Comparison of Different Hydraulic Fracture Growth Models Based on a Carbonate Reservoir in Iran

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Abstract

There has been little interest in the application of hydraulic fracture treatment in Iranian oil fields, thanks to the primarily suitable production rates of the vast oil fields. In this paper, hydraulic fracturing treatment was simulated by different models for a carbonate reservoir in the southwest of Iran. Suitable pay zones were nominated based on the lithology, water-oil saturation, geomechanical properties, and finally in-situ stress conditions – with the optimum option chosen based on a pseudo three-dimensional (P3D) model. In this work, modeling with P3D, finite different method (FDM), and the methods proposed by Perkins, Kern, and Nordgren (PKN) and Khristianovic, Geertsma, and de Klerk (KGD) were performed in order to determine and compare fracture growth geometrical aspects and the required pressure. Comparison of the above-mentioned models confirmed that P3D and FDM provides more reasonable results, while neither of PKN and KGD models was suitable for such a complex condition. Eventually, sensitivity analysis of input data, such as in-situ stress, injection rates, and reservoir geomechanical properties, was performed to evaluate the variation influence of these factors on fracture growth aspects, such as required pressures and geometrical specifications. The results showed that successful hydraulic fracturing treatment not only depended on the controllable parameters like fluid and proppant specifications, but also uncontrollable parameters such as reservoir properties and in-situ stress had to be taken into account. This study can help to select the optimum model in future hydraulic fracture design and implement it in carbonate reservoirs with similar conditions.

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1. Introduction

Since the first fracturing experiment performed in 1947, 2.5 million fractures have been pumped worldwide [1]. Many hydraulic fracturing operations have been performed in carbonate reservoirs and numerous investigations have been done to determine pressure and geometry of hydraulic fracturing [2]. However, the high production rate of the naturally fractured carbonate reservoirs in Iran has led to neglecting such studies in this country [3].

Hydraulic fracture modeling is a complex nonlinear mathematical problem that involves the mechanical interaction of the propagating fracture with the injected slurry. Several assumptions are commonly made to simulate propagation [4]: plane fractures symmetric with respect to the wellbore, elastic formation, linear fracture mechanics for fracture propagation predictions, power law behavior of fracturing fluids or slurries, and simplification of fracture geometry and its representation by few geometric parameters [5]. Hydraulic fracture modeling can be categorized in three-dimensional (3D), pseudo three-dimensional, and two-dimensional models. In the two-dimensional models, assuming a constant height of cracks and lack of fluid flow in the vertical direction, the crack propagates only in the longitudinal direction. On the other hand, pseudo three-dimension (P3D) models consider the vertical growth of crack, but ignore vertical fluid flow. In addition, by considering a single vertical cross-section, which shows the maximum height of crack, or a discrete number of elastically independent vertical cross sections or cells, the three-dimensional response can be estimated. By making three-dimension models and lateral expansion of the crack, the limitations are removed. In this regard, the three-dimensional and two-dimensional models have respectively the most and the least complexity compared with others [6].

Hydraulic fracture modeling is performed for many different purposes. Many studies have been conducted on different hydraulic fracturing models by comparing many of the available simulators to provide some guidance [5]. Aguilar-Razo used 3D fracture model to study the fracture initiation pressure and proppant distribution in a carbonate gas field. In this analysis, a radial multiple fracture geometry was defined and validated through reservoir simulation. The combined effects of perforation phase, the length of the perforated interval, tortuosity, high leakoff, and multiple fractures caused the premature screenout [7]. Also, modeling was conducted in an oil-producing fractured carbonate reservoir in Saudi Arabia to assess the impact of infusion rate on the behavior of hydraulic fracturing [8]. Based on the results of this work, it was concluded that fracture dimensions increased in general with increase in injection rates. The hydraulic fracture was modeled for carbonate reservoir to verify the fracture non-proliferation in the unwanted zones and the necessary input data required to guarantee a more accurate estimation of the mechanical properties and stress for fracture design were outlined [9]. Hashemi et al. applied FracPORO software to model the effect of different input parameters on the fracture half-length, which has a direct relation with zone productivity [10]. Zhang et al. used P3D fracture extended model in the coal seam [11]. This paper reported a novel numerical method that incorporated fracture mechanics principles and the FRANC3D and ANSYS numerical tools to investigate the three-dimensional initiation and propagation behavior of hydro-fracturing cracks in shale rock [12].

