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DUAL-LAYER MULTIPLE TUNED MASS DAMPERS FOR VIBRATION CONTROL OF STRUCTURES

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The performance of dual-layer multiple tuned mass dampers (DL-MTMD) with uniformly distributed natural frequencies is investigated. The DL-MTMD consists of one large tuned mass damper (L-TMD) and an arbitrary number of small tuned mass dampers (S-TMD). The primary structure is represented as a single degree-of-freedom system which corresponds to a specific vibration mode to be controlled in a real structure. The response of the structure with DL-MTMD is studied under harmonic excitation acting at the primary main system. The performance criterion used for assessing the optimum parameters and effectiveness of the DL-MTMD is selected as the minimization of the maximum dynamic magnification factor (DMF) of the displacement response of main structure. Two dynamic models of the DL-MTMD are proposed in the present study. The Model-I consists of S-TMD having the same mass and damping ratio and uniform distribution of natural frequencies. The Model-II consists of S-TMD with same stiffness and equal damping ratio and uniform distribution of natural frequencies. The comparative performance of the two models indicated that the performance of Model-II in comparison with Model-I is superior with respect to reduction in the displacement DMF.

Keywords: Tuned mass damper, harmonic, dual-layer multiple tuned mass damper, dynamic magnification factor, comparative performance

1. Introduction

Since the 1970s the tuned mass dampers (TMD) have been extensively studied and applied to suppress the wind-induced vibration of building structures (Wirsching and Campbell 1974; Mc-

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Namara 1977; Luft 1979; Warburton 1982). The major efforts of these studies were devoted towards developing the design procedure and optimizing the TMD parameters for improved performance. In most of the real applications, only a single TMD is installed at the top floor of structure and it is tuned to the fundamental mode of vibration. The inherent limitations of a single TMD are the suppression of response to very narrow band of frequencies, sensitivity to the fluctuation in the tuning frequency with respect to the controlled frequency and the offset in the optimum damping of the TMD. The mistuning or off-optimum damping will reduce the effectiveness of the TMD significantly. Iwanami and Seto (1984) proposed the dual tuned mass dampers and made research on the optimum design of two TMD for harmonically forced oscillation of the structure. It was shown that the two TMD are more effective than a single TMD. However, the effectiveness was not significantly improved. Since then employing more than one TMD with different dynamic characteristics has been proposed to further improve the effectiveness and robustness of the TMD. The multiple tuned mass dampers (MTMD) with the distributed natural frequencies were proposed by Xu and Igusa (1992) was also studied by Yamaguchi and Harnpornchai (1993), Abe and Fujino (1994), Abe and Igusa (1995), Jangid (1995, 1999), Joshi and Jangid (1997), Li (2000), Bakre and Jangid (2004) and Han and Li (2006). The MTMD is shown to possess better effectiveness and higher robustness in mitigating the oscillations of structures in comparison with corresponding single TMD. Studies by Xu and Igusa (1992) and Igusa and Xu (1994) demonstrated that a series of lightly damped oscillators, whose frequencies are distributed over a small range around the natural frequency of SDOF system can be more effective and more robust than a single TMD with same ratio when the system is excited by a wideband random disturbance. It was found that the multiple oscillators are equivalent to a single "moderately damped" TMD. The concept of distributed TMD was further developed by Abe and Fujino (1994) and applied the perturbation technique to derive analytically the critical frequency band-width of the oscillators to make a SDOF system multiply tuned and to establish a robustness criterion for the frequency tuning corresponding to a given bandwidth. It was shown that the multiple oscillators are efficient when at least one of the oscillators is strongly coupled with the SDOF system. The oscillators become much less efficient when the band-width of MTMD is large, while they are less robust when the bandwidth is extremely small. Kareem and Kline (1995) conducted a comprehensive parametric study on the effect of the number of dampers, damping ratio of individual damper, mass distribution, and frequency bandwidth. It was concluded that the performance of a multiple oscillators system is nearly identical regardless of the mass distribution and frequency spacing of the oscillators. Abe and Igusa (1995) also investigated the effectiveness of multiple oscillators in reducing the response of structures of closely spaced natural frequencies. All the studies summarized above mainly focused on the vibration suppression of a single mode or closely spaced modes of a structural system under a wideband random input.

Recently, Li and Zhu (2006) proposed the dual layer tuned mass dampers (DL-TMD), consisting of one L-TMD and one S-TMD) to study effectiveness and robustness for undesirable vibration of structures subjected to the ground acceleration. The numerical results indicated that the DL-TMD can render better effectiveness and higher robustness by changing the drift frequency ratio (DFR) as compared with TMD. The robustness to the natural frequency tuning (NFT), measured by the frequency band width coefficient (FBWC), the DL-TMD is significantly better than the MTMD thus indicating that the DL-TMD is an advanced control device. To improve further response reduction, Li (2006) proposed DL-MTMD with for achieving better effectiveness and robustness for controlling the undesirable vibrations of structures under the ground acceleration. The numerical results indicate that the DL-MTMD can render better effectiveness and higher robustness to the change in the NFT in comparison with the MTMD with the distributed natural frequencies with equal total mass ratio.

It is to be noted that the most of research work on MTMD system have used the total number of the TMD units constituting the MTMD as an odd number, referred to as the odd number based MTMD, by targeting at the central natural frequency. The arbitrary integer based MTMD have also been proposed by Li and Zhang (2005) for the purpose of convenience in application of the MTMD by abandoning the central natural frequency hypothesis. Evidently, the idea of arbitrary integer, compared with odd number, should be more versatile in accommodating the requirements in practical situations. In view of this, the performance DL-MTMD for a main system structure subjected to external harmonic excitation is studied in this paper. The structure is represented as SDOF system corresponding to a specific mode of vibration to be controlled of a real structure. The criterion used for assessing the optimum parameters and effectiveness of the DL-MTMD is selected as the minimization of the maximum DMF of the displacement response of main structure.

2. Structural Model

The schematic structural arrangement for DL-MTMD is shown in the Figure 1. The main structure is modeled as SDOF system characterized by generalized stiffness, k_s, damping coefficient, c_s and mass, m_s. Each S-TMD in the DL-MTMD system with different dynamic characteristics is also modeled as a SDOF system. As a result, the total number of degrees-of-freedom of the structural system is $n + 2$ (where n denotes the number of S-TMD unit in the DL-MTMD). The natural frequencies of the S-TMD in the DL-MTMD are arranged in increasing order. The mass and stiffness of L-TMD in the DL-MTMD system is represented as m_b and k_b, respectively. The mass of the i^{th} S-TMD in the DL-MTMD is m_i and the stiffness is k_i.

