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# CELLULAR AUTOMATA IN OPTIMUM SHAPE OF BRICK MASONRY VAULT UNDER DYNAMIC LOADS

Kaveh Kumarci[1], Afsaneh Banitalebi Dehkordi[2] and Pooya Khosravyan Dehkordi[2]

[1] *Sama Technical and Vocational Collage, Islamic Azad University, Shahr-e kord Branch, Shahr-e kord, Iran*

[2] *Islamic Azad University, Shahr-e-kord Branch, Shahr-e-kord, Iran*

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This paper deals with optimum shape brick masonry vaults under dynamic loads by cellular automata. In this paper, the vaults are modeled. Then, they are analyzed and optimized under acceleration–time components of Elcentro earthquake. For vault response optimization, the results were used in cellular automata computational model. Then vaults are analyzed and optimized by modeling rules. The results of error range and time of analysis in automata cellular model and FEM software compared. Finally, comparing the results of CA (Cellular Automata) method and FEM (Finite Element Method) method, shows that although precision is less in CA method, but the time of analysis and optimization is so much smaller.

*Keywords*: optimum shape, vault, brick masonry, dynamic load, tensile stress, cellular automata

## 1. Introduction

Traditionally, vault is defined as a part of circle or bow, but our particular definition of vault is as follows: it is a curve surface for covering that its span is higher than its depth (Heyman, 1982). Among the structural components in masonry buildings, arches and vaults deserve particular attention. They are very widespread in historical centers and their preservation as part of the

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[1] Lecturer
[2] Graduate Student

**Correspondence to:** Kaveh Kumarci, Sama Technical and Vocational Collage, Islamic Azad University, Shahr-e-kord Branch, Shahr-e-kord, Iran, E-mail: kumarci_kaveh@yahoo.com

cultural heritage is very topical subject. Because of their ages or for accidental causes (such as earthquakes), these structures can offer several types of damage.

These structures represented a significant structural capacity, which were used in the past in spanning wide opening with adobe or brick masonry. Structural efficiency is attributed to the curvature of the vault, which transfers vertical loads laterally along the vault to the abutments at each end (Blasi and Foraboschi ,1990). Transferring vertical forces gives a rise to both horizontal and vertical reactions at the abutments. The curvature of the vault and its restraint by the abutments cause a combination of flexural stress and axial compression. The depth of vault as well as its rise and configuration can be manipulated to keep stresses primarily compressive and because the brick masonry is very strong in compression, so brick masonry vault can support a considerable load (Brickwork, 1989).

The increasing interest in historic architectural heritage and need for conservation of historical structure has led to the continuous development of many methods for analysis of masonry vaults. However, some types of vaults have not been thoroughly analyzed, mainly because of the problem of applying simplified theories to their complicated shapes (J.P.Szolomicki, 2009).

Regarding the importance and application of vault in traditional structures, vault optimization has been considered (Huerta, 2001). There have been some researches on brick masonry under dynamic loads (Kumarci et al, 2008). Dynamic or time history analysis is an analytical method for determining reflections during the earthquake in structures. Through this analysis, the response of structure under loadings which are related to time has been studied (Hughes, 1987). Dynamic analysis and optimization of vault need to consume a long time; it is necessary to use a proper computational model such as cellular automata to analyze and optimize the vault in less time and also for more acceptable results. Cellular automata are a decentralized computation model. It is an appropriate method for computation and simulation of complicated behaviors by local data (Wolfram, 2002). The present research goals are: modeling, analyzing and optimizing complicated behaviors of semi-circular, obtuse angel, four-centered pointed, Tudor, ogee, equilateral, catenaries, lancet and four-centered vault, under dynamic load using cellular automata. Cellular Automata rules often produce spatial patterns which make them recognizable by human observers. Nevertheless, it is generally difficult, if not impossible, to identify the characteristic(s) that make a rule produce a particular pattern. Discovering rules that produce spatial patterns that a human being would find “similar” to another given pattern is a very important task, given its numerous possible applications in many complex systems models (Bandini et al., 2009).The importance of this research is to show the ability of analyzing and optimizing of every vault after one time of modeling in a so much shorter time.

## 2. Modeling, Analyzing and Optimizing Vault Shape Using FEM

At the first step vault modeling has been conducted by FEM. Furthermore, dynamic analysis has been conducted applying north-south horizontal accelerations of Elcentro earthquake in which the time, maximum acceleration, maximum velocity and maximum displacement are 31.98(s), 0.31(g), 33 (cm/sec) and 21.4 (cm), respectively Figure 1 and SOLID65 is used for analysis in this stage. Vault shape optimization emphasized on the minimizing of vault weight. In FEM software, the base and top thickness, maximum tensile stress and weight of structure have been defined as design variables, state variables and objective function, respectively. For example, optimum shape of semicircular vault in FEM software has been shown Figure 2. Regarding the extra time for analysis and optimization, the optimization has been conducted in design optimum processor by means of Sub-problem Approximation Method. This is an estimating method for variable designing, state and objective function via curve fitting tool. It is a general method for solving many engineering problems (Crisfield, 1985).

### *2.1. Geometrical Modeling*

According to shape optimization design variables, such as base thickness ($t_0$) and top thickness ($t_1$) as parameters, all the key points are defined as follows (Figure.3): Point 1: (0, 0); Point 2: (R, 0); Point 3: (-R, 0); Pint 4: (0, R); Point 5: (R+t0, 0); Point 6: (-R-$t_0$, 0); Point 7: (0, R+$t_1$).

In vault modeling, the tolerance increases because the thickness decreases from base to top (Abruzzese et al, 1995). It should be mentioned that in modeled vault, the thickness decreases from base ($t_0$) to top ($t_1$) linearly and also vault thickness of axis is 100 (cm) in the length direction. The motion of support nodes is zero and dynamic force has no effect on them. In addition, brick masonry is made by brick and mortar as homogenous material Table 1. These results are very important because the structure behavior of masonry vaults and their collapse mechanisms depend on the material property forming them. So, optimum shape in brick masonry vaults is supported on the assumption of large compressive strength for blocks, no tension transmitted across the joints and finite friction.

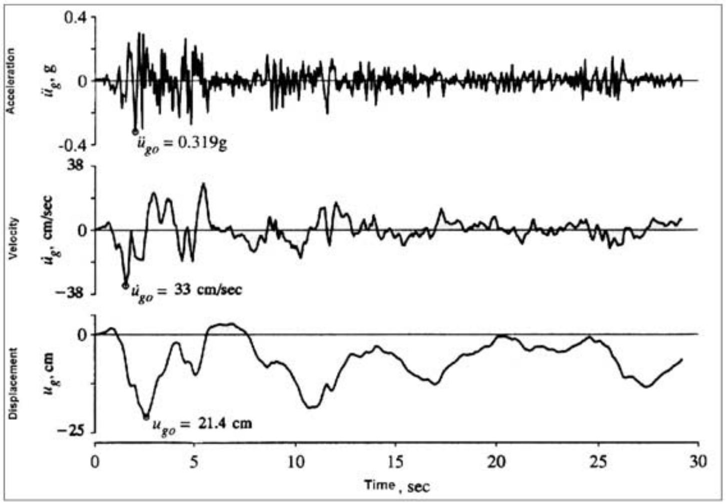


Figure 1. North-south horizontal component of Elcentro earthquake

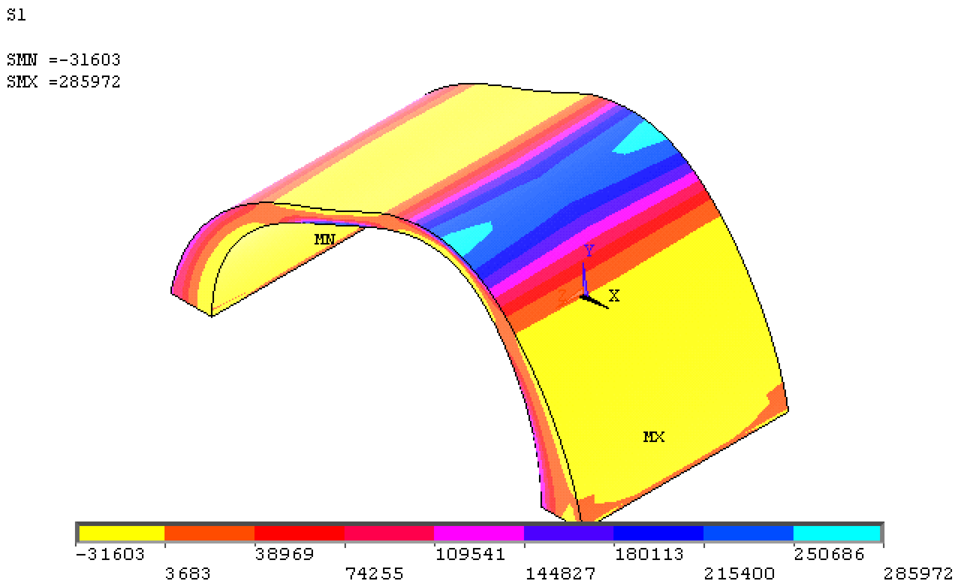


Figure 2. Optimum shape of semicircular vault using FEM software

Table 1. Brick masonry characteristics (Bsthe, 1996)

| density( $\rho$ ) ( Kg/m$^3$) | Elastic modulus ( $N/m^2$ ) | Poisson ratio ( $\upsilon$ ) |
|---|---|---|
| 1460 | $5\times10^8$ | 0.17 |

