International Journal of Advanced Structural Engineering, Vol. 3, No. 2, Pages 153-163, December 2011

Published online December 2011 at (http://journals.azad.ac.ir/IJASE)

STUDY OF DYNAMIC INFINITE ELEMENT USED FOR SOIL STRUCTURE INTERACTION

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Received 1 August 2011
Revised 18 October 2011
Accepted 29 October 2011

Starting from two-dimensional (2D) equations of motion, discretized formulations for transient behavior of soil-structure interaction problems have been derived. Two different dynamic infinite elements taking into account single and two-wave types are presented in transformed space. By coupling the infinite elements with standard finite elements, an ordinary finite element procedure is used for simulation of wave propagation in an unbounded foundation due to external forces.

Keywords: dynamic infinite element, finite element method, soil structure interaction

1. Introduction

The simulation of unbounded domains in numerical methods is a very important topic in dynamic soil-structure interaction and wave-propagation problems. Historically, unbounded media problems have been treated by finite-difference method (FDM) and finite elements together with transmitting boundaries. The finite element formulation with standard viscous boundary conditions gives approximate results since some of the wave energy is trapped in closed region. The use of dynamic infinite elements has been introduced as an alternative tool to transmitting boundaries for unbounded domain problems. All of these works are concerned about harmonic loading alone. The usual method for treating dynamic soil-structure interaction problems is to

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divide the unbounded medium into two regions: (1) near field; and (2) far field. The near field part is discretized using standard finite elements and the far field part is discritized using dynamic infinite elements. The use of coupled finite and infinite elements is in the context of standard FEM procedure.

More recently, boundary element method (BEM) has been used for the analysis of soil-structure interaction problems. However, for systems with complicated geometry and material properties, application of BEM proves to be difficult. Although BEM is suitable for unbounded soil domain problems, even in the presence of simple structure, BEM cannot be applied directly for the analysis of soil-structure interaction system alone. It is to be accompanied by FEM.

The objective of the study is to obtain the transient behavior of a soil-structure interaction system under the action of an arbitrary dynamic loading. Two-dimensional new transient infinite elements taking into account multiwave types are presented in Laplace transform domain. Solution in time domain is obtained by using an appropriate numerical inverse Laplace transform technique (Durbin 1974).

1.1. Dynamic Finite and Infinite Elements

While solving a soil-structure interaction problem numerically, finite element network is established by discretizing the solution domain into elements. This network contains some finite elements in the near field and some infinite elements in far field.

In this study, a standard eight node isoparametric, quadratic plane element is chosen as the finite element. For discritization of far field, two types of infinite elements with decay function, which can cover single or multiwave components, have been derived. The first one is an infinite element that includes a single-wave character. The second one is an infinite element that takes into account any two of the three different wave types: pressure (P); shear (S); Rayleigh (R) waves.

2. Formulation

2.1. Infinite Element with Single-wave Type

For this case, a five-node infinite element is used. One direction extends to infinity (ξ- direction, $0 \le \xi \le \infty$), while the other is finite (η-direction, $-1 \le \eta \le +1$).
The mapping relationship between real and reference elements is given as:

$$x = \sum_{i=1}^{5} M_i \, x_i \tag{1a}$$

$$y = \sum_{i=1}^{5} M_i \, y_i \quad \text{Where } M_i \text{ (i=1-5) = mapping shape functions} \tag{1b}$$

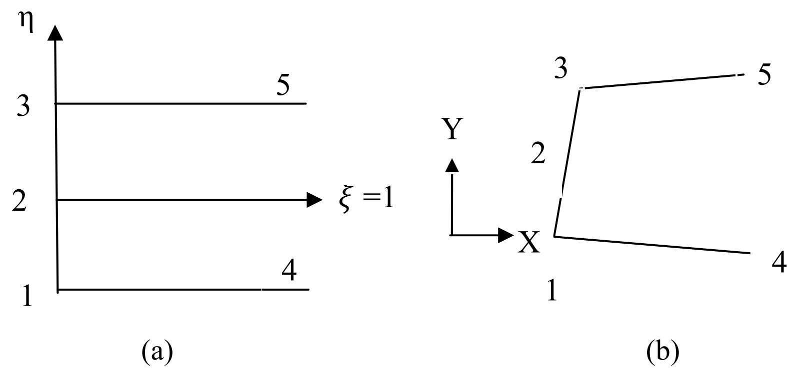

Figure 1. Mapping relationship between elements: a) Reference element; b) Real element