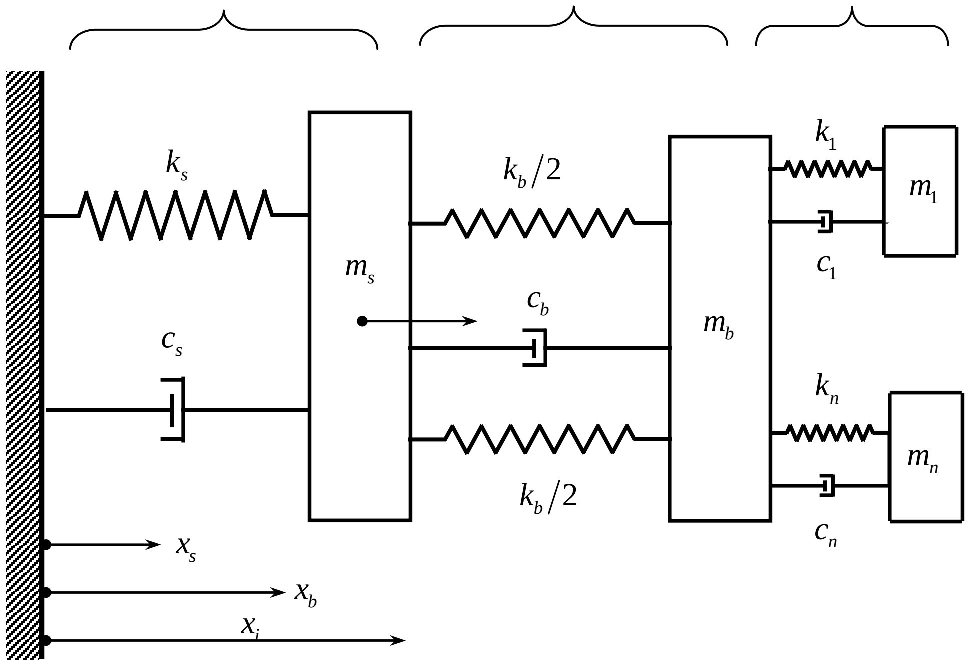

Figure 1. Schematic arrangements of structure with DL-MTMD

The frequency spacing factor or band-width of S-TMD units is:

$$\beta = (\omega_n - \omega_1) / \omega_s \tag{1}$$

The average natural frequency of S-TMD units is:

$$\omega_T = \sum_{i}^{n} \omega_i / n \tag{2}$$

The tuning frequency ratio of the S-TMD units, is the ratio of the average natural frequency of the S-TMD units to the controlled natural frequency of the structure, which is expressed as:

$$f_d = \omega_T / \omega_s \tag{3}$$

And the tuning frequency ratio of the L-TMD, namely the ratio of the natural frequency ($\omega_b = \sqrt{k_b / m_b}$) of the L-TMD to the controlled natural frequency of the structure is:

$$f_b = \omega_b / \omega_s \tag{4}$$

The average damping ratio of the S-TMD units is:

$$\xi_T = \sum_{i=1}^{n} \xi_i / n \tag{5}$$

The mass ratio of the S-TMD units to the L-TMD is:

$$\mu_H = \sum_{i=1}^{n} m_i / m_b \tag{6}$$

The total mass ratio of the DL-MTMD to the controlled structure is:

$$\mu = \frac{\sum_{i=1}^{n} m_i + m_b}{m_s} \tag{7}$$

$$\sum_{i=1}^{n} m_i = \frac{\mu\mu_H}{1+\mu_H} m_s \tag{8}$$

$$m_b = \frac{\mu}{1+\mu_H} m_s \tag{9}$$

With the installation of the MTMD, the frequency response curve of the structure can be flattened over an increasingly wide frequency range with the increase in the optimum frequency spacing by suppressing the secondary peaks induced by the MTMD either by a larger optimum damping ratio or a larger number of the TMD units involved.

The natural frequencies of the S-TMD in the DL-MTMD, are assumed to be distributed equidistantly as:

$$\omega_1, \omega_2, ..., \omega_n = \left[\left[f - \beta/2 \right] : \left[\beta / (n-1) \right] : \left[f + \beta/2 \right] \right] \omega_s \tag{10}$$

The notation $\left[f - \beta/2 \right] : \left[\beta / (n-1) \right] : \left[f + \beta/2 \right]$ represents an arithmetic progression with the first term being $\left[f - \beta/2 \right]$, an interval being $\left[\beta / (n-1) \right]$, and the last term being $\left[f + \beta/2 \right]$.

The natural frequency of the i^{th} S-TMD in the DL-MTMD can then be expressed as:

$$\omega_i = \left[f - \frac{\beta}{2} + \frac{i-1}{n-1}\beta \right] \omega_s \tag{11}$$

The equations of motion for the main structure with DL-MTMD subjected to a harmonic excitation are expressed by:

$$[M]\{\ddot{X}\} + [C]\{\dot{X}\} + [K]\{X\} = \{1\} f(t) \tag{12}$$

where $\{X\} = \{x_s, x_b, x_1, ..., x_n\}^T$ is the displacement vector of the structural system; x_s and x_b are the displacement of the main system and large block, respectively with respect to base; x_i is the displacement of the i^{th} TMD in S-TMD; $[M]$, $[C]$ and $[K]$ are the mass, damping and stiffness matrices for the structural system, respectively; {1}={1, 0, 0,….., 0}^{T} ; and f (t) is the external wind type force acting on the main system.

The $[M]$, $[C]$ and $[K]$ matrices are expressed as:

$$[M]=\begin{bmatrix} m_s & 0 & 0 & .. & 0 \\ 0 & m_b & 0 & .. & 0 \\ 0 & 0 & m_1 & .. & 0 \\ . & . & . & . & 0 \\ 0 & 0 & 0 & .. & m_n \end{bmatrix} \tag{13}$$

$$[C]=\begin{bmatrix} c_s+c_b & -c_b & 0 & 0 & ... & 0 \\ -c_b & c_b+\sum c_i & -c_1 & -c_2 & ... & -c_n \\ 0 & -c_1 & c_1 & 0 & ... & 0 \\ 0 & -c_2 & 0 & c_2 & ... & 0 \\ . & . & . & . & . & . \\ 0 & -c_n & 0 & 0 & ... & c_n \end{bmatrix} \tag{14}$$

$$[K]=\begin{bmatrix} k_s+k_b & -k_b & 0 & 0 & ... & 0 \\ -k_b & k_b+\sum k_i & -k_1 & -k_2 & ... & -k_n \\ 0 & -k_1 & k_1 & 0 & ... & 0 \\ 0 & -k_2 & 0 & k_2 & ... & 0 \\ . & . & . & . & . & . \\ 0 & -k_n & 0 & 0 & ... & k_n \end{bmatrix} \tag{15}$$

For the present study, the external excitation force acting on the main system is modeled by harmonic force expressed as $f(t)=f_0e^{i\omega t}$; where f_0 is the amplitude of excitation; ω is the circular frequency; t denote the time; and $i=\sqrt{-1}$. The corresponding steady-state harmonic response of the system to the harmonic excitation will be:

$$\{X\}=X(\omega)e^{i\omega t} \tag{16}$$

The $X(\omega)$ indicates the amplitude vector of the steady-state response of the combined system which is expressed by:

$$X(\omega)=\left(-\omega^2[M]+i\omega[C]+[K]\right)^{-1}\{1\}\, f_0 \tag{17}$$

The first term of amplitude vector represents displacement of main system while the second term represents displacement of large block of DL-MTMD. Subsequent terms represent displacement of ith damper unit in the S-TMD.