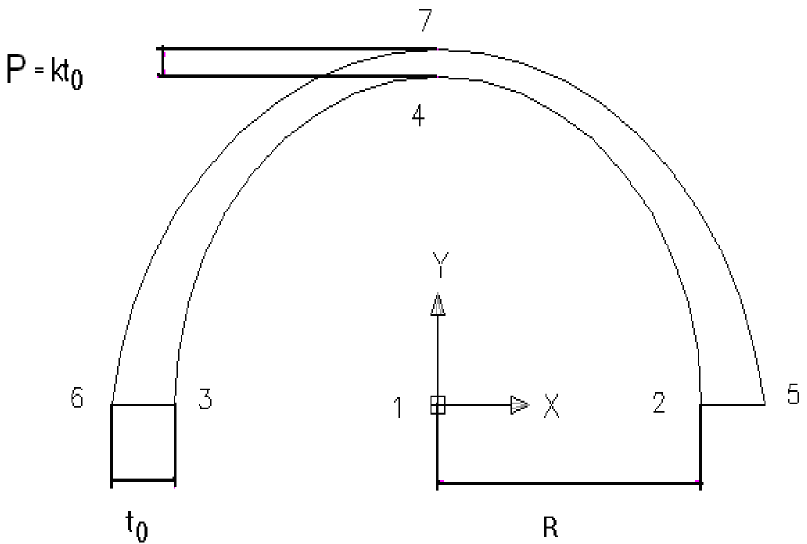


Figure 3. Geometrical model of semicircular vault

Non-linear and elastic analysis relies on determination of effective coefficients of vaults which have been reported in Table 2 and have been taken in agreement with the experimental data available. In the present paper, vault radius limit (R), maximum tensile stress, base and top thickness in optimum state are considered as 4-8 (m), 49000-5100 (KN/m$^3$), 0.8- 1.44 (m) and 0.2-0.35(m) respectively for all modeled vault.

## 3. Cellular Automata

At the beginning of 1950, cellular automata (CA) have been proposed by Von Neumann. He was interested to male relation between new computational device - automata theory - and biology. His mind was preoccupied with generating property in natural events (Neumann, 1993).

He proved that CA can be general. According to his findings, CA is a collection of cells with reversible states and ability of computation for everything. Although Van rules were complicated and didn't strictly satisfy computer programs, he continued his research in two parts: for decentralizing machine which is designed for simulation of desirable function and designing of a machine which is made by simulation of complicated function by CA (Neumann,1996).

Wolfram has conducted some research on problem modeling by the simplest and most practicable method of CA too. In 1970,"The Game of Life" introduced by Conway and became very widely known soon. At the beginning of 1980, Wolfram studied one-dimension CA rules and demonstrated that these simple CAs can be used in modeling of complicated behaviors (Wolfram, 1983, 1984).

### *3.1. Definitions*

Cellular Automata rules often produce spatial patterns which make them recognizable by human observers. Nevertheless, it is generally difficult, if not impossible, to identify the characteristic(s) that make a rule produce a particular pattern. Discovering rules that produce spatial patterns that a human being would find "similar" to another given pattern is a very important task, given its numerous possible applications in many complex systems models (Bandini et al., 2009). CA is characterized by (a) cellular space (b) transfer rule (Moore, 2003).

Cellular automata (CA) are an important modeling paradigm in the natural sciences and an extremely useful approach in the study of complex systems (Terrazas et al, 2007).

Table 2. Effective coefficient in non elastic and nonlinear analysis (Baggio and Trovalusci, 2000)

| motion coefficient for open crack | motion coefficient for close crack | allowable tension stress $N/m^2$ | allowable compressive stress $N/m^2$ |
|---|---|---|---|
| 0.1 | 0.9 | $5\times10^4$ | $5\times10^5$ |

### *3.2. Change State Rules*

Each cell changes its state, spontaneously. The primary quality of cells depends on primary situation of problem. By these primary situations, CA is a system which has certain behavior by local rules. The cells which are not neighbors have no effect on each other. CA has no memory, so the present state defines the next state (Wolfram, 2002).

A cellular automata (CA) has an n-dimensional cellular space and consists of a regular rigid cells. Neighborhood of a cell consists of the surrounding adjacent cells, including the cell itself (Figure 4).

Quadruple CA is as CA= (Q, d, V and Φ), where Q, d, V and Φ are collection of possible state, CA dimension, CA neighborhood structure and local transferring rule, respectively.

For 1-d CA, amount of i cell ($1\leq i\leq n$) at t is shown by $a_i(t)$ and is calculated by this formula:

$$a_i(t+1)= \Phi\,[a_{i-1}(t),\, a_i(t),\, a_{i+1}(t)] \tag{1}$$

In this formula, if Φ is affected by the neighbors, it is general. If Φ is a function of neighbor's cell collection and central cell, it is totalistic.

## 4. Vault Modeling Using CA

In this stage, regarding the definition of neighborhood radius and state reversal rule in a three state 1-d CA, the data for each vault will be analyzed to find the rules of simulation of vault behavior. To achieve this aim, 100 samples of each vault radius, base and top thickness and maximum tensile stress were chosen and analyzed by two and three state algorithm (Figure 5 defines a two state algorithm completely). After one billion accomplishments, for 256 two-state rules and one million three-state rules, some models were provided for each vault. For example, Figures 6 defines semicircular rules, respectively.

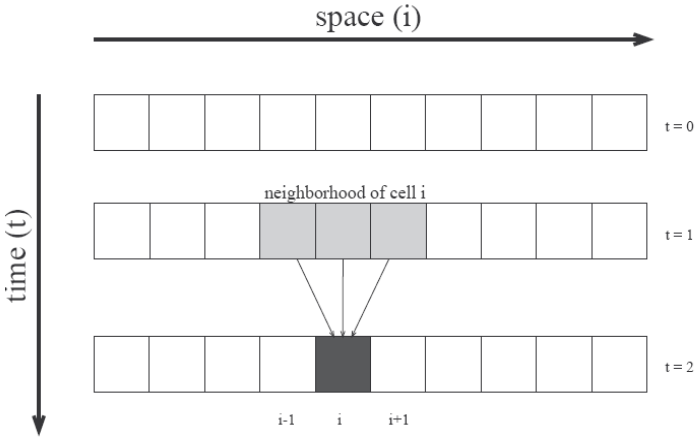


Figure 4. Neighborhood space of Von Numann in 1-D CA

## 5. Testing Cellular Automata Models

Maximum tensile stress for 50 samples (according to algorithm in Figure 7) has been provided. The error percent has been compared with another analyzed model in FEM software.

### *5.1. Test of CA Model for Semicircular Vault*

Maximum tensile stress was achieved for 50 samples of semicircular vault by CA. Figure 8 defines the comparison between maximum tensile stress in FEM and CA models**.** The mean of error percent in semicircular vault is 14.37%. Figure 9 represents error percent of each sample. Moreover, Figure 10 illustrates the diagram of the comparison between time of maximum tensile stress computation using CA and FEM software, respectively.

## 6. Vault Optimization Using CA

In this stage, by means of CA model for each vault top and base thickness were optimized. Considering optimized maximum tensile stress which is 51000(N/m$^2$), the range of radius, top thickness and maximum tensile stress in each vault are considered as input, so vault base thickness will be provided. In the next stage, the size of vault radius, base thickness and maximum tensile strain are considered as input. So vault top thickness will be provided (vault base thickness optimization is defined in Figure 11). It is obvious that vault thrust depends fundamentally on the thickness and height of vault.

