opinions and efficiency intervals . In this table, DMUs are ranked in decreasing order.

Since the expert opinions are introduced in evaluation, the ranking is possible to be different by AP, MAJ or SZMJ ranking.

DMU	CCR	BCC	EXP. OP.	[Lj.Uj]	Rank
S13	1	1	[0.90,0.98]	[0.90,1.00]	1
S14	0.65	0.99	[0.70,0.80]	[0.65,0.99]	2
S15	0.64	1	[0.70,0.80]	[0.64,1.00]	3
S11	1	1	[0.70,0.90]	[0.70,1.00]	4
S10	0.80	0.85	[0.50,0.80]	[0.50,0.85]	5
S08	1	1	[0.80,0.95]	[0.80,1.00]	6
S06	0.78	1	[0.80,0.90]	[0.78,1.00]	7
S04	0.96	1	[0.70,0.80]	[0.70,1.00]	8
S02	0.91	0.99	[0.60,0.80]	[0.60,0.99]	9
S09	0.86	0.90	[0.40,0.60]	[0.40,0.90]	10
S07	0.52	0.92	[0.30,0.40]	[0.30,0.93]	11
S01	0.43	0.83	[0.80,0.90]	[0.43,0.90]	12
S03	0.76	0.79	[0.60,0.70]	[0.60,0.79]	13
S12	0.38	0.39	[0.30,0.40]	[0.30,0.41]	14
S05	0.37	0.43	[0.10,0.30]	[0.10,0.43]	15

Table 2: The intervals and final ranking

5. Conclusion

DEA standard models are linear programming procedure for a frontier analysis of inputs and outputs. There may be several reasons why DEA result are not dependable .Firstly, the obtained efficiency are sensitive to changes in sample size , input –output size , reference technology , etc. In fact, the result of DEA is an efficiency interval rather than a single efficiency score. Secondly, in this analysis, no formal performance judgments including qualitative aspects, for example, in the form of various export evaluations are considered. In this paper, for ranking DMUs , a procedure is proposed which not only considers the result of DEA standard models , but also asks for expert opinions . Then, bu combining these results, it makes an efficiency interval for each DMU. These intervals are ranked by a fuzzy number ranking method.

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