Two models are proposed by varying mass, m_i, spring stiffness, k_i, and damping coefficient, c_i, of each ith S-TMD in the DL-MTMD system. The Model-I consists of S-TMD with its entire unit with same mass and equal damping ratio, ξ_i along with uniform distribution of natural frequencies. The Model-II consists of S-TMD with its entire unit with same stiffness and equal damping ratio, ξ_i along with uniform distribution of natural frequencies. The above stated system parameters for the S-TMD basis are reported in the Table 1.

Table 1. Determination of system parameters for the DL-MTMD

S-TMD system parameters	Model -I	Model-II
Stiffness of damper	$k_1 \neq k_2 \neq .... \neq k_n$	$k_1 = k_2 = ... = k_n = k_T$
Damping coefficient	$c_1 \neq c_2 \neq .... \neq c_n$	$c_1 \neq c_2 \neq .... \neq c_n$
Mass of damper	$m_1 = m_2 = ... = m_n$	$m_1 \neq m_2 \neq .... \neq m_n$
Damping ratio	$\xi_1 = \xi_2 = ... = \xi_n = \xi_T$	$\xi_1 = \xi_2 = ... = \xi_n = \xi_T$

Determination of system parameters

$k_i = m_i \omega_i^2$	$k_i = k_T = \dfrac{\mu \mu_H}{\left[1+\mu_H\right]\sum_{i=1}^{n}\left(1/\omega_i^2\right)} m_s$
$c_i = 2m_i\omega_i\xi_i = 2m_T\omega_i\xi_T$	$c_i = 2m_i\omega_i\xi_i = 2m_i\omega_i\xi_T$
$m_i = m_T = \dfrac{\mu\mu_H}{n(1+\mu_H)} m_s$	$m_i = \dfrac{k_i}{\omega_i^2} = \dfrac{k_T}{\omega_i^2}$

The optimum parameters and effectiveness of the DL-MTMD will be assessed using the following criterion: First for a fixed value of frequency ratio ω/ω_s (set with in the range of 0.1 to 2.5) each parameter is varied keeping other parameters constant which yields minimum of maximum DMF. Finally, the smallest mini-maxes can be selected, together with the corresponding tuning frequency ratio, average damping ratio, frequency spacing factors and mass ratio as the optimum values. With the optimum parameters for the DL-MTMD obtained in terms of the optimum criterion the DL-MTMD strokes, including those for the L-TMD and S-TMD can be simultaneously evaluated.

The concept of arbitrary integer based MTMD is used instead of only odd number based MTMD. The number of S-TMD are varied from 1 to 7. The mass ratio for the study is chosen as 1 %, 3 % and 5 %. The main system damping is considered as 2 %. The optimum parameters namely mass ratio, μ_H, damper damping, ξ_d, band-width, β and tuning frequency for large block, f_b and for small dampers, f_d in DL-MTMD are chosen to satisfy the condition of minimizing maximum DMF and are reported in the Tables 2 to 4 for Model-I and in Tables 5 to 7 for the Model-II. The results in these tables indicate that that the DL-MTMD performs better than single TMD. The corresponding strokes of S-TMD are reported in the Tables 8 to 10. It is concluded form these tables that the stroke displacement of the S-TMD in the DL-MTMD system decreases with the increase of the number of dampers.

Table 2. Optimum parameters for mass ratio μ = 0.01 and main system damping ξ_s = 2 %

Model-I (all small dampers with same mass)

Number of additional	μ_H	f_b	f_d	β	ξ_b	ξ_d	R	R_b
1	0.01865	0.99995	0.9899	---	0.0	0.102	7.6092	23.07
2	0.0145	0.99995	0.989	0.0751	0.0	0.089	7.5197	24.05
3	0.0140	0.99993	0.9895	0.0900	0.0	0.0755	7.4375	23.63
4	0.013	0.99989	0.9885	0.105	0.0	0.0601	7.3468	23.23
5	0.0125	0.99989	0.9880	0.115	0.0	0.0550	7.3443	23.18
6	0.012	0.99989	0.9880	0.121	0.0	0.0451	7.2513	22.80
7	0.012	0.99989	0.9878	0.125	0.0	0.0400	7.2233	22.40

Table 3. Optimum parameters for mass ratio μ = 0.03 and main system damping ξ_s = 2 %

Model-I (all small dampers with same mass)

Number of additional	μ_H	f_b	f_d	β	ξ_b	ξ_d	R	R_b
1	0.03865	0.99995	0.9689	---	0.0	0.125	5.5609	15.10
2	0.0375	1.00550	0.9685	0.0842	0.0	0.110	5.4635	13.20
3	0.0375	1.00550	0.9675	0.0971	0.0	0.099	5.3725	13.20
4	0.0375	1.00850	0.9665	0.1110	0.0	0.095	5.3720	13.10
5	0.0375	1.00900	0.9660	0.1200	0.0	0.090	5.3655	13.10
6	0.0375	1.00990	0.9660	0.1280	0.0	0.085	5.3615	13.13
7	0.0375	1.00990	0.9660	0.1350	0.0	0.080	5.3548	13.18

Table 4. Optimum parameters for mass ratio μ = 0.05 and main system damping ξ_s = 2 %

Model-I (all small dampers with same mass)

Number of additional small dampers, n	μ_H	f_b	f_d	β	ξ_b	ξ_d	R	R_b
1	0.05965	1.009	0.9509	---	0.0	0.145	5.0766	12.10
2	0.05870	1.0088	0.9505	0.105	0.0	0.130	4.8250	10.53
3	0.05860	1.0075	0.9501	0.115	0.0	0.125	4.8058	10.50
4	0.05860	1.0075	0.9500	0.130	0.0	0.118	4.7946	10.445
5	0.05860	1.0075	0.9500	0.145	0.0	0.107	4.7762	10.35
6	0.05860	1.0065	0.9500	0.154	0.0	0.101	4.7760	10.332
7	0.05860	1.0065	0.9500	0.165	0.0	0.098	4.7725	10.277

Table 5. Optimum parameters for mass ratio μ = 0.01 and main system damping ξ_s = 2 %

Model-II (all small dampers with same stiffness)

Number of additional small dampers, n	μ_H	f_b	f_d	β	ξ_b	ξ_d	R	R_b
1	0.01865	0.9985	0.99999	---	0.0	0.102	7.2933	23.655
2	0.01700	1.0027	0.9865	0.080	0.0	0.0895	7.1742	22.08
3	0.01700	1.0026	0.9865	0.0900	0.0	0.0804	7.0216	21.50
4	0.01700	1.0026	0.9865	0.0998	0.0	0.0745	6.9656	21.12
5	0.01700	1.0026	0.9855	0.108	0.0	0.0695	6.9267	20.993
6	0.01700	1.0026	0.9855	0.120	0.0	0.0645	6.8862	20.922
7	0.01700	1.0025	0.9855	0.135	0.0	0.0595	6.8831	20.865