Different modeling approaches have recently been developed to simulate complex fracture networks in naturally fractured formations [13]. In this study, PKN, KGD, P3D, and FDM models are used for such simulation. Although PKN and KGD are two-dimensional (2D) models with high simplifications, they are most popular among the models used in the petroleum industry [14]. These models are used when layering of the reservoir is not complex and the possibility of extending fractures in the upper and lower layers is not a problem since height in these models is fixed. The PKN geometry is normally used when the fracture length is much greater than the fracture height, while the KGD geometry is used if fracture height is more than the fracture length [15]. To illustrate how certain variables affect fracture propagation, Eqs. 1-3 present PKN and Eqs. 4-6 present the KGD fracture geometry and pressure assumptions [16].

$$L \propto \left[\frac{EQ^{-n'}V^{2n'+2}}{(1-\upsilon^2)k'H^{n'+2}} \right]^{\frac{1}{2n'+3}} \tag{1}$$

$$w \propto \left[\frac{(1-\upsilon^2)k'Q^{n'}V}{EH^{n'}} \right]^{\frac{1}{2n'+3}} \tag{2}$$

$$\Delta p \propto \left[\frac{E^{2n'+2}K'Q^{n'}V}{(1-\upsilon^2)^{2n'+2}H^{3n'+3}} \right]^{\frac{1}{2n'+3}} \tag{3}$$

$$L \propto \left[\frac{EQ^{-n'}V^{2n'+2}}{(1-\upsilon^2)k'H^{n'+2}} \right]^{\frac{1}{2n'+4}} \tag{4}$$

$$w \propto \left[\frac{(1-\upsilon^2)k'Q^{n'}V}{EH^{n'+2}} \right]^{\frac{1}{2n'+4}} \tag{5}$$

$$\Delta p \propto \left[\frac{E^{n'+1}K'Q^{n'}}{(1-\upsilon^2)^{n'+1}V^{n'}} \right]^{\frac{1}{n'+2}} \tag{6}$$

In these equations L, w, H, and Δp are fracture's half length, width, half height, and net fracture pressure, respectively. Also, n', k', E, Q, V, and υ are flow behavior index, slurry consistency index, Young's modulus, total injection flow rate, fracture volume, and Poisson's ratio, respectively.

P3D model computes the fracture's height, width, and length distribution with the data for the pay zone and all the rock layers above and below the perforated interval [17]. It modifies the 2D (PKN) model by adding height variation along the fracture length and its effect on the fracture width [14].

P3D models are built on the basic assumption that the reservoir elastic properties are homogeneous and averaged over all layers containing the fracture height. Since confining stress dominates elastic properties when computing fracture width, this assumption is reasonable in many cases [14].

There are 2 categories: cell-based and lumped models [18]. In the lumped approach, the fracture geometry at each time step consists of 2 half ellipses joined at their centers in the fracture length direction. The fracture length, top tip (top half-ellipse), and bottom tip (bottom half-ellipse) are calculated at each time step. In the cell-based approach, the fracture length is sub-divided into a series of PKN-like cells, each with its own computed height.

Currently, application of the full three-dimensional model has undergone a handful of experiments and research. FDM model is a planar three-dimensional (3D) geometry fracture simulator with a fully coupled fluid/solid transport simulator. The fracture extension and deformation models are based on a formulation that expects the formation to fail in shear and be essentially coupled [19]. The model is based on a regular grid structure used for the elastic rock displacement calculations as a planar 2D finite-difference grid for the fluid flow solutions. The solution is general enough to allow modeling of multiple fracture initiation sites simultaneously and is applicable to any planar 3D geometry from perfect containment to uncontrolled height growth [5].