Where:

$$M_1 = \frac{1}{2}(1-\xi)(1-\eta)\;; \tag{2a}$$

$$M_2 = 0 \tag{2b}$$

$$M_3 = \frac{1}{2}(1-\xi)(1+\eta)\;; \tag{2c}$$

$$M_4 = \frac{1}{2}(\xi)(1-\eta); \tag{2d}$$

$$M_5 = \frac{1}{2}(\xi)(1+\eta) \tag{2e}$$

In the Laplace transform domain the displacement field for this dynamic infinite element can be written as:

$$\bar{u} = \sum_{i=1}^{3} \bar{N_i}\,\bar{u_i}\;,\;\; \bar{v} = \sum_{i=1}^{3} \bar{N_i}\,\bar{v_i} \tag{3}$$

Where N_i (i=1-3) = displacement shape functions of this infinite element expressed as:

$$\overline{N_1}(\xi,\eta,s) = P(\xi,s)\left[\frac{1}{2}\eta(\eta-1)\right] \tag{4a}$$

$$\overline{N_2}(\xi,\eta,s) = P(\xi,s)[(1-\eta^2)] \tag{4b}$$

$$\overline{N_3}(\xi,\eta,s) = P(\xi,s)\left[\frac{1}{2}\eta(\eta+1)\right] \tag{4c}$$

Where $P(\xi,s)$ = wave propagation function of the element

$$P(\xi,s) = exp\left[-\left(\alpha + \frac{s}{c}L\right)\xi\right] \tag{5}$$

Where α = decay coefficient; s= Laplace transform parameter; c= one of the wave velocities (cp, cs or cr); L= distance between Nodes 1 and 4(or 3 and 5).

2.2. Infinite Element with Two-wave Type

$$\bar{u} = \sum_{i=1}^{5} \bar{N}_i \, \bar{u}_i \; ; \; \bar{v} = \sum_{i=1}^{5} \bar{N}_i \, \bar{v}_i \tag{6}$$

Where N_i (i=1-5) = displacement shape functions of this infinite element expressed as:

$$\overline{N_1}(\xi,\eta,s) = P(\xi,s)\left[\frac{1}{2}\eta(\eta-1)\right] \tag{7a}$$

$$\overline{N_2}(\xi,\eta,s) = P(\xi,s)[(1-\eta^2)] \tag{7b}$$

$$\overline{N_3}(\xi,\eta,s) = P(\xi,s)\left[\frac{1}{2}\eta(\eta+1)\right] \tag{7c}$$

$$\overline{N_4}(\xi,\eta,s) = P(\xi,s)\left[\frac{1}{2}(1-\eta)\right] \tag{7d}$$

$$\overline{N_5}(\xi,\eta,s) = P(\xi,s)\left[\frac{1}{2}(1+\eta)\right] \tag{7e}$$

Where $P(\xi,s)$ = wave propagation function of the element

$$P(\xi,s) = \sum_{i=1}^{2} a_i \, exp\left[-\left(\alpha + \frac{s}{c_i}L\right)\xi\right] \tag{8}$$

Where

α = decay coefficient; s= Laplace transform parameter; s_k = a + i.(2πk/T) ; c_i= wave velocities (cp, cs or cr); a_i= undetermined constants; L= distance between Nodes 1 and 4(or 3 and 5); a_i is determined by considering eq.6. Equating nodal displacements on any infinite side of element to displacements expressed by eq. 6.For this purpose considering one side of element with nodes 1 and 4 (or 3 and 5) the following relationships are obtained:

$$\begin{Bmatrix} P_1 \\ P_4 \end{Bmatrix} = \begin{bmatrix} 1 & 1 \\ exp[-(\alpha+\beta_1)\xi] & exp[-(\alpha+\beta_2)\xi] \end{bmatrix} \begin{Bmatrix} a_1 \\ a_2 \end{Bmatrix} = [A] \begin{Bmatrix} a_1 \\ a_2 \end{Bmatrix} \tag{9}$$

and the solution of this Equation gives

$$\begin{Bmatrix} a_1 \\ a_2 \end{Bmatrix} = [E] \begin{Bmatrix} P_1 \\ P_4 \end{Bmatrix} ; [E] = [A]^{-1} \tag{10}$$