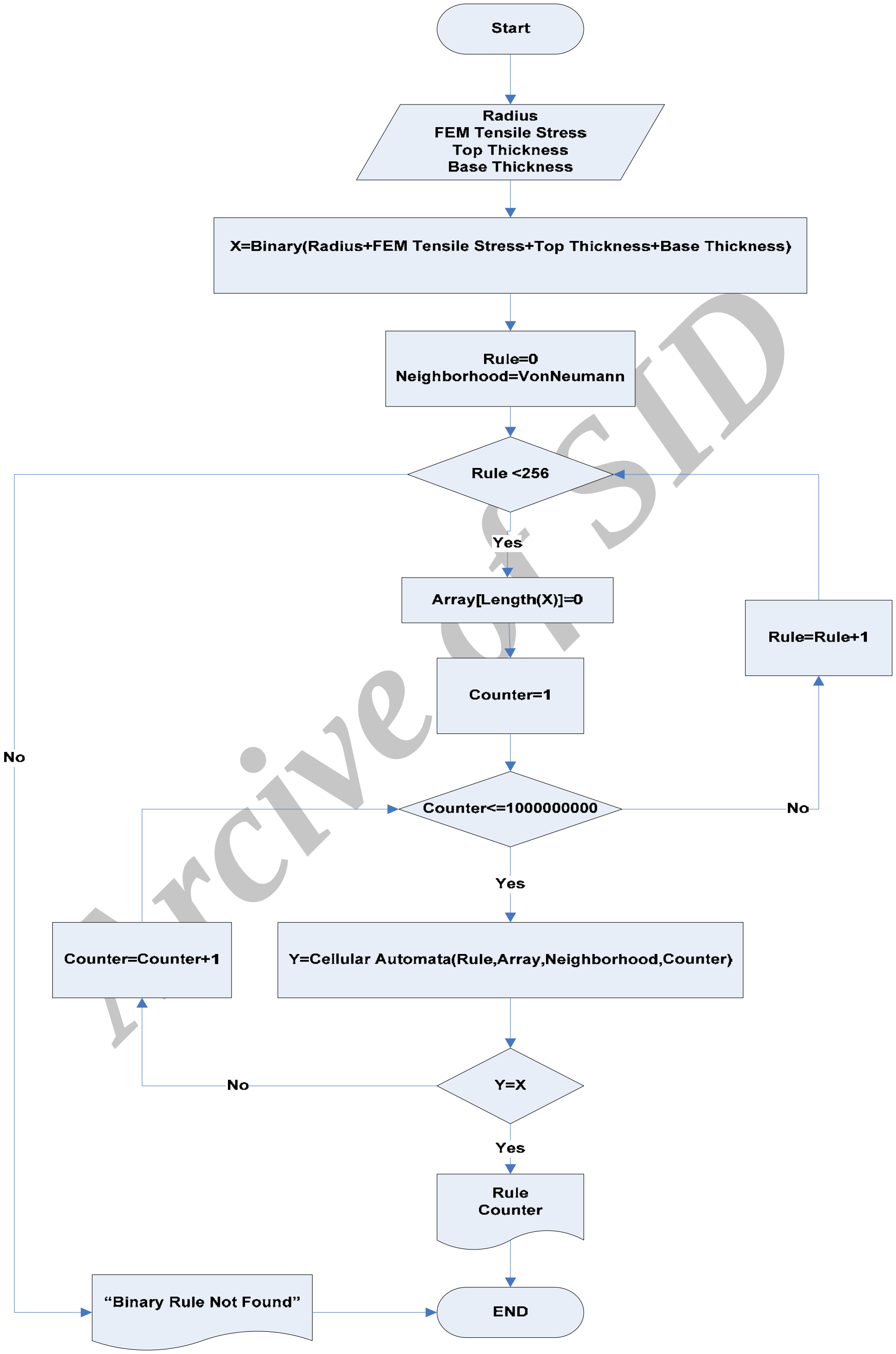


Figure 5. Algorithm for finding two- state 1-D cellular automata model for vault behavior modeling

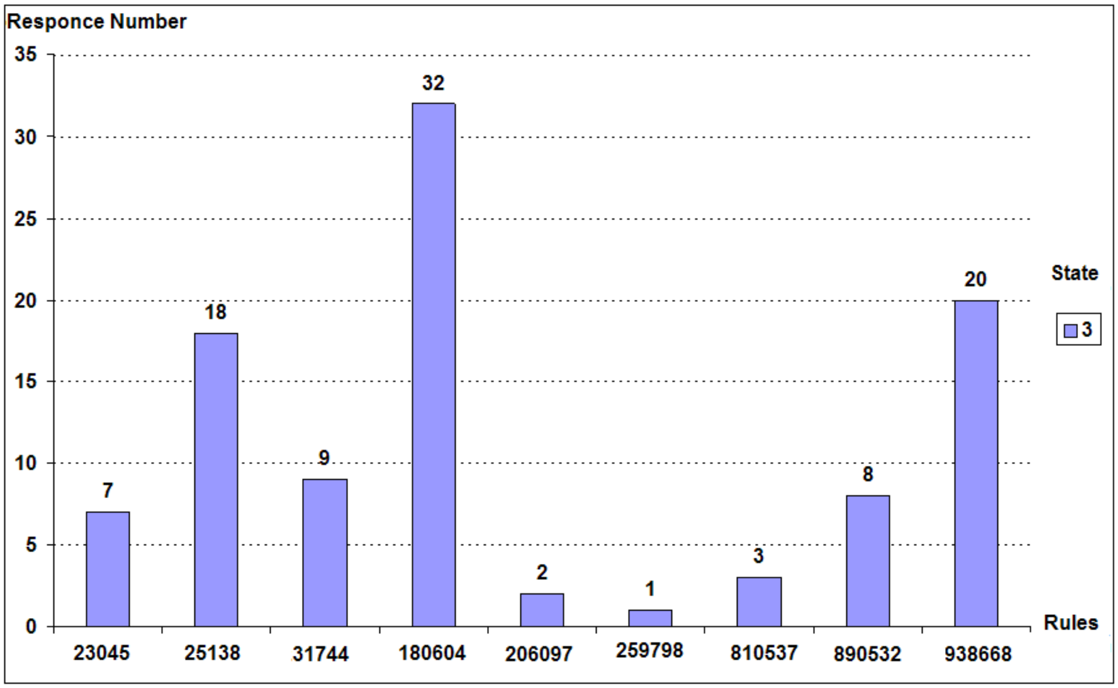


Figure 6. Diagram of three-state rules of cellular automata and some of samples in semicircular vault

### *6.1. Top Thickness Optimization in Semicircular Vault Using CA*

In this stage, 50 semicircular vault samples were chosen for top thickness optimization. Their optimum maximum tensile stress range, vault radius and base thickness were 49000 to 51000 (KN/m$^3$), 4~8 meter and 0.8 to 1.44, respectively. Afterward, the top thickness was calculated and compared with top thickness in FEM software (Figure 14). The mean of error percent of top thickness calculation was 14.34%. Figures 12 and 13 show error percent of each sample in CA toward FEM software and the comparison of optimization time of top thickness optimization in semi circular vault.

### *6.2. Base Thickness Optimization in Semicircular Vault Using CA*

In this section, 50 semicircular vault samples were chosen for top thickness optimization. Their optimum maximum tensile stress range, vault radius and base thickness were 49000 to 51000(KN/ m$^3$), 4~8 meter and 0.2 to 0.35, respectively. After calculation of base thickness-according to algorithm in Figure 12, the results were compared with base thickness in FEM software (Figure 17). The mean of error percent of base thickness calculation was 13.40%. Figures 15 and 16 show error percent of each sample in CA toward FEM software and the comparison of optimization time of base thickness optimization in semi circular vault:

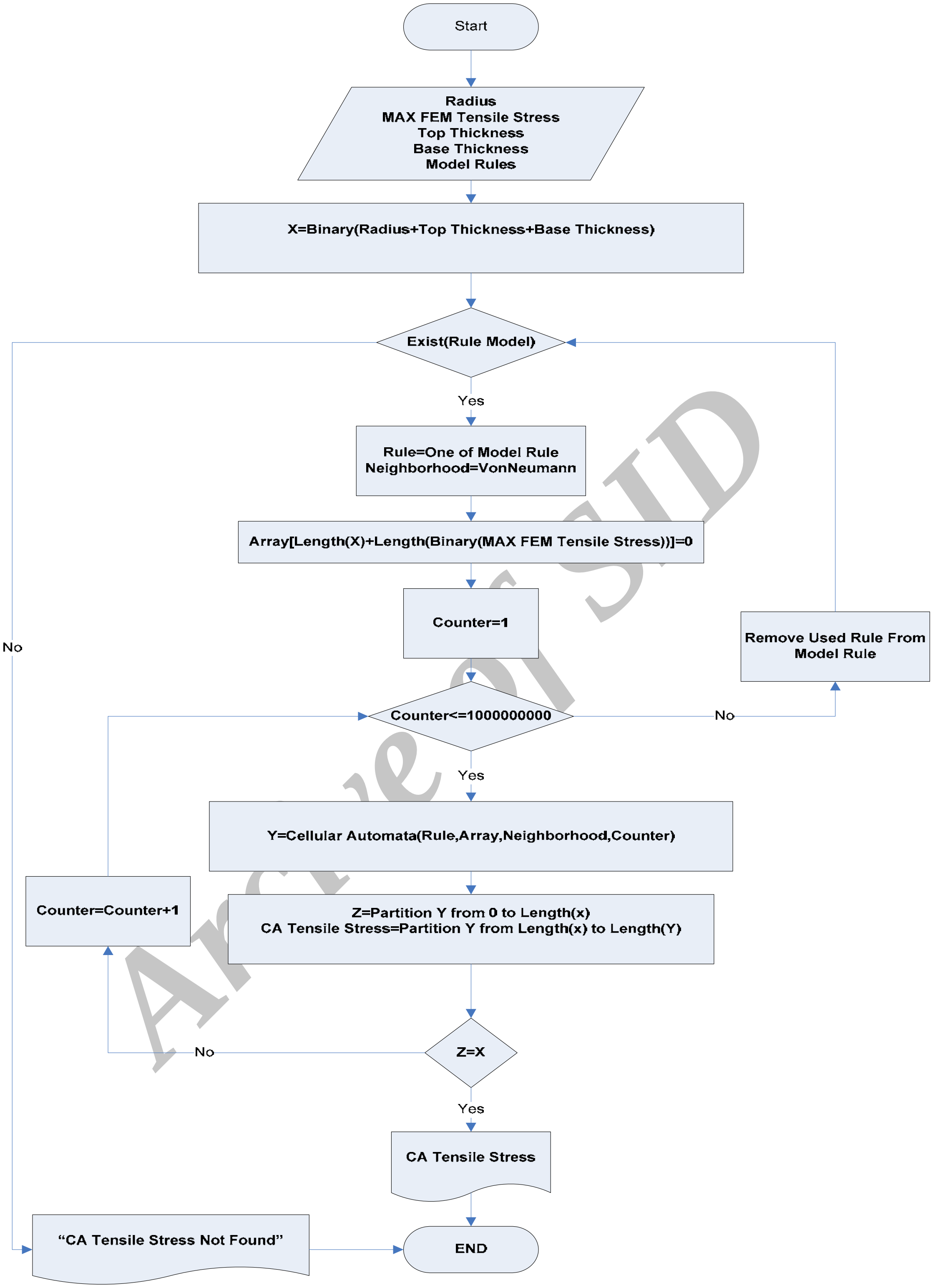


Figure 7. Algorithm for analysis of vault behavior using a two-state 1-D CA for maximum tensile stress

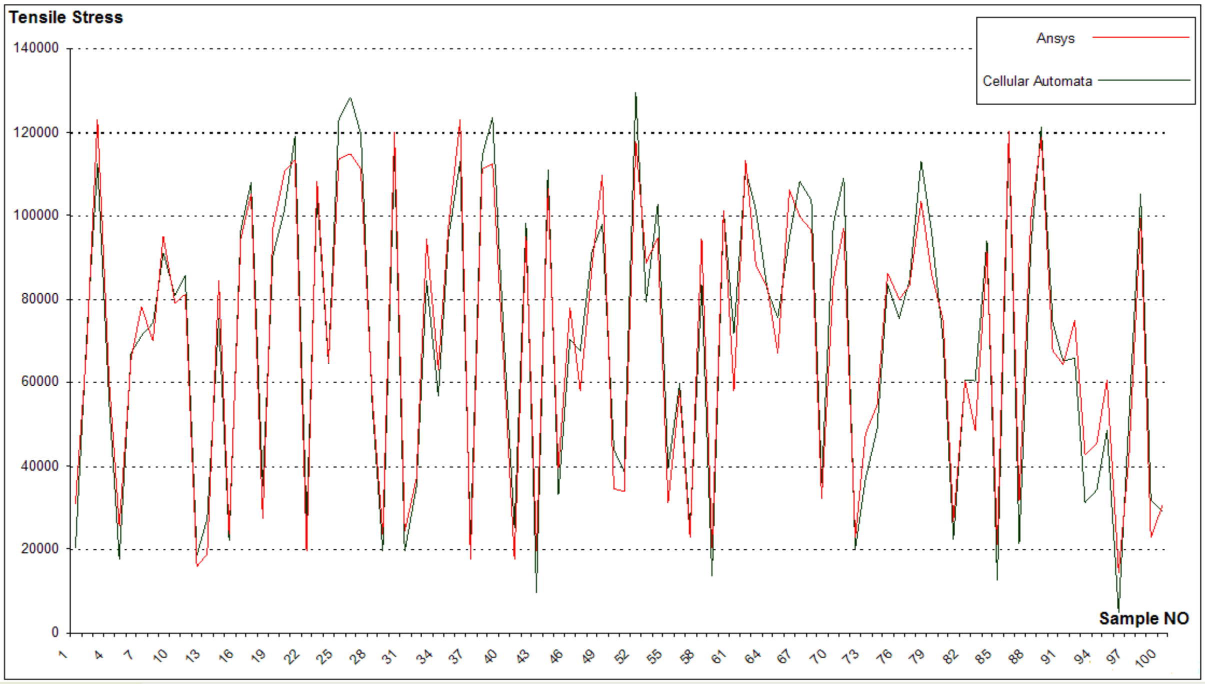


Figure 8. Comparison between maximum tensile stress using
FEM software and CA model in semicircular vault

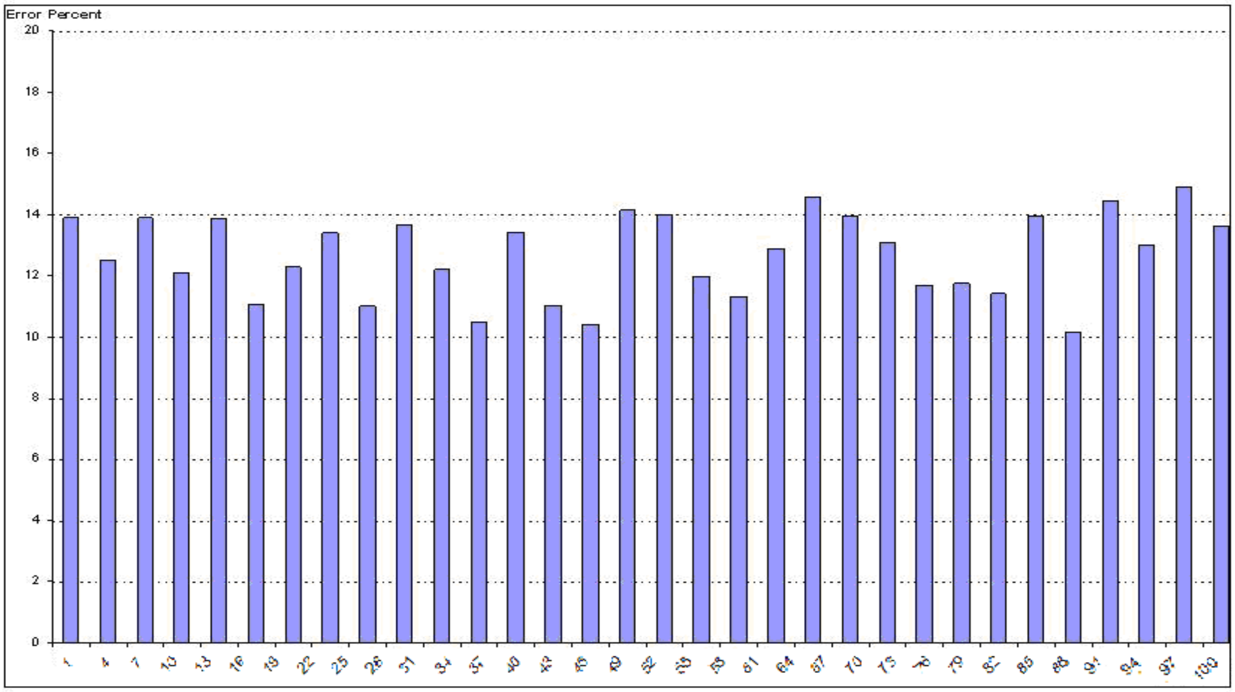


Figure 9. Error percent of maximum tensile stress computation
by CA to FEM software in semicircular vault