In this study, hydraulic fracturing is simulated in one of the Iranian carbonate oil fields. First, oil field and some physical, mechanical, and fluid reservoir properties are introduced and the possibility of hydraulic fracturing is studied. Then, acceptable zone for designing hydraulic fracture is chosen based on the lithology, water-oil saturation, geomechanical properties, and finally in-situ stress conditions – with the optimum option chosen based on a pseudo three-dimensional (P3D) model. Hydraulic fracture modeling to determine the geometry and pressure is performed in different ways. Eventually, sensitivity analysis of input data is performed to evaluate the influence of variation of parameters on fracture growth aspects.

2. Conditions of Field and Reservoir

The oil field studied in this work is located in the southwest of Iran, with the in-place oil reserve amounting to 6.5 billion barrels, from which 330 million barrels of crude oil are recoverable, accounting for 6% recovery factor. In such cases, the mechanism of natural drift provides no conductivity and energy to extract hydrocarbon from the reservoir. Therefore, due to the closure of discontinuities or reduction in the pressure, well stimulation methods such as hydraulic fracturing are required to enhance oil recovery. Well was drilled vertically in Sarvak, Kazhdumi, and Gadvan formations to the depth of 4175m for oil production. The main layers for production were Sarvak, Kazhdumi, and Gadvan [20], which belonged to Bangestan group that was the dominant

rock of the reservoir well. According to drilling reports, these formations consisted of limestone with marl and shale.

2.1. Ability to perform hydraulic fracturing

The key to any model is to have a complete and accurate data set that describes the layers of the formation to be fracture treated, plus the layers of rock above and below the zone of interest. According to reports, oil in Sarvak formation in this well is only from depth 2760 down to 3140 m. Table 1 shows some of the reservoir fluid properties. For better accuracy, Sarvak layer was differentiated based on its oil content and rock characteristics. Physical and geomechanical properties used for modeling are shown in Table 2. Porosity, elasticity modulus, Poisson's ratio, and compressive and tensile strengths were evaluated using petrophysical logs. Pore pressure data were estimated from empirical relations and calibrated by

the help of DST test. The amount of horizontal in-situ stress was evaluated using poroelastic equation and corrected by image logs. Fault regime in this study area was normal, where tectonic strains along the minimum and maximum horizontal stress directions were computed as 0.23 and 0.87, respectively.

Table 1. Some reservoir fluid properties

Property	Quantity	Test manner
Water content (volume percent)	<0.05	ASTM D-4006
API (degree)	20	ASTM D-1298
Viscosity at 40°C (c.St)	66	ASTM D-445
Compressibility (1/kPa)	5.25	In 3982 psi
	5.89	In 2985 psi

Table 2. Physical and mechanical properties of reservoir rock

Property	Quantity						
Depth (m)	2751 to 2760	2760 to 2800	2800 to 2866	2866 to 2878	2878 to 2913	2913 to 2925	2925 to 2936
Thickness (m)	9	40	66	12	35	12	11
Porosity (%)	30.1	9.7	9.7	9.5	9.6	11.0	12.0
Young's modulus (GPa)	6.5	17.6	15.9	15.3	16.0	15.1	14.6
Tensile strength (MPa)	2.2	4.4	4.0	3.9	4.0	3.9	3.8
UCS (MPa)	22.0	43.8	40.7	39.2	40.5	38.6	37.8
Pore pressure (MPa)	45.6	24.6	35.9	35.7	32.5	33.3	33.8
The minimum principal in-situ stress (MPa)	51.8	41.5	49.2	49.0	47.0	47.3	47.5
Oil level grading*	0	4	4	3	4	5	1

*Oil in the reservoir is categorized from 0 (without oil) to 5 (maximum oil)

In the last few years, reservoirs in formations such as Bangestan, Fahlian, and Arvand have gained more attention for oil production in Iran. There is low productivity index in Bangestan (Ilam and Sarvak layers) due to the weak specification of rock and field. Main properties of Bangestan group are low permeability, limited fracture distribution, and near closed fissures, which make it suitable for hydraulic fracturing treatment.