Case 1

For node 1 (ξ=0) to satisfy (1), P_1 and P_4 must be equal to 1 and 0.

$$\begin{Bmatrix} a_1 \\ a_2 \end{Bmatrix} = \begin{bmatrix} E_{11} & E_{12} \\ E_{21} & E_{22} \end{bmatrix} \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \Longrightarrow \quad \begin{matrix} a_1 \\ a_2 \end{matrix} = \begin{matrix} E_{11} \\ E_{21} \end{matrix} \tag{11}$$

Case 2

For node 4 (ξ=1) to satisfy (1), P_1 and P_4 must be equal to 0 and 1.

$$\begin{Bmatrix} a_1 \\ a_2 \end{Bmatrix} = \begin{bmatrix} E_{11} & E_{12} \\ E_{21} & E_{22} \end{bmatrix} \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} \quad \Longrightarrow \quad \begin{matrix} a_1 \\ a_2 \end{matrix} = \begin{matrix} E_{12} \\ E_{22} \end{matrix} \tag{12}$$

By substituting these values of shape functions, Equation for displacements u and v can be obtained. For obtaining stiffness matrix, the [B] is obtained by differentiating the shape functions containing the decay functions.

$$\begin{aligned}
N1 &= \frac{e^{-a*L/C2} * e^{(-\xi*(\alpha+(a*\frac{L}{C1}))} - e^{-a*L/C1} * e^{(-\xi*(\alpha+(a*\frac{L}{C2}))}}{e^{-a*L/C2} - e^{-a*L/C1}} * 0.5\eta(\eta-1) \\
N2 &= \frac{e^{-a*L/C2} * e^{(-\xi*(\alpha+(a*\frac{L}{C1}))} - e^{-a*L/C1} * e^{(-\xi*(\alpha+(a*\frac{L}{C2}))}}{e^{-a*L/C2} - e^{-a*L/C1}} * (1-\eta*\eta) \\
N3 &= \frac{e^{-a*L/C2} * e^{(-\xi*(\alpha+(a*\frac{L}{C1}))} - e^{-a*L/C1} * e^{(-\xi*(\alpha+(a*\frac{L}{C2}))}}{e^{-a*L/C2} - e^{-a*L/C1}} * 0.5\eta(\eta+1) \\
N4 &= \frac{-e^{\alpha} * (-e^{(-\xi*(\alpha+(a*\frac{L}{C1}))} + e^{(-\xi*(\alpha+(a*\frac{L}{C2}))})}{e^{-a*L/C2} - e^{-a*L/C1}} * 0.5(1-\eta) \\
N5 &= \frac{e^{\alpha} * (-e^{(-\xi*(\alpha+(a*\frac{L}{C1}))} + e^{(-\xi*(\alpha+(a*\frac{L}{C2}))})}{e^{-a*L/C2} - e^{-a*L/C1}} * 0.5\eta(\eta+1)
\end{aligned} \tag{13}$$

Derivative of shape functions (Equation 13) are given as follows. The variable a = 6/T.