Table 6. Optimum parameters for mass ratio μ = 0.03 and main system damping ξ_s = 2 %

Model-II (all small dampers with same stiffness)

Number of additional small dampers, n	μ_H	f_b	f_d	β	ξ_b	ξ_d	R	R_b
1	0.03765	0.9999	0.9705	---	0.0	0.125	5.4556	13.677
2	0.0375	1.0022	0.9765	0.101	0.0	0.113	5.355	13.112
3	0.0365	1.0022	0.9765	0.112	0.0	0.0999	5.2933	13.145
4	0.0365	1.0022	0.9765	0.121	0.0	0.0935	5.2838	13.1396
5	0.0365	1.0022	0.9765	0.130	0.0	0.0885	5.2786	13.057
6	0.0365	1.0022	0.9765	0.138	0.0	0.0815	5.2583	13.154
7	0.0365	1.0022	0.9765	0.145	0.0	0.0725	5.2312	13.093

Table 7. Optimum parameters for mass ratio μ = 0.05 and main system damping ξ_s = 2 %

Model-II (all small dampers with same stiffness)

Number of additional small dampers, n	μ_H	f_b	f_d	β	ξ_b	ξ_d	R	R_b
1	0.05965	0.99999	0.9509	---	0.0	0.146	4.5912	10.42
2	0.05945	1.0099	0.9509	0.105	0.0	0.125	4.4640	9.9275
3	0.05945	1.0099	0.9509	0.118	0.0	0.110	4.4586	9.7935
4	0.05945	1.0099	0.9504	0.127	0.0	0.102	4.4449	9.7100
5	0.05945	1.0095	0.9501	0.140	0.0	0.0975	4.4410	9.6383
6	0.05945	1.0095	0.9500	0.151	0.0	0.0945	4.4400	9.6266
7	0.05945	1.0095	0.9500	0.165	0.0	0.0905	4.4398	9.554

3. Evaluation of Optimum Parameters of DL-MTMD System

Tables 2 to 10 show the results of the study for two proposed models of the DL-MTMD system (i.e. Model-I with same mass of dampers in S-TMD and Model-II with same stiffness of dampers in S-TMD). For both the models, the damper damping is kept constant for all dampers. As stated and proved by Li (2006), it is confirmed in this study also that the optimum value of linking dashpot, cb, between the structure and L-TMD in the DL-MTMD is equals zero. This makes the fabrication of the DL-MTMD system easier as we do not require any damper device and stiffness device is good enough for the L-TMD.

Figure 2 shows the comparison of two models of DL-MTMD system along with the corresponding single TMD. It is seen that the performance of the system improves significantly by addition of S-TMD in the system. The DMF for single optimum TMD which is 9.465 reduces to 7.5197 by addition of two S-TMD to a large block for Model-I and 7.1742 for the Model-II with mass ratio as 1% remains constant. The improvement in DMF is of the order of 24% by using the DL-MTMD system. The performance of Model-II type DL-MTMD system is found to be better as compared to the corresponding Model-I for the same mass of dampers in the S-TMD.

Figures 3 and 4 show the dynamic response of DL-MTMD with n = 4 for both Model-I and Model-II and for mass ratios equal to 1 %, 3 % and 5 %. It is seen that the stroke of S-TMD in Model-I is more than that for Model-II. As the mass ratio increases, the stroke of S-TMDs in the DL-MTMD decreases for both types of systems.

The optimum parameters for Model-I and Model-II for the mass ratio equal to 1 %, 3 % and 5 % are plotted in the Figures 5 and 6. It is observed that as the mass ratio increases from 1 to 5 %, the optimum frequency band-width of S-TMD increases making the system more robust for both types of models. Also, as number of dampers in S-TMD increases the band-width also increases and thus, makes the system more robust. The optimum damping ratio decreases as number of S-TMD dampers increases and makes the system more fabrication friendly due requirements of smaller size of damping devices. However, for Model-II the optimum damper damping required is relatively more as seen from the Figure 7. It is to be noted that for MTMD system the optimum band-width required is more as compared to DL-MTMD. The reduction in the displacement DMF of main system is relatively more for Model –II in comparison with Model-I. Further, it is to be noted that for both models, reduction in DMF is not substantial as *n* approaches to four. The stroke of L-TMD reduces as the number of S-TMD increases for both the models. The optimum tuning frequency ratio, f_d for dampers decreases as the number of dampers increase and the off tuning with respect to large block required is more. The optimum tuning frequency ratio, f_b remains practically constant with respect to number of S-TMD. The optimum mass ratio, μ_H of the S-TMD to the L-TMD decreases as number of dampers increase from one to three, which implies that more mass is required for S-TMD as number of dampers increase.

Table 8. Stroke of S-TMD for mass ratio =1%, main system damping = 2 %

Model-I (all small dampers with same mass)	
Number of additional small dampers, *n*	Stroke of each small TMD arranged in increasing order of frequency from left to right
1	57.01
2	57.55, 53.358
3	61.244, 57.60, 57.20
4	67.975, 64.914, 60.0, 63.95
5	69.49, 69.05, 64.05, 63.117, 67.15
6	75.294, 75.306, 71.20, 68.63, 69.487, 73.56
7	78.438, 78.94, 75.86, 73.02, 72.25, 73.32, 77.258

Model-II (all small dampers with same stiffness)	
Number of additional small dampers, *n*	Stroke of each small TMD arranged in increasing order of frequency from left to right
1	49.94
2	52.629, 50.353
3	53.945, 52.243, 52.0216
4	54.89, 54.28, 52.66, 52.26
5	55.5788, 55.52, 54.55, 52.755, 52.866
6	56.395, 56.974, 56.539, 54.946, 53.54, 53.972
7	56.35, 58.565, 59.23, 57.705, 55.94,55.222, 55.223

Table 9. Stroke of S-TMD for mass ratio =3%, main system damping = 2%

Model-I (all small dampers with same mass)	
Number of additional small dampers, *n*	Stroke of each small TMD arranged in increasing order of frequency from left to right
1	32.20
2	28.40, 25.12
3	30.28, 27.50, 25.85
4	30.24, 28.65, 26.40, 25.80
5	30.95, 29.60, 27.72, 26.58, 26.37
6	31.65, 30.49, 28.75, 27.32, 27.20, 26.95
7	32.42, 31.32, 29.71, 28.05, 27.98, 27.89, 27.64

Model-II (all small dampers with same stiffness)	
Number of additional small dampers, *n*	Stroke of each small TMD arranged in increasing order of frequency from left to right
1	25.71
2	27.4, 23.98
3	29.288 , 26.295, 25.375
4	30.086, 27.78, 25.84, 25.318
5	30.64, 28.735, 26.52, 25.59, 25.52
6	31.88, 29.955, 27.85, 26.71, 26.16, 26.09
7	33.39, 31.224, 28.96, 28.07, 27.70, 27.07, 27.071