Although formation type is limestone with little marl and shale, it is a good candidate for proppant fracturing. Minimum in-situ stress in this

formation is more than 40 MPa, which is recommended for proppant fracturing [21]. Also, the permeability of reservoir is one of the most important parameters in order to predict the production rate before and after hydraulic fracturing operation and whether to use such treatment operation. The average permeability of this zone is 7.6 millidarcy that, according to the other research [22, 23], is acceptable for using proppant fracturing. Brinell hardness of chalk limestone rock is about 66 MPa, which is less than 100 and, thus, is recommended [24] for hydraulic fracturing as soft rock.

3. Selecting a Pay Zone

It is very important that the upper and lower zones resist fracture propagation and keep the fracture in the interval. The difference between the upper and lower zones in in-situ stress amounts and layer change are the most important parameters for choosing suitable HF zone. Roshanai et al. suggested in-situ stress differences up to 500 psi for this purpose [23]. The amounts and variations of horizontal in-situ stress for the whole pay zone are shown in Fig. 1. As presented in the figure, the in-situ stress noticeably increases from depth 2750 to 2760 m and seems to be a good fracturing controller to the upside. However, it is negligible for hydraulic fracturing after 2936 m because of very low oil content. The lower limit for HF is the depth 2913 m, at which fracturing will not propagate in the non-oil zone. If fractures propagate in the non-oil area, there will be proppant and fluid loss as well as increase in the operation cost.
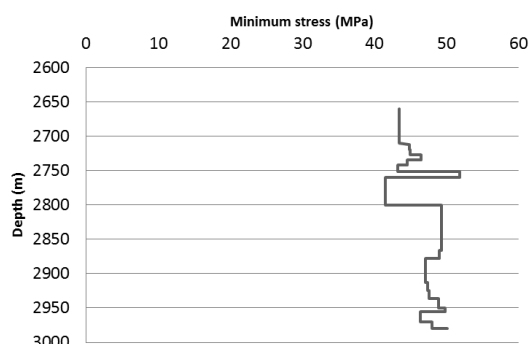

Figure 1. Variations of average minimum horizontal in-situ stress

4. HF Modeling

Formation properties and fracture requirements, such as the fluid and proppant type, the volume of proppant, and geomechanical parameters, are used to build HF simulation in PKN, KGD, P3D, and FDM models. The same fluid and proppant properties and a pumping rate of 12 m3/min are applied for all models. As the first step (with results shown in Fig. 2), the model is run using a P3D method for the depth 2760 to 2913 m. As it can be seen, the fracture propagates from depth 2760 to 2800 m and, thereafter, by increasing the fluid fracture, it is not expanded to 2800 m downside. The fracture from 2760 to 2800 m is expanded until the 10-meter thick layer at the top side of the zone was broken. Proppant and fluid start to be loosened in the non-oil zone. It is well established that for fracture interval selection, the fluid pumping schedule must be the same for different intervals. Low thickness of the top barrier does not permit fracture to expand in the ideal zone, even in the lower pumping rates.

The new zone could not be chosen from 2800 down to 2878 m, since there is a high horizontal in-situ stress, which could cause the fracture expansion to the upper depth of 2760 m, again. Thus, the depths 2760 to 2800 m and 2878 to 2913 m seem to be suitable choices. On the other hand, the depth 2878 to 2913 m is not suitable interval, because after 2913 m, the minimum horizontal in-situ stress barrier and layer change are not observed. Therefore, hydraulic fracture simulation is performed in the range of 2760 to 2800 m.
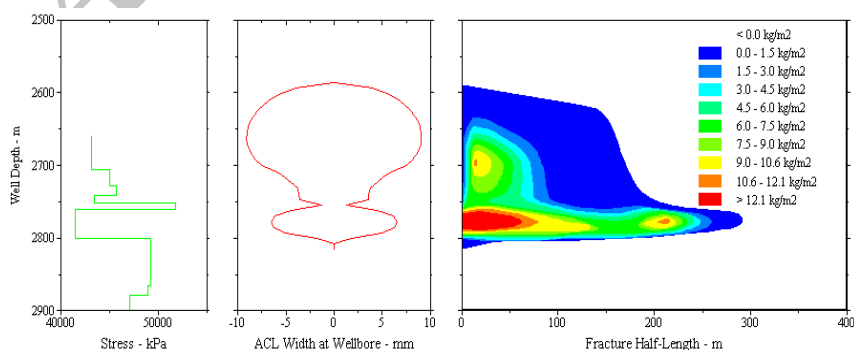