$$\begin{aligned}
\frac{\delta N1}{\delta\xi} &= 0.5*\eta*(\eta-1)*\frac{e^{-a*L/C2*e^{(-\xi*(\alpha+(-a*\frac{L}{C1}))}*-\alpha+(a*\frac{L}{C1})} - e^{-a*L/C1*e^{(-\xi*(\alpha+(-a*\frac{L}{C2}))}*-\alpha+(a*\frac{L}{C2})}}{e^{-a*L/C2} - e^{-a*L/C1}} \\
\frac{\delta N1}{\delta\eta} &= 0.5*(2\eta-1)*\frac{e^{-a*L/C2*e^{(-\xi*(\alpha+(-a*\frac{L}{C1}))}} - e^{-a*L/C1*e^{(-\xi*(\alpha+(-a*\frac{L}{C2}))}}}{e^{-a*L/C2} - e^{-a*L/C1}} \\
\frac{\delta N2}{\delta\eta} &= -(2\eta)*\frac{e^{-a*L/C2*e^{(-\xi*(\alpha+(-a*\frac{L}{C1}))}} - e^{-a*L/C1*e^{(-\xi*(\alpha+(-a*\frac{L}{C2}))}}}{e^{-a*L/C2} - e^{-a*L/C1}} \\
\frac{\delta N2}{\delta\xi} &= (1-\eta^2)*\frac{e^{-a*L/C2*e^{(-\xi*(\alpha+(-a*\frac{L}{C1}))}*-\alpha+(a*\frac{L}{C1})} - e^{-a*L/C1*e^{(-\xi*(\alpha+(-a*\frac{L}{C2}))}*-\alpha+(a*\frac{L}{C2})}}{e^{-a*L/C2} - e^{-a*L/C1}} \\
\frac{\delta N3}{\delta\eta} &= 0.5*(2\eta+1)*\frac{e^{-a*L/C2*e^{(-\xi*(\alpha+(-a*\frac{L}{C1}))}} - e^{-a*L/C1*e^{(-\xi*(\alpha+(-a*\frac{L}{C2}))}}}{e^{-a*L/C2} - e^{-a*L/C1}}
\end{aligned} \tag{14}$$

$$\frac{\delta N3}{\delta \xi} = 0.5 * \eta * (\eta + 1) * \frac{e^{-a*L}/_{C2*e^{(-\xi*(\alpha+\left(-a*\frac{L}{C1}\right))}}*-\alpha+\left(a*\frac{L}{C1}\right)-e^{-a*L}/_{C1*e^{(-\xi*(\alpha+\left(-a*\frac{L}{C2}\right))}}*-\alpha+\left(a*\frac{L}{C2}\right)}{e^{-a*L}/_{C2-e^{-a*L}/_{C1}}}$$

$$\frac{\delta N4}{\delta \xi} = 0.5(1 - \eta) * \frac{-e^{\alpha}*(-e^{(-\xi*(\alpha+\left(a*\frac{L}{C1}\right))}*-\alpha+\left(a*\frac{L}{C1}\right)+e^{(-\xi*(\alpha+\left(a*\frac{L}{C2}\right))}*-\alpha+\left(a*\frac{L}{C2}\right))}{e^{-a*L}/_{C2-e^{-a*L}/_{C1}}}$$

$$\frac{\delta N4}{\delta \eta} = -0.5 * \frac{-e^{\alpha}*(-e^{(-\xi*(\alpha+\left(a*\frac{L}{C1}\right))}+e^{(-\xi*(\alpha+\left(a*\frac{L}{C2}\right))})}{e^{-a*L}/_{C2-e^{-a*L}/_{C1}}} \tag{14}$$

$$\frac{\delta N5}{\delta \eta} = 0.5 * \frac{-e^{\alpha}*(-e^{(-\xi*(\alpha+\left(a*\frac{L}{C1}\right))}+e^{(-\xi*(\alpha+\left(a*\frac{L}{C2}\right))})}{e^{-a*L}/_{C2-e^{-a*L}/_{C1}}}$$

$$\frac{\delta N5}{\delta \xi} = 0.5(1 + \eta) * \frac{-e^{\alpha} * (-e^{(-\xi*(\alpha+\left(a*\frac{L}{C1}\right))} * -\alpha + \left(a * \frac{L}{C1}\right) + e^{(-\xi*(\alpha+\left(a*\frac{L}{C2}\right))} * -\alpha + \left(a * \frac{L}{C2}\right))}{e^{-a*L}/_{C2} - e^{-a*L}/_{C1}}$$

$$[k] = h_e \int_{\eta=-1}^{+1} \int_{t=0}^{\infty} [\bar{B}]^T \, [D][\bar{B}] det[J] d\, \xi d\eta \tag{15}$$

h_e = thickness $det[J]$ is obtained from the mapping relationship.

Based on the above formulation, two cases were analyzed. A ramp load applied on the soil model and a soil frame structure, where the frame is carrying a transient loading.