Table 10. Stroke of S-TMD for mass ratio = 5 %, main system damping = 2%

Model-I (all small dampers with same mass)	
Number of additional small dampers, *n*	Stroke of each small TMD arranged in increasing order of frequency from left to right
1	24.094
2	20.66, 17.98
3	21.17, 19.7, 18.09
4	21.668, 20.50, 19.058, 17.762
5	22.495, 21.38, 19.95, 18.62, 18.53
6	23.045, 22.033, 20.69, 19.39, 19.025, 19.011
7	23.32, 22.48, 21.26, 20.014, 19.225, 19.30, 19.254

Model-II (all small dampers with same stiffness)	
Number of additional small dampers, *n*	Stroke of each small TMD arranged in increasing order of frequency from left to right
1	24.094
2	19.016, 17.048
3	20.302, 18.20, 18.05
4	21.236, 19.466, 18.54, 18.244
5	21.271, 19.728, 18.375, 18.356, 17.90
6	21.53, 20.168, 18.80, 18.475, 18.30, 17.83
7	21.82, 20.56, 19.2, 18.68, 18.65, 18.35, 18.355

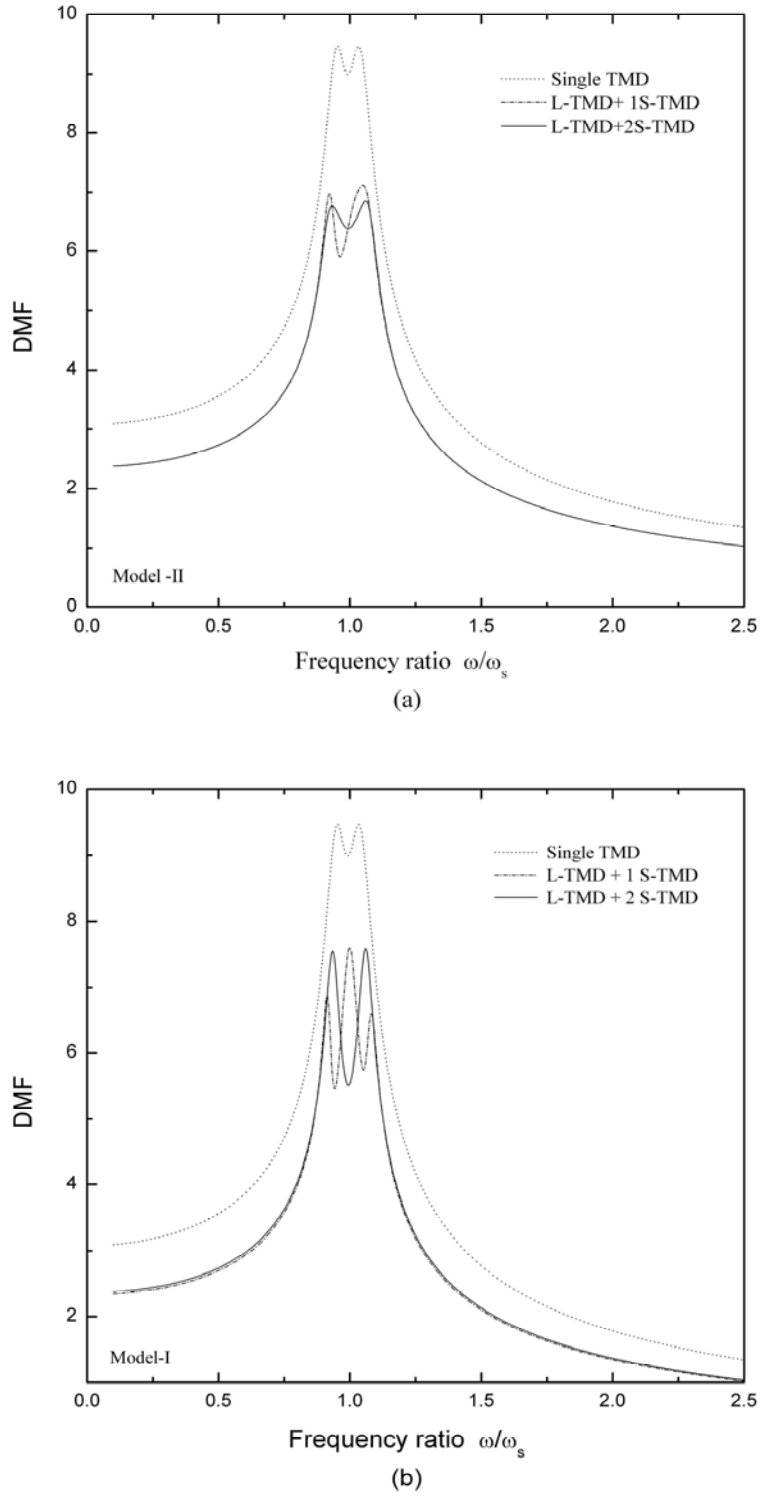

Figure 2. Comparison of dynamic response of Model-I and Model-II for mass ratio = 0.01 and main system damping = 2 %