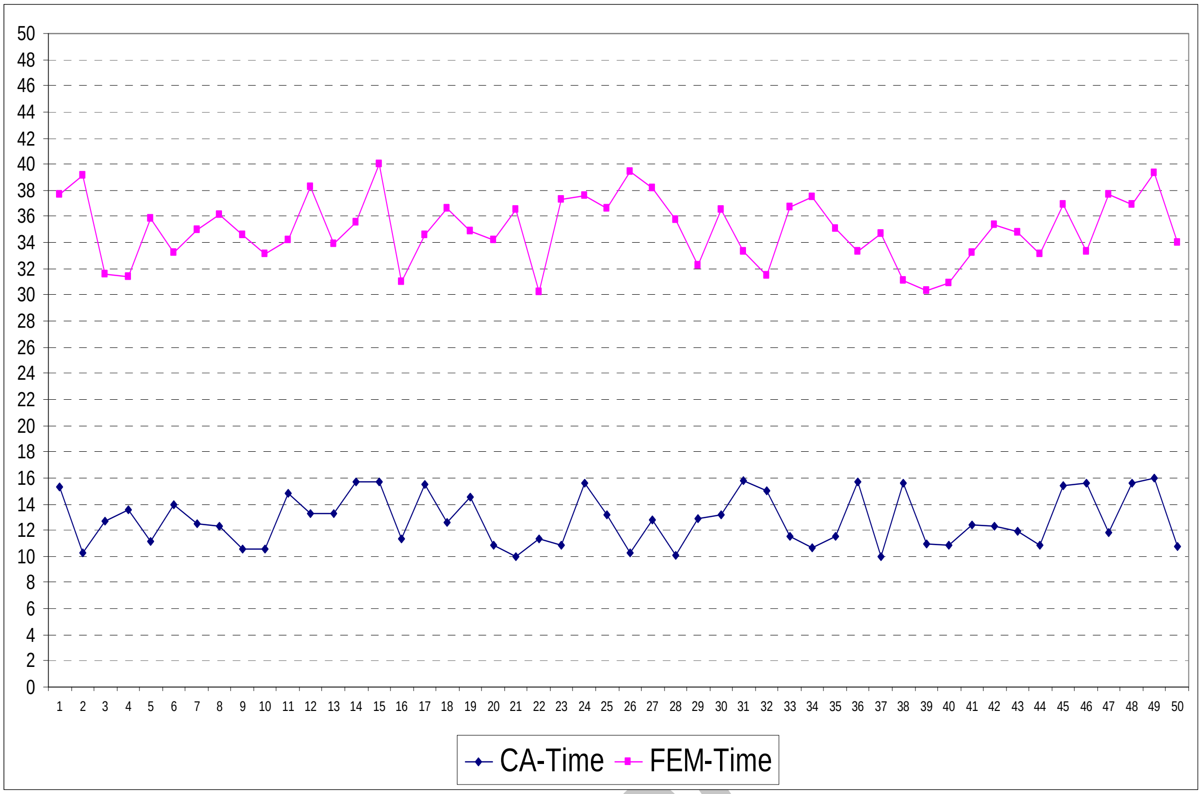


Figure 10. Comparison between time of computation of maximum tensile stress by FEM software and CA

## 7. Conclusions

The analysis of ancient masonry structures is a challenging task for engineers. It has been proved that masonry vault structures are most stable when they are optimized. In the last decades, accurate and reliable tools for the analysis of these structures have been developed. In the present paper, nine vaults - semi-circular, obtuse angels, four-centered pointed; Tudor, ogee, equilateral, catenaries, lancet and four-centered vault- were modeled using FEM software and CA models. Figures 18, 19 and 20 show the analysis and optimization time, the results which are provided by CA in vault modeling and the mean of error percent for vault analysis and its optimization, respectively.

Considering the obtained results, CA model can be used in simulation of all vaults. Therefore, the calculation time decreases. Also, it can be used in dynamic response, natural frequency and response of structure under different dynamic loads. To increase models precision, the rules which are larger than 1000000 and repeated more than 1000000000 times are needed.

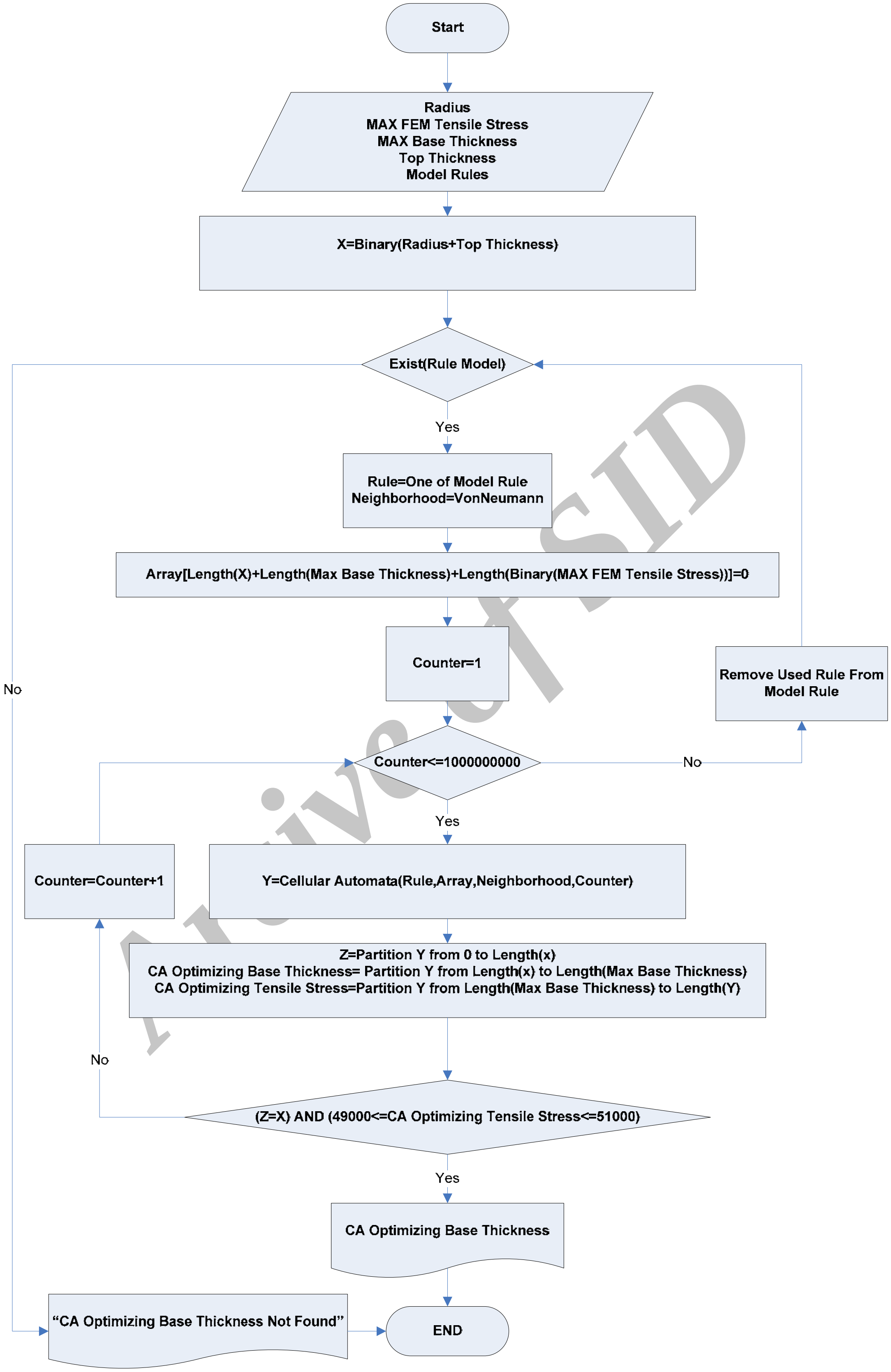


Figure 11. Algorithm of vault thickness optimization using a two-state -1-D cellular automata

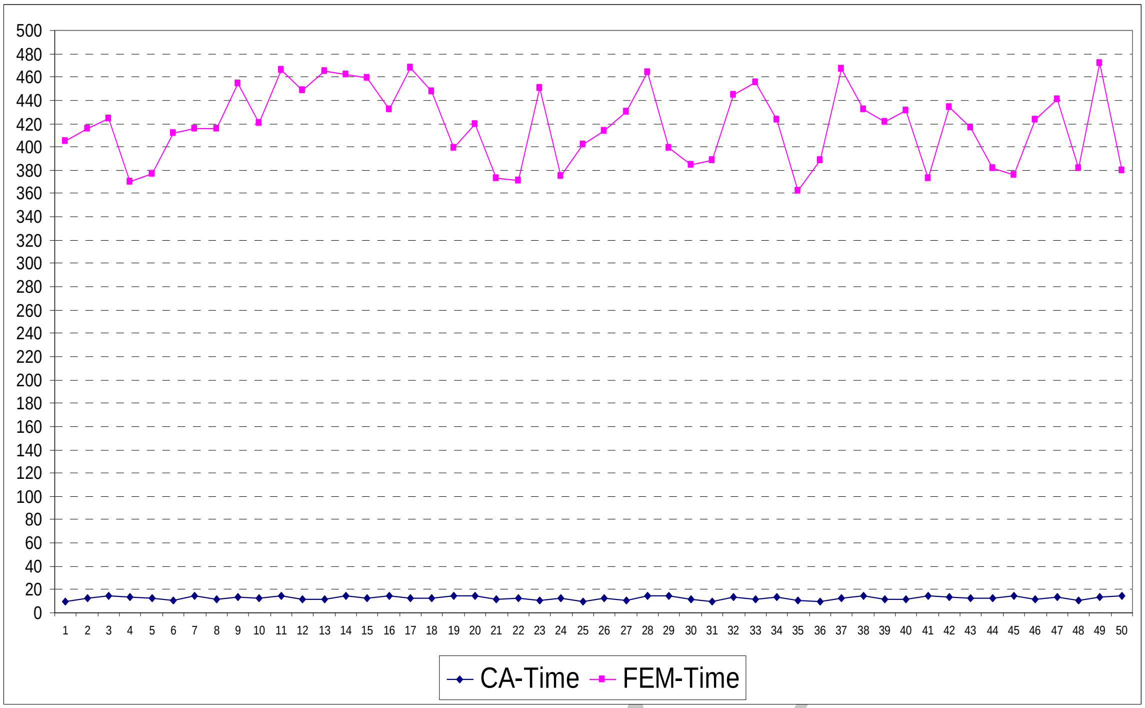


Figure 12. Comparison between maximum tensile stress of semicircular vault using FEM software and CA models