Figure 2. Geometry predicted by the P3D model for the 2760 to 2913 m interval

Less fluid volume is used for the new zone than for 2760 down to 2913 m. Fluid volume changes

such that the fracture is expanded only in the desired zone. The required pressure for the men-

tioned methods is also different. Minimum required pressure for initiating fracture growth in KGD, PKN, P3D, and FDM models are respectively 42.4, 42.7, 43.1, and 45.2 MPa. The estimated KGD and PKN pressures are presented in Eqs. 6 and 3, respectively. The estimated pressure for the P3D method is calculated based on PKN method, except that upper and lower layers are considered in the calculation. An increase in the pressure may occur due to more required stress for upper and lower layers of the selected area.

The effective pressure in FDM model is related to the total fluid pressure in the fracture, P_f, the least principal earth stress, σ_h, and the pore pressure in a permeable rock bed, P_p, by the following equation [25].

$$P = P_f - \sigma_h - P_p \qquad (7)$$

It is assumed that the plane of the fracture, which is also the plane of the numerical grid, is normal to the direction of the least principal earth stress, as shown by Hubbert and Willis [26]. The complete calculation has been explained by Barree [27].

The fracture initiation pressure (FIP) in a poroelastic permeable rock is estimated using Eq. 7 [26].

$$P = T_0 + (\sigma_H + \sigma_h) - 2(\sigma_H - \sigma_h) - \alpha P_p \qquad (8)$$

where T_0 is the tensile strength, σ_H and σ_h are the maximum and minimum horizontal stresses, respectively, α is Biots factor, and P_p is pore pressure. Using Eq. 7, the fracture initiation pressure is calculated to be 49.4MPa. The estimated value from the equation is closer to the one calculated using FDM models.

Distribution of proppant for the P3D model is shown in Fig. 3. According to this model, the fracture is expanded 1m to the upper layer and 2 m to the lower layer. The geometry of the fracture for FDM model is shown in Fig. 4, wherein a basic difference with other three models is noticed. Fracture expansion to the upper layer is at most 1 m, while it is expanded 13 m to the lower layer. This expansion, which is because of low in-situ stress in the lower layer, could be desirable since there is an oil zone down to 2800 m. Fracture expansion to the upper layer is very low and, because there is no underground water, this expansion causes no problem. PKN and KGD models are

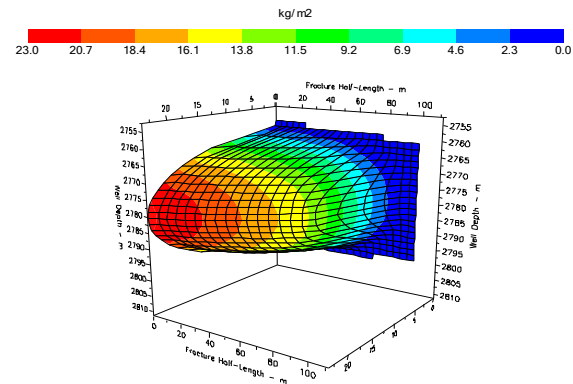2D models with fracture height as the input parameter (which is from 2760 down to 2800 m).

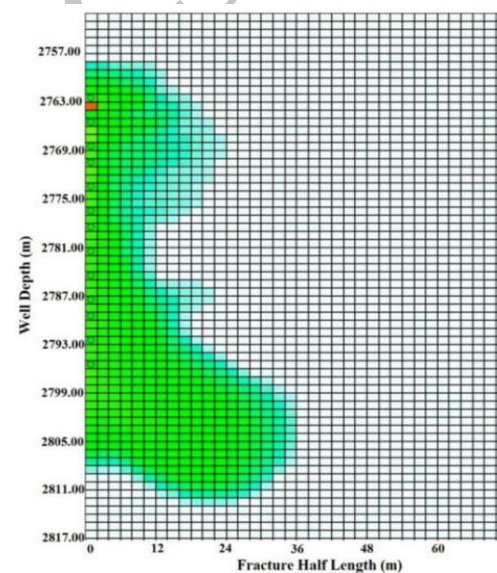**Figure 3.** Fracture proppant distribution in the P3D method