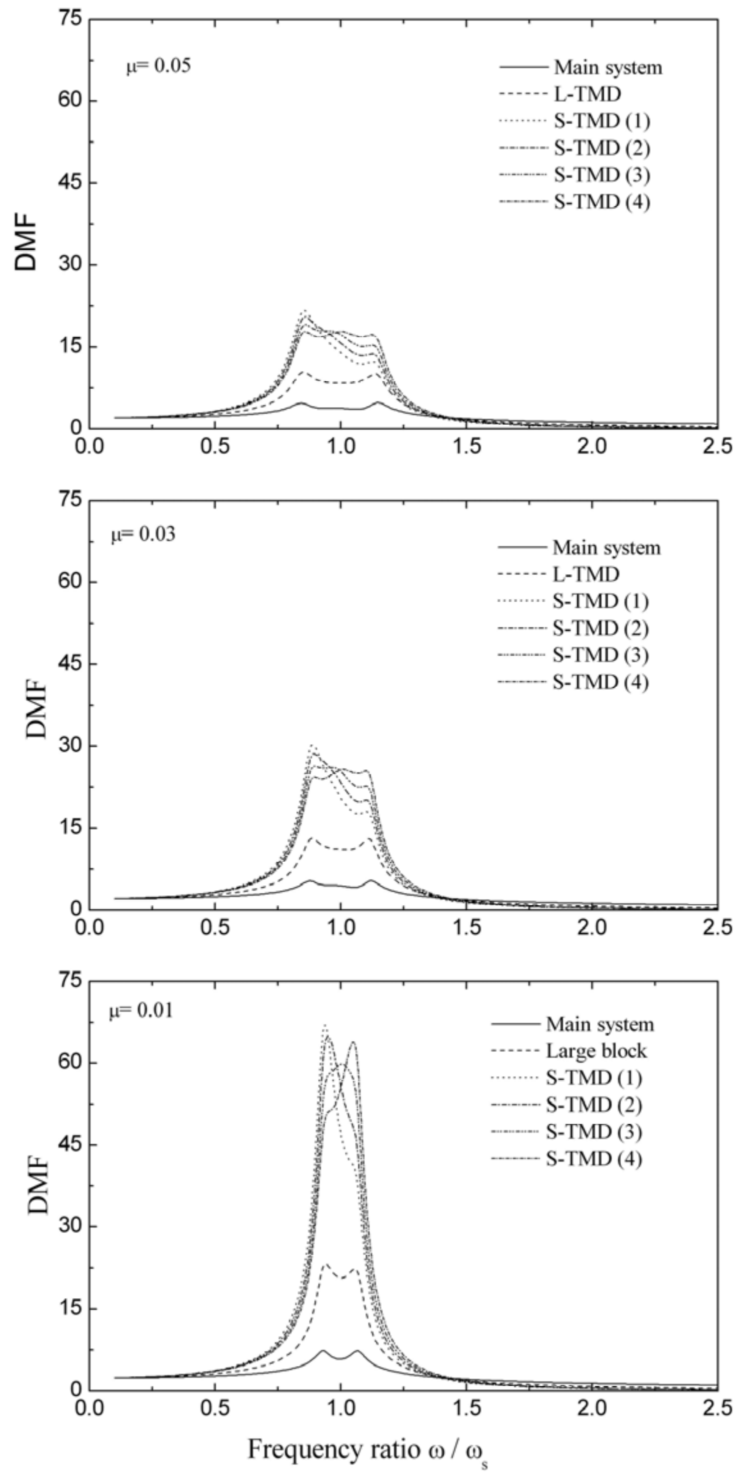

Figure 3. Dynamic response of DL-MTMD (n = 4) for Model-I for mass ratio 1, 3 and 5 %

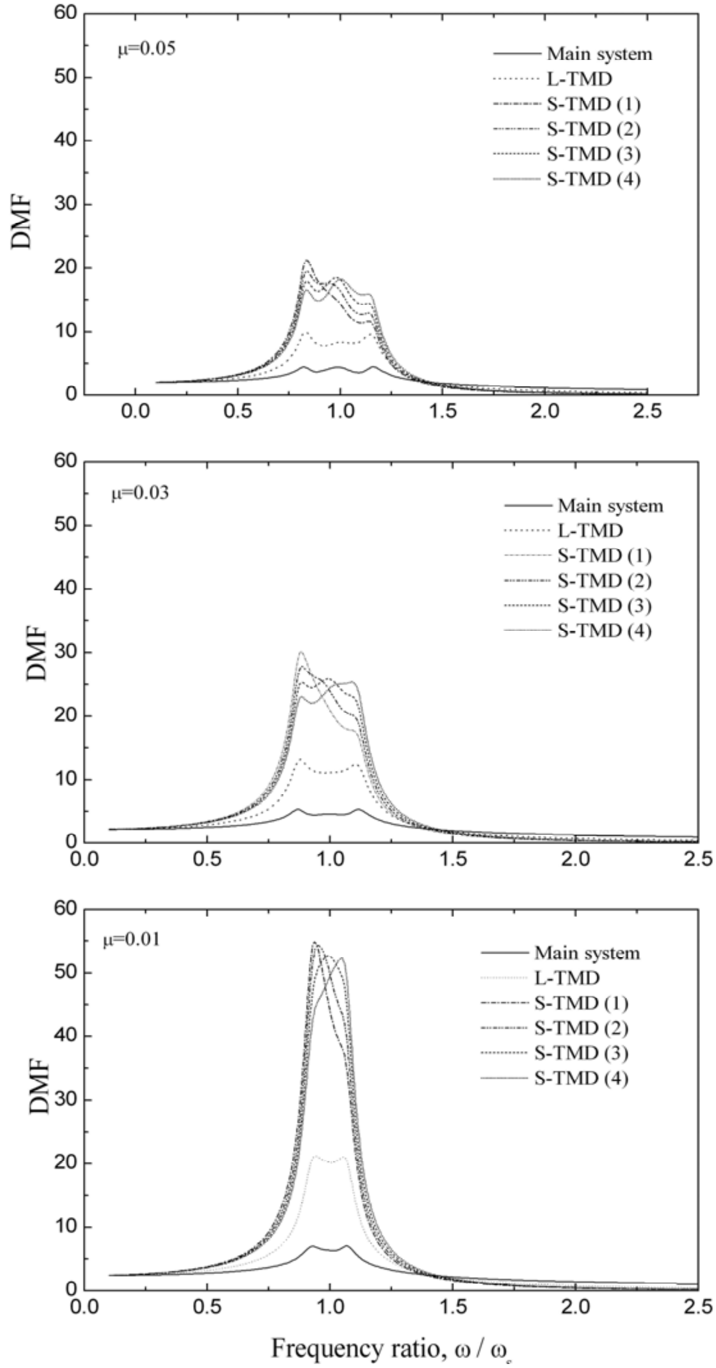

Figure 4. Dynamic response of DL-MTMD (n = 4) of Model-II for mass ratio= 1, 3 and 5 %