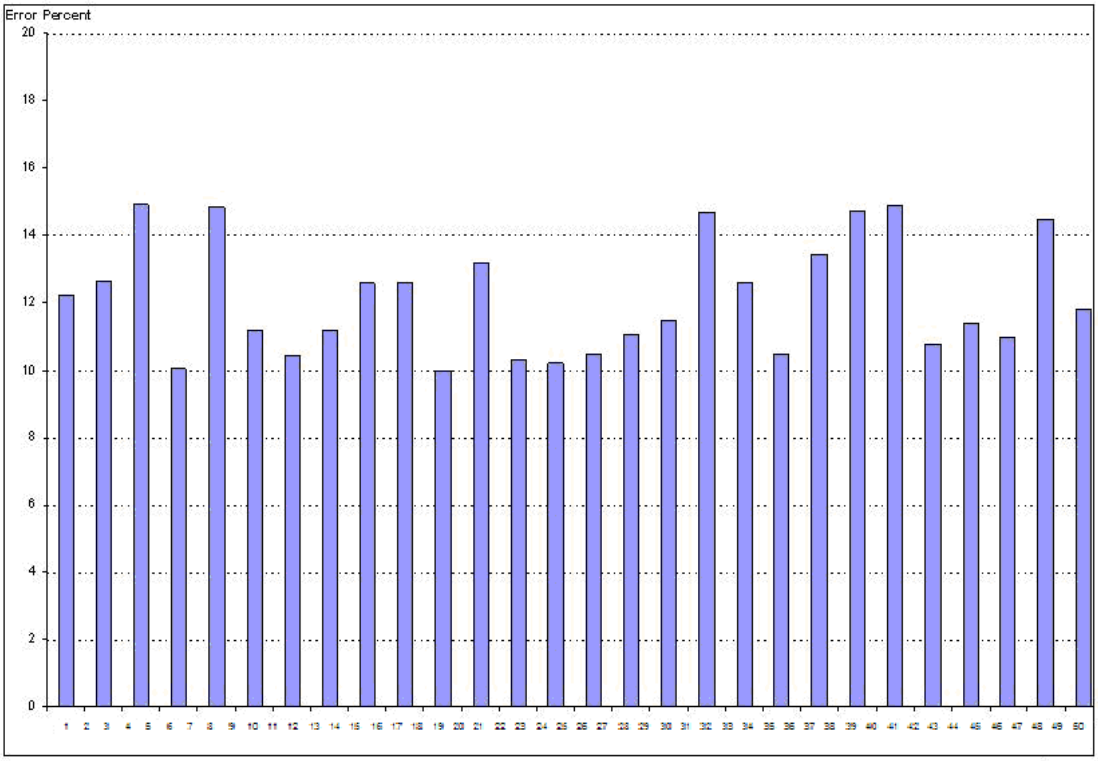


Figure 13. Error percent of top thickness optimization in semicircular vault using CA model towards FEM software

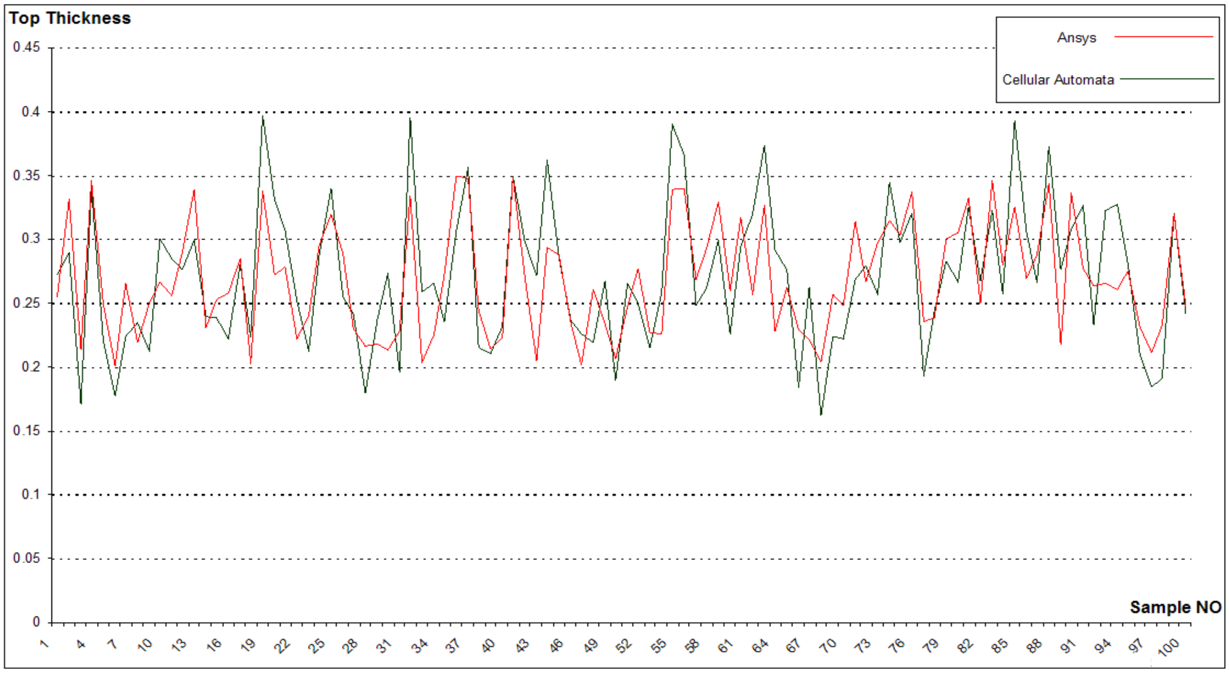


Figure 14. The comparison of optimum range of vault top thickness using CA model and FEM software

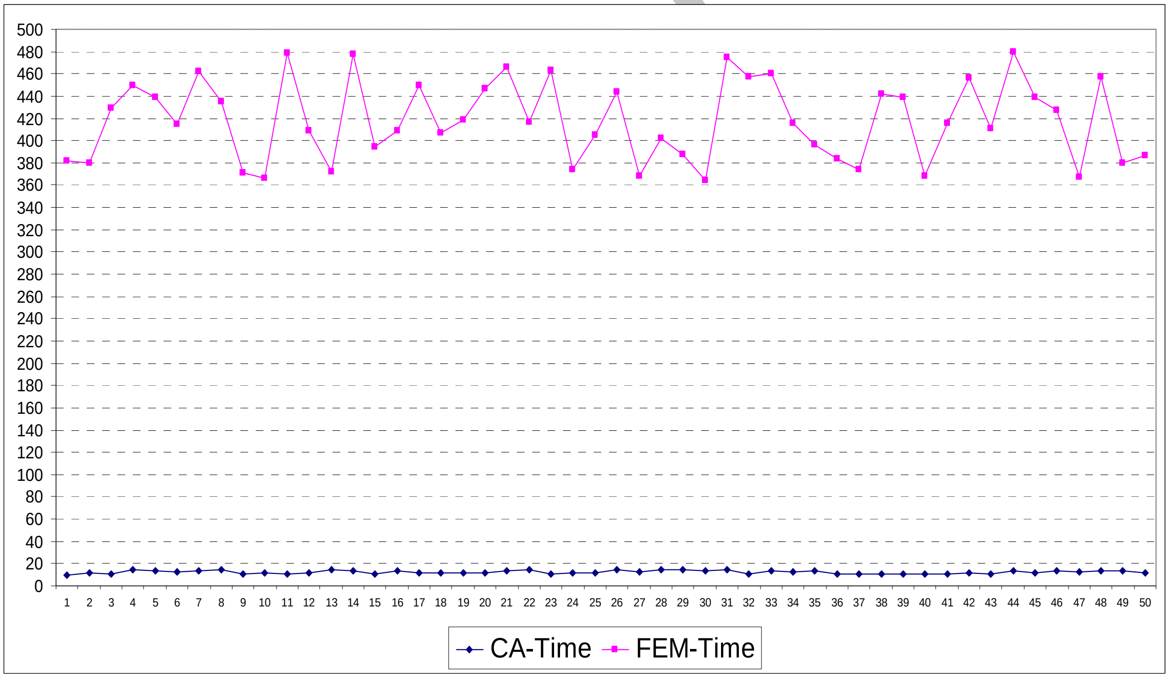


Figure 15. The comparison between optimization time of base thickness in semicircular vault using FEM software and CA models