3. Analysis of 2D Soil Model with Ramp and Triangular Load

The B-spline impulse response analysis concept as developed by Rizos is shown in Figure 2. A unit B-spline impulse of the form is applied to the elastodynamic system and the time history of the response is computed by the BEM. This response is a unique characteristic of the elastodynamic system and represents the B-spline impulse response function (BIRF) in a discrete form that is independent of the type of external excitation. In addition, the BIRF functions are only calculated for the degrees of freedom, M, that is expected to be excited by an external force at any time during the response of the system to arbitrary excitations.

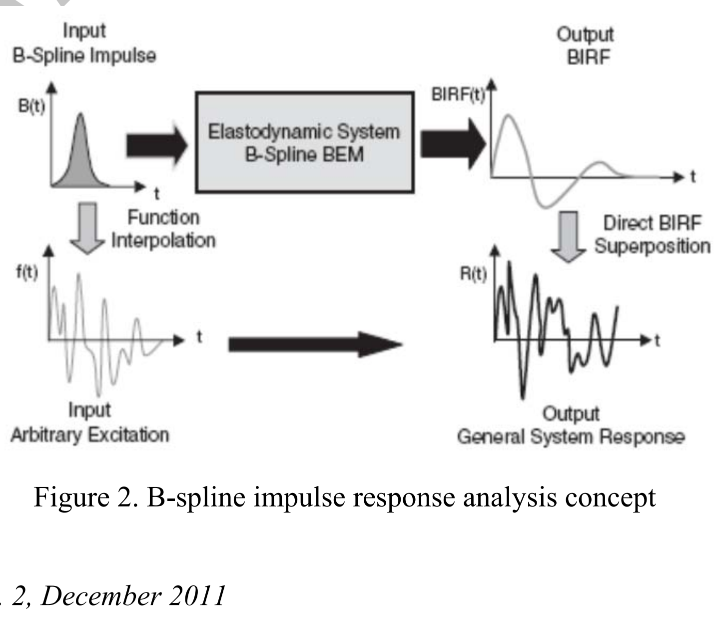

Figure 2. B-spline impulse response analysis concept

The foundation BIRF functions are calculated for a massless, rigid, square foundation, of side w = 3.048 m (120 in.), resting on a homogeneous elastic half-space. This system is considered as a reference system. The properties of the half space are shown in Table 1.

Table 1. Input data for BEM rigorous solution

Property	Value SI (USA)
Square foundation side, w_r	3.048 m
Lame's λ	$4.57x10^{+07}$kPa
Shear modulus, G	$2.286x10^{+07}$ kPa
Density, ρ	3.017 kg/m^3
B-spline support, Δt_r	0.00010 s
Time step, δt_r	0.000025 s
Pressure wave velocity, v_p	5507.83 m/s
Shear wave velocity, vs_r	2753.92 m/s

The boundary of the half space is discretized into 8-node boundary elements as shown in Figure 3. The foundation is assumed to remain always in contact with the soil and the rigid surface boundary element introduced by Rizos is adopted in this work. The motion of the rigid foundation is expressed by the 3 translations and 3 rotations

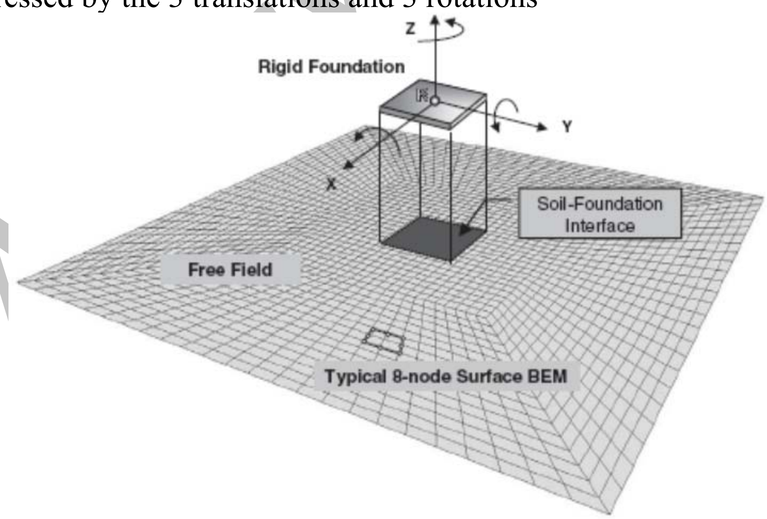

Figure 3. Discretized free field and contact (Interface) surface with 8 node boundary elements

of a reference node, R, at the foundation centre, as shown in Figure (3). Each of the six degrees of freedom (DOF) of the discrete soil–foundation system is excited by a 4th order B-spline impulse, of duration Δ=1x10^{-4} s, and the associated time histories of the response of all DOFs are computed using the BEM method at discrete times t_n=nΔt/4.