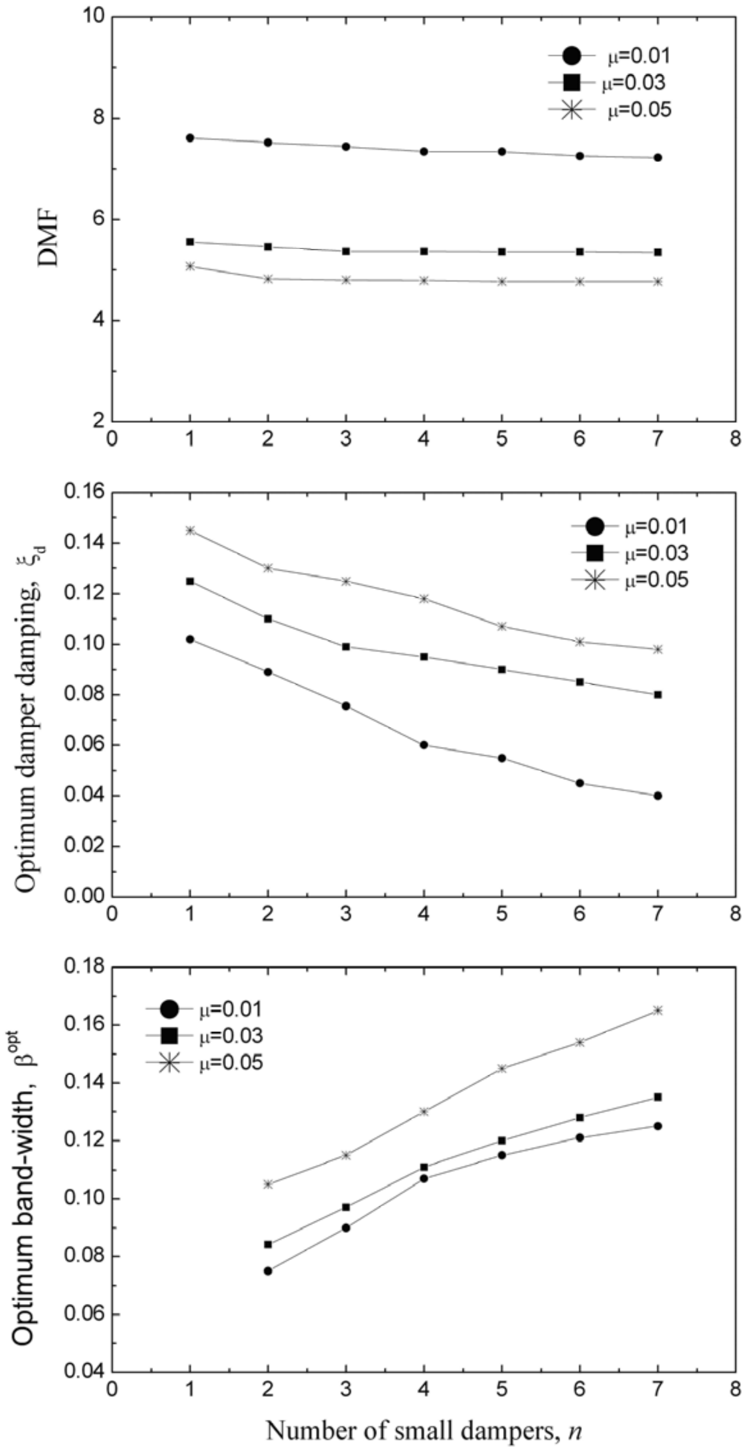

Figure 5. Comparison of optimum parameters β^{opt} and ξ_d and DMF of Model-I for different mass ratios

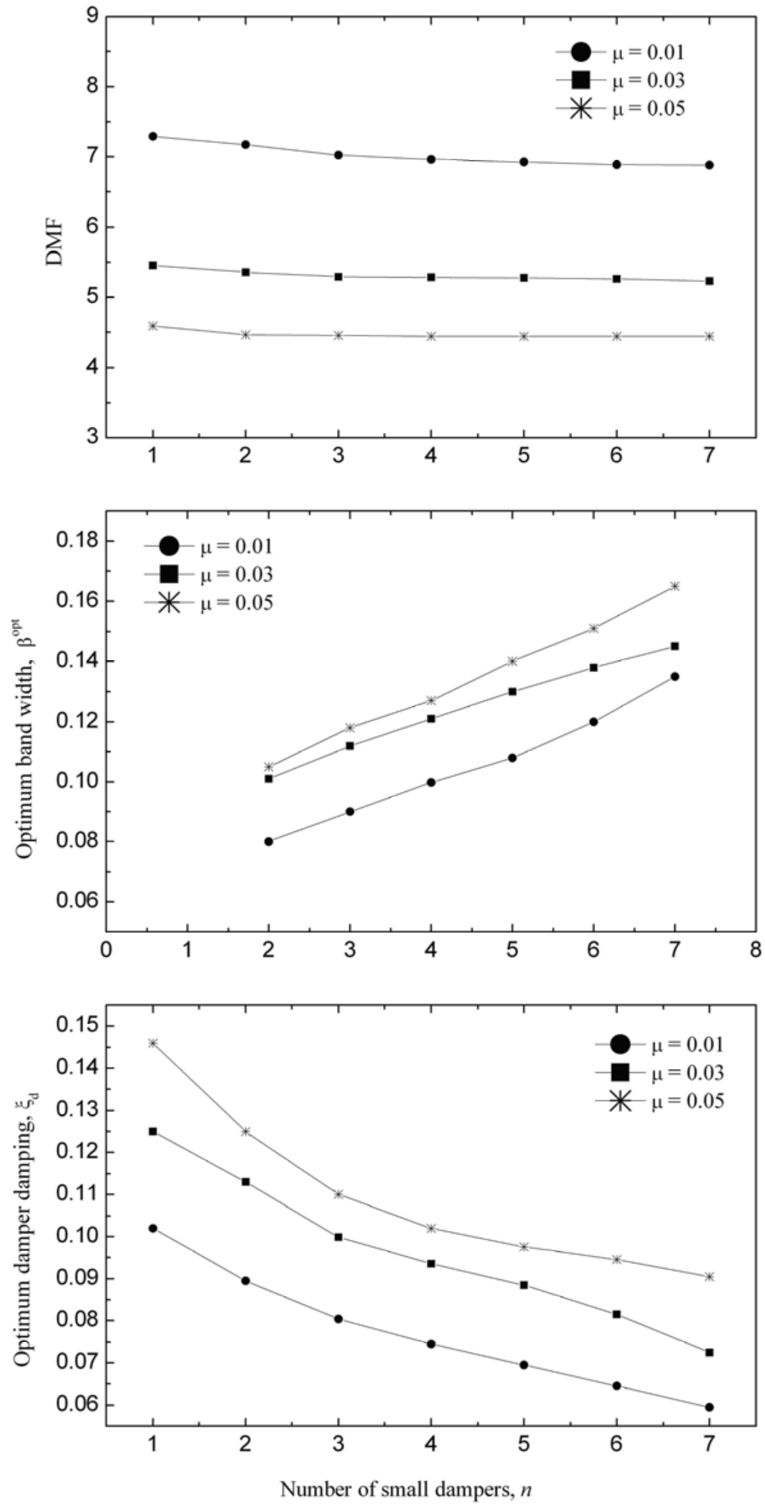

Figure 6. Comparison of optimum parameters β^{opt} and ξ_d and DMF of Model-II for different mass ratios