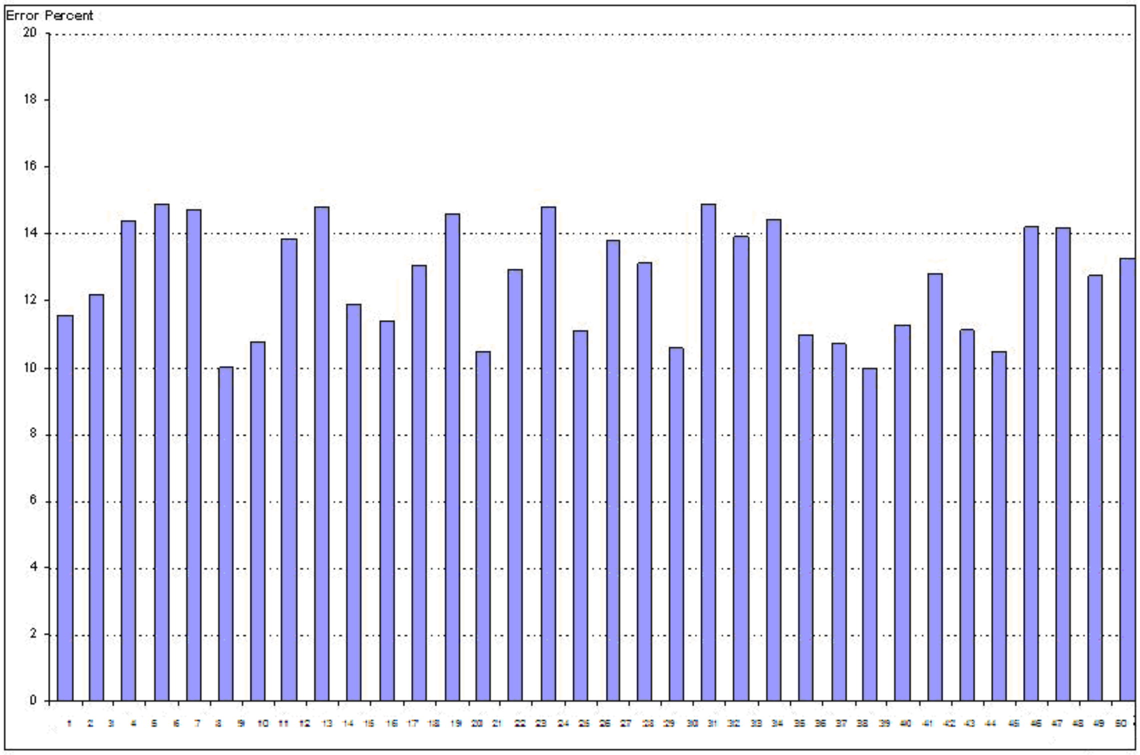


Figure 16. Error percent of base thickness optimization in semicircular vault using CA model towards FEM software

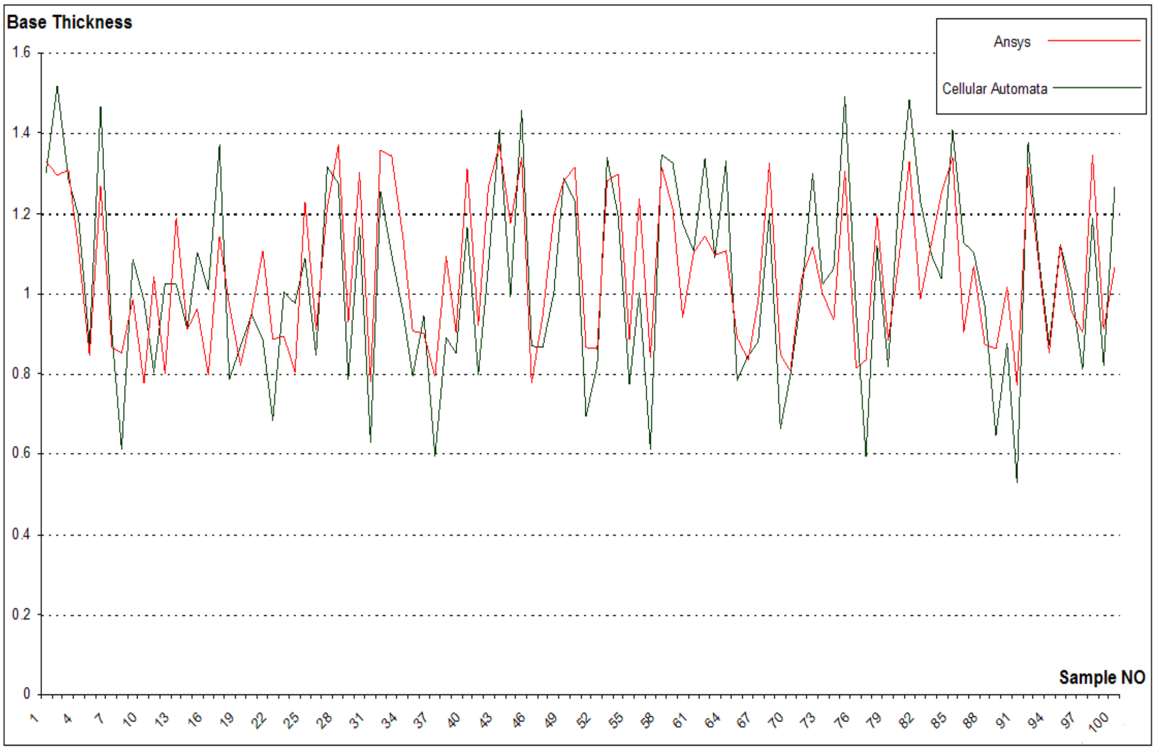
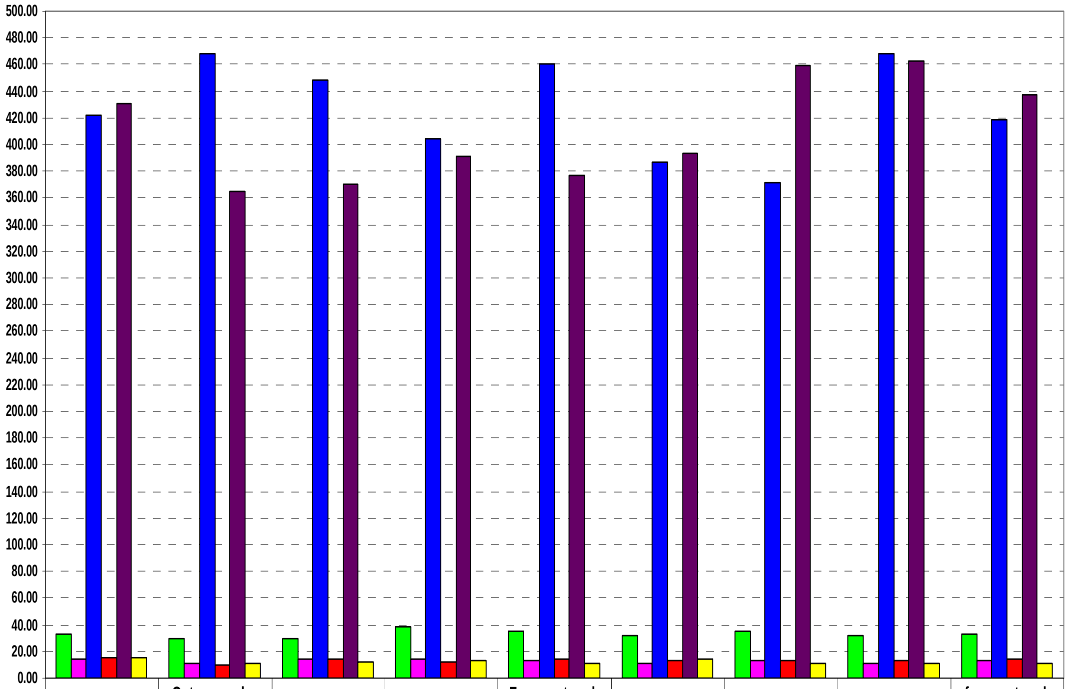
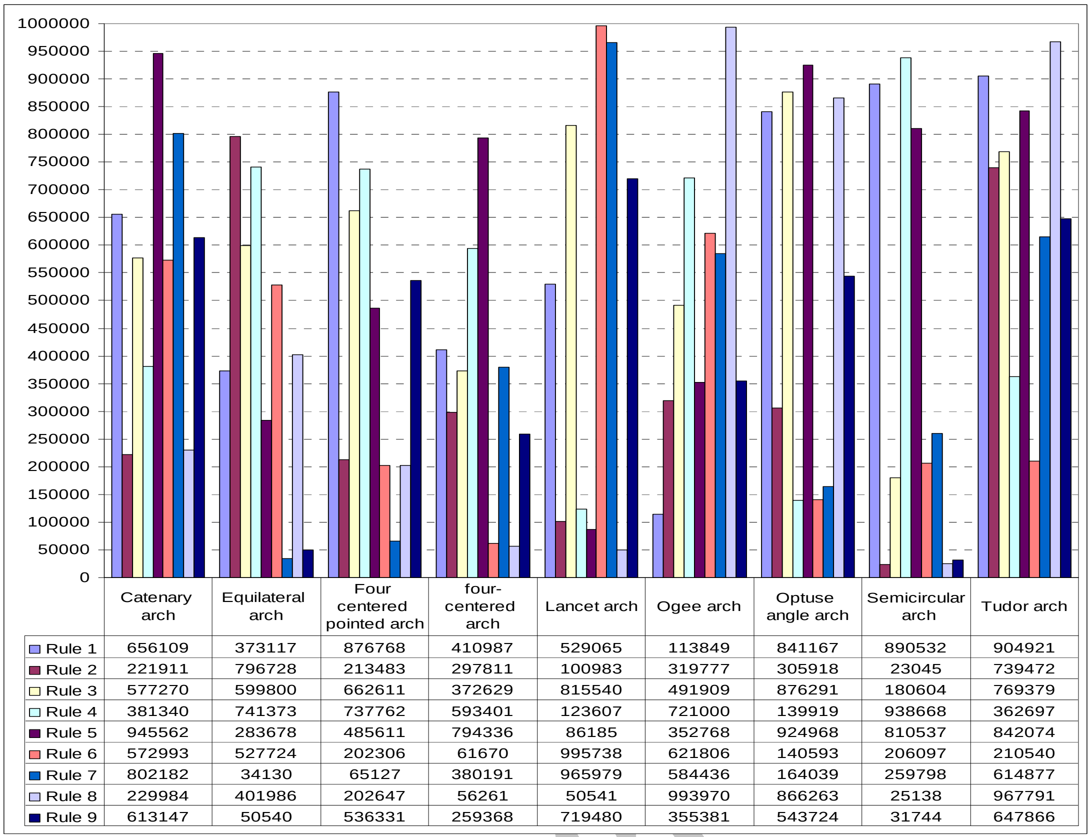