Considering the parameters mentioned in table 1, the following plots for vertical displacement using time domain analysis of a system subjected to a step impulse with finite rise time and triangular load pulse are given:

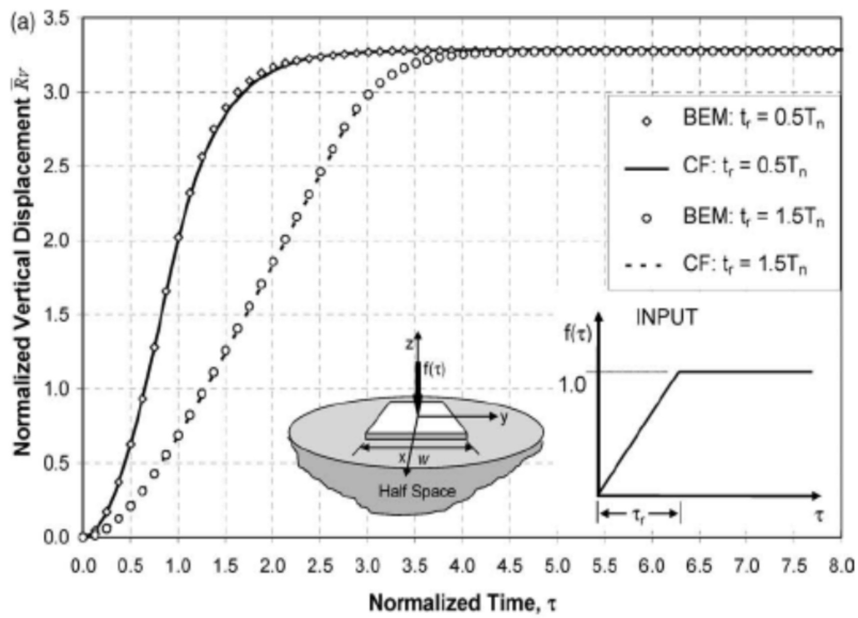

Figure 4. Time domain analysis of a system subjected to a step impulse with finite rise time