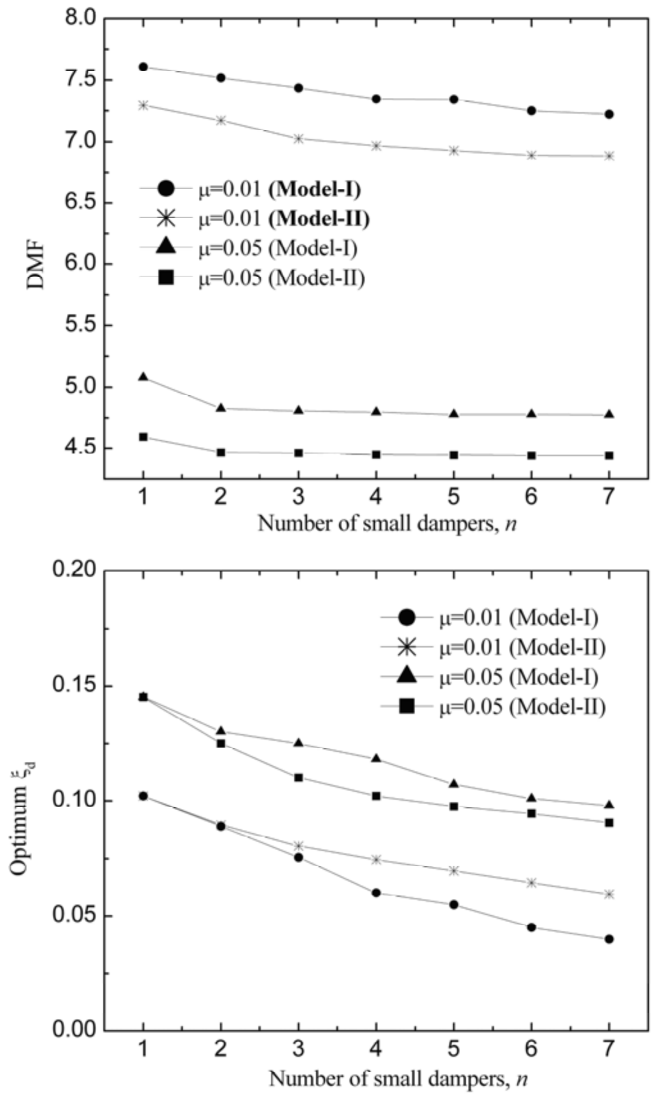

Figure 7. Comparison of DMF and optimum ξ_d for Model-I and Model-II

Tables 8 to 10 report stroke of S-TMD in DL-MTMD arranged in the increasing order of frequency. For a damper unit in S-TMD with lowest frequency exhibits more value of stroke as compared to damper unit with higher frequency. As number of dampers in S-TMD increases, the stoke length of S-TMD increases. Lesser stoke is reported for S-TMD with same stiffness for all dampers as compared to S-TMD with same mass. As the mass ratio increases the stroke of individual TMD in MTMD reduces. The DL-MTMD reduces the vibration of the structure through large relative motion between L-TMD and S-TMD by which the dashpot of each S-TMD is activated to dissipate input energy to the system.

4. Conclusion

The DL-MTMD consisting of one L-TMD to which an arbitrary number of S-TMD are attached, is proposed as a structural system to reduce the displacement DMF of a primary system subjected to external harmonic excitation. The two models with equal damper damping ratio are proposed namely the Model-I with same mass for all dampers in S-TMD and Model-II with same stiffness for all dampers of S-TMD. The S-TMD is considered with uniformly distributed natural frequencies. An arbitrary integer TMD approach is used instead of odd number TMDs. From the trend of the results of the present study, the following conclusions are drawn:

1- An arbitrary integer based S-TMD in the DL-MTMD system shows better effectiveness for controlling the displacement DMF of the main system as compared with conventional MTMD system.

2- The optimum value of the linking dash-pot provided between the structure and L-TMD in the DL-MTMD system is found to be zero.

3- As the number of dampers in the S-TMD approach to four, the improvement in the performance of the DL-MTMD system is marginal.

4- As the mass ratio and number of dampers increases, the optimum band–width also increases making the DL-MTMD system more robust.

5- The optimum damping ratio of the DL-MTMD decreases as number of dampers increases. Further, the optimum damper damping required for Model-II is relatively more than that for Model-I. The proposed Model-II is more effective for reducing the displacement DMF of the main system with Model-I.

6- The optimum tuning frequency ratio for dampers decreases as the number of dampers increase, also the off tuning with respect to large block required is more, whereas optimum tuning frequency ratio remains practically constant.

7- The stroke displacement of the L-MTMD in the DL-MTMD system decreases with the increase of the number of dampers.

References

Abe, M. and Fujino, Y. (1994), “Dynamic characterization of multiple tuned mass dampers and some design formulas”, *Earthquake Engineering and Structural Dynamics*, Vol. 23, Pages 813-835.

Abe, M. and Igusa, T. (1995), “Tuned mass dampers for structures with closely spaced natural frequencies”, *Earthquake Engineering and Structural Dynamics*, Vol. 24, Pages 247 -261.

Bakre, S.V. and Jangid, R.S. (2004), "Optimum multiple tuned mass dampers for base excited damped main system", *Journal of Structural Stability and Dynamics*, Vol. 4, Pages 527-542.

Han, B.K. and Li, C.X. (2006), "Evaluation of multiple dual tuned mass dampers for structures under harmonic ground acceleration", *International Journal of Structural Stability and Dynamics*, Vol. 6, Pages 59-75.

Igusa, T. and Xu, K. (1994), "Vibration control using multiple tuned mass dampers", *Journal of Sound and Vibration*, Vol. 175, Pages 491-503.

Iwanami, K. and Seto, K. (1984), "Optimum design of dual tuned mass dampers and their effectiveness", *Proceedings of the Japan Society of Mechanical Engineering*(C), Vol. 50, Pages 44-52.

Jangid, R.S. (1995), "Dynamic characteristics of structures with multiple tuned mass dampers", *Structural Engineering and Mechanics*, Vol. 28, Pages 1041-1049.

Jangid, R.S. (1999), "Optimum multiple tuned mass dampers for base excited un-damped system", *Earthquake Engineering and Structural Dynamics*, Vol. 28, Pages 1041-1049.

Joshi, A.S. and Jangid R.S. (1997), "Optimum parameters of multiple tuned mass dampers for base excited damped systems", *Journal of Sound and Vibration*, Vol. 202, Pages 657-667.

Kareem, A. and Kline, S. (1995), "Performance of multiple mass dampers under random loading", *Journal of Structural Engineering, ASCE*, Vol. 121, Pages 348-361.

Li, C. (2000), "Performance of multiple tuned mass dampers for attenuating undesirable oscillations of structures under the ground acceleration", *Earthquake Engineering and Structural Dynamics*, Vol. 29, Pages 1405-1421.

Li, C. (2006), "Performance of dual-layer multiple tuned mass dampers for structures under ground excitations", *Journal of Structural Stability and Dynamics*, Vol. 6, Pages 541-557.

Li, C. and Zhang, J. (2005), "Evaluation of arbitrary integer based multiple tuned mass dampers for structures", *Journal of Structural Stability and Dynamics*, Vol. 5, Pages 475-488.

Li, C. and Zhu, B. (2006), "Estimating double tuned mass dampers for structures under ground acceleration using a novel optimum criterion", *Journal of Sound and Vibration*, Vol. 298, Pages 280-297.

Luft, R.W. (1979), "Optimum tuned mass dampers for buildings", *Journal of Structural Division, ASCE*, Vol. 105, Pages 2766-2772.

McNamara, R.J. (1977), “Tuned mass dampers for buildings”, *Journal of Structural Division, ASCE*, Vol. 103, Pages 1785-1798.

Warburton, G.B. (1982), “Optimum absorber parameters for various combinations of response and excitation parameters”, *Earthquake Engineering and Structural Dynamics*, Vol. 10, Pages 381 - 401.

Wirsching, P.H. and Campbell, G.W. (1974), “Minimal structural response under random excitation using vibration absorber”, *Earthquake Engineering and Structural Dynamics*, Vol. 2, Pages 303-312.

Xu, K. and Igusa, T. (1992), “Dynamic characteristics of multiple substructures with closely spaced frequencies”, *Earthquake Engineering and Structural Dynamics*, Vol. 21, Pages 1059-1070.

Yamaguchi, H., and Harnpornchai, N. (1993), “Fundamental characteristics of multiple tuned mass dampers for suppressing harmonically forced oscillations”, *Earthquake Engineering and Structural Dynamics*, Vol. 22, Pages 51-62.