Figure 17. The comparison of optimum range of vault base thickness using CA model and FEM software



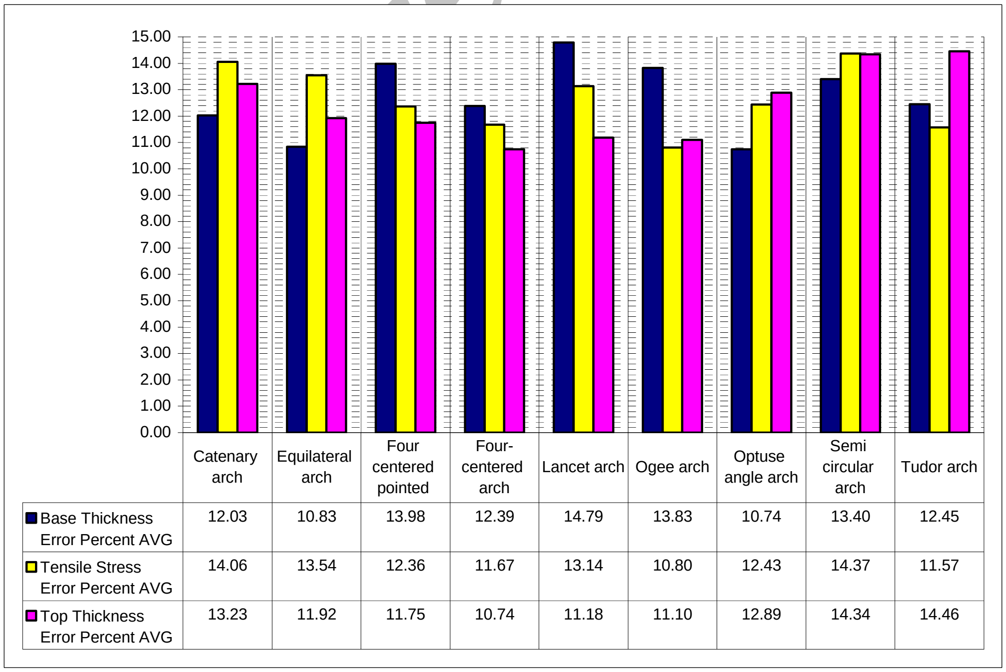
| | Semicircular arch | Optuse angle arch | Ogee arch | Tudor arch | Four centered pointed arch | Equilateral arch | Catenary arch | Lancet arch | four-centered arch |
|---|---|---|---|---|---|---|---|---|---|
| 🟩 FEM Analyzing Average Time | 33.07 | 30.10 | 30.16 | 38.03 | 35.49 | 31.98 | 34.74 | 32.40 | 33.23 |
| 🟪 CA Analyzing Average Time | 14.70 | 11.14 | 14.81 | 14.72 | 13.58 | 11.23 | 13.36 | 10.94 | 13.00 |
| 🟦 FEM Top Thickness Optimizing Average Time | 421.84 | 467.97 | 448.17 | 404.42 | 460.59 | 386.51 | 371.56 | 468.63 | 418.41 |
| 🟥 CA Top Thickness Optimizing Average Time | 14.94 | 10.38 | 14.28 | 12.20 | 14.53 | 13.65 | 13.56 | 13.34 | 13.78 |
| 🟪 FEM Base Thickness Optimizing Average Time | 430.69 | 364.33 | 370.78 | 390.70 | 376.42 | 393.77 | 458.86 | 462.39 | 437.91 |
| 🟨 CA Base Thickness | 14.85 | 11.39 | 11.93 | 12.88 | 10.79 | 14.47 | 11.27 | 11.24 | 10.94 |

Figure 18. The comparison between mean of analysis and optimization time of discussed vaults using FEM software and CA model



| | Catenary arch | Equilateral arch | Four centered pointed arch | four-centered arch | Lancet arch | Ogee arch | Optuse angle arch | Semicircular arch | Tudor arch |
|---|---|---|---|---|---|---|---|---|---|
| Rule 1 | 656109 | 373117 | 876768 | 410987 | 529065 | 113849 | 841167 | 890532 | 904921 |
| Rule 2 | 221911 | 796728 | 213483 | 297811 | 100983 | 319777 | 305918 | 23045 | 739472 |
| Rule 3 | 577270 | 599800 | 662611 | 372629 | 815540 | 491909 | 876291 | 180604 | 769379 |
| Rule 4 | 381340 | 741373 | 737762 | 593401 | 123607 | 721000 | 139919 | 938668 | 362697 |
| Rule 5 | 945562 | 283678 | 485611 | 794336 | 86185 | 352768 | 924968 | 810537 | 842074 |
| Rule 6 | 572993 | 527724 | 202306 | 61670 | 995738 | 621806 | 140593 | 206097 | 210540 |
| Rule 7 | 802182 | 34130 | 65127 | 380191 | 965979 | 584436 | 164039 | 259798 | 614877 |
| Rule 8 | 229984 | 401986 | 202647 | 56261 | 50541 | 993970 | 866263 | 25138 | 967791 |
| Rule 9 | 613147 | 50540 | 536331 | 259368 | 719480 | 355381 | 543724 | 31744 | 647866 |

Figure 19. The comparison between provided rules for discussed vault using cellular automata



| | Catenary arch | Equilateral arch | Four centered pointed | Four-centered arch | Lancet arch | Ogee arch | Optuse angle arch | Semi circular arch | Tudor arch |
|---|---|---|---|---|---|---|---|---|---|
| Base Thickness Error Percent AVG | 12.03 | 10.83 | 13.98 | 12.39 | 14.79 | 13.83 | 10.74 | 13.40 | 12.45 |
| Tensile Stress Error Percent AVG | 14.06 | 13.54 | 12.36 | 11.67 | 13.14 | 10.80 | 12.43 | 14.37 | 11.57 |
| Top Thickness Error Percent AVG | 13.23 | 11.92 | 11.75 | 10.74 | 11.18 | 11.10 | 12.89 | 14.34 | 14.46 |

Figure 20. The comparison between the mean of error percent of analysis of tensile stress and optimization of base and top thickness for discussed vault using CA model toward FEM software

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