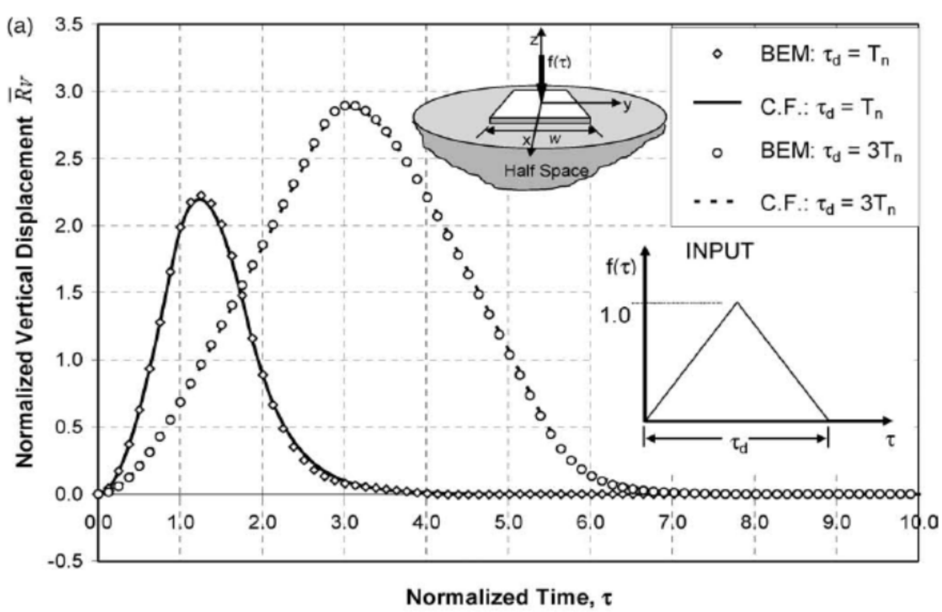

Figure 5. Time domain analysis for a system subjected to a triangular load pulse

4. Study Problems

A soil model with ramp loading, as shown in Figure (6), is analyzed using FEPro (computer program). Vertical displacement at a point within the soil is noted. In order to validate the nature of vertical displacement, the results of the problem (mentioned in Figure 6, 7), are compared with the results of aforesaid BIRF problem (Figure 6, 4).

Then 2D soil model having infinite element with two wave type is analyzed. To simulate the effect of foundation, point loads are applied at few points as shown in Figure (6). Later on parameters are considered for the analysis of this 2D model in FEPro.

E_{soil} (modulus of elasticity of soil) = 2x10^6 kN/m^2; ν (Poisson's ratio) = 1/3 ; ρ_{soil}(density of soil) = 3 kg/m^3 ; wave velocity = 2754 m/sec; point loads = 100 KN.

Table 2. Time history input for ramp load

Time (sec)	P_x(kN)	P_y(kN)	M_z(kN-m)
0.5	0	100	0
1	0	500	0
1.5	0	900	0
2	0	1000	0
2.5	0	1000	0
3	0	1000	0
3.5	0	1000	0
4	0	1000	0
4.5	0	1000	0
5	0	1000	0

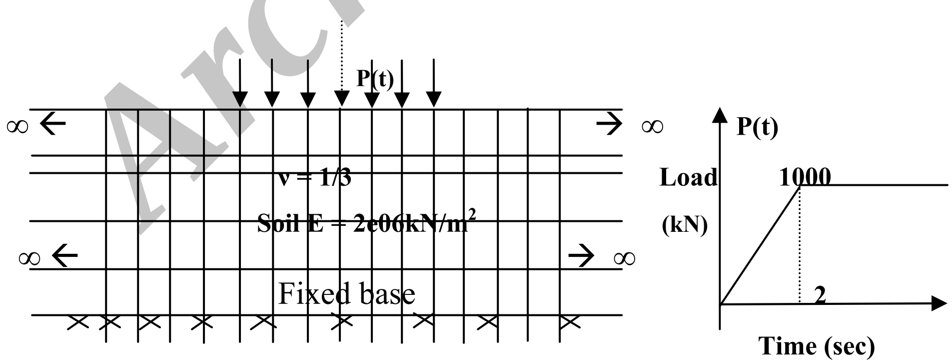

Figure 6. 2D soil model with ramp loading

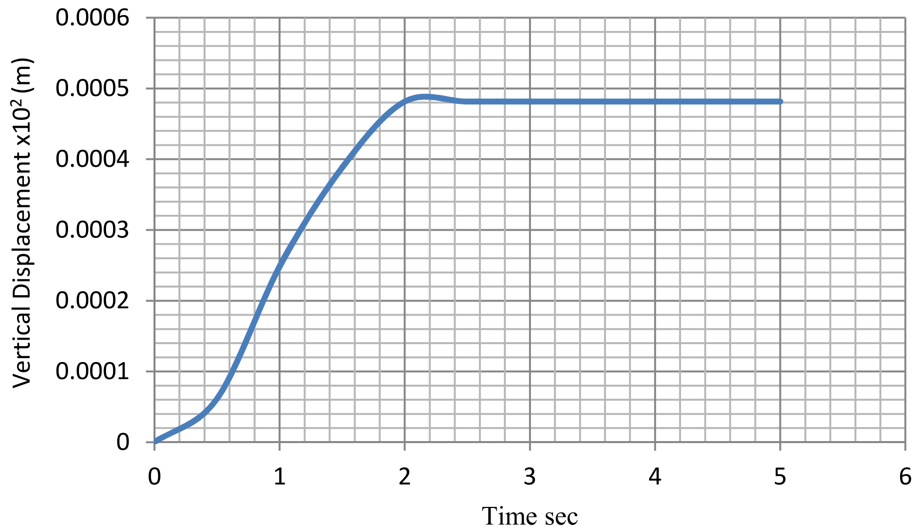

Figure 7. Vertical displacement plot for ramp loading

Keeping all other parameters the same, the following (Table 3) input time history is used to analyze 2D soil model for a particular triangular loading.

Table 3. Time history input for triangular load

Time (sec)	P_x(kN)	P_y(kN)	M_z(kN-m)
1	0	100	0
2	0	500	0
3	0	1000	0
4	0	500	0
5	0	100	0
6	0	0	0
7	0	0	0
8	0	0	0
9	0	0	0
10	0	0	0

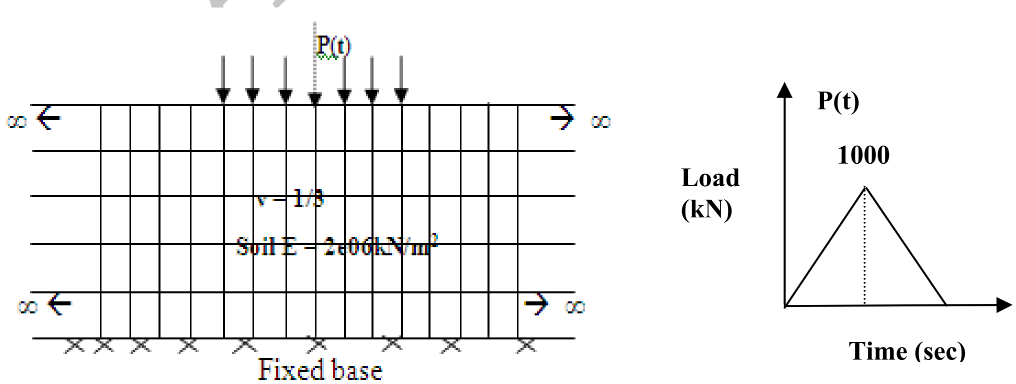

Figure 8. Soil model with triangular loading

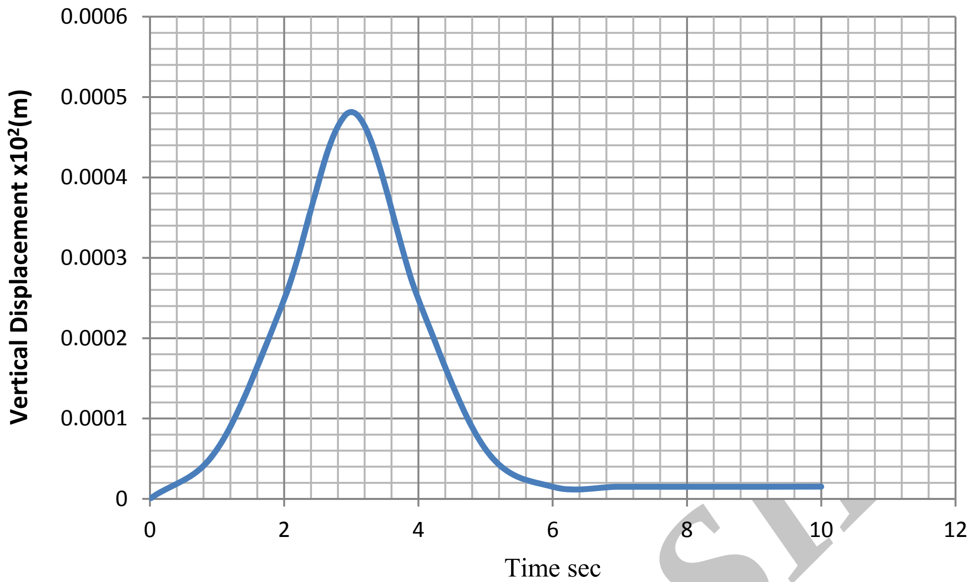

Figure 9. Vertical displacement plot for triangular loading

5. Conclusions

From Figures 4, 5, 7 and 9, it can be concluded that the nature of the vertical displacement of nodes, where transient load is applied, is the same. Variation of peak values is mainly due to different values of loading. This concept can be extended to analyze the transient response of soil-structure interaction problems.

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