Figure 4. Fracture geometry in FDM model

Fracture length for each model is shown in Fig. 5. The length of fracture is ideally calculated in PKN, KGD, and P3D models and is firmly expanded according to the average values considered for the reservoir. Fracture expansion in FDM method depends on reservoir properties made from the created elements. The variations of fracture length and width are shown in Fig. 5 for all methods.
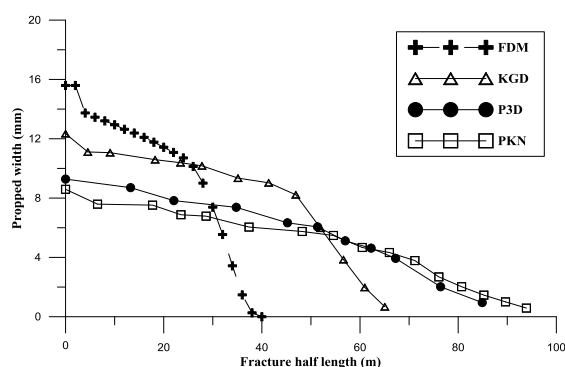
Figure 5. Fracture width and half-length in the P3D model

Fracture width inversely varies with fracture length, i.e., the fracture is wider in a model with a shorter length. Moreover, the fracture has a shorter length than usual in FDM model because of the applied greater height. The width of fracture is reduced as it gets far from the well. Moreover, the width varies inversely with the length of the fracture. Calculated width for FDM model is the maximum and, thereupon, it shows the least length. PKN and P3D models show almost the same widths and lengths for the fracture since they have common bases. The width of fracture in

KGD model has been calculated based on the length and, since fracture length is more than height, it indicates a larger width than P3D and PKN models do.

5. Sensitivity Analysis

Sensitivity study is a good approach to evaluating the effect of input data on the obtained final results. Based on this analysis, input data, such as injection rate (12 m³/min), viscosity, leak-off, permeability, the height of interval, in-situ stress, Young's modulus, and uniaxial compressive strength are examined on fracture geometry and pressure. The results of sensitivity analysis performed by P3D are shown in Table 3. As presented in the table, the minimum in-situ stress is the most important parameter that affects the fracture initiation pressure. The results also show that in-situ stress has lower effects on fracture geometry, probably due to the stress change in all directions. In fact, since it is not possible to change in-situ stress only in one direction, the ratio of in-situ stress remains constant. Compared with the minimum in-situ stress, elasticity has very low effect on fracture initiation pressure.

Table 3. Variations of the minimum horizontal in-situ stress for fracture initiation pressure and geometry

Geometry			Pressure (MPa)	In-situ stress (MPa)
Height (m)	Length (m)	Width (mm)		
43.3	79.0	10.0	42.4	41.5
42.1	79.5	10.0	52.8	51.9
41.5	79.2	10.0	63.2	62.6
41.1	79.1	10.0	73.3	72.6
40.9	79.0	10.0	84.1	83.0

Effect of parameters on fracture geometry is shown in Figs. 6-8. The vertical axis of the graph shows the variations of fracture length, width, and height while the horizontal axis presents the variation percentages of these parameters. Elasticity modulus and height of interval are the most important parameters on fracture geometry. Based on these figures, height and length of fractures considerably increase with increase in modulus of elasticity, but the changes of the width are much lower. In other words, fracture becomes thinner and wider with increase in modulus of elasticity. This result is consonant with the one reported in [28]. As shown in figures, the width and length of fracture become smaller with increase in height. Also, the effect of

height variations on the ratio of the height of fractured zone to the height of selected zone is evident. It is observed that the ratio declines with increase in the interval. This result is in agreement with investigation of [29].

By increasing the injection rate, the length of fracture decreases, while its width and height increase; this result is also in agreement with that of [8]. Varying the injection rate, leak-off, and viscosity results in no change in reservoir properties. Besides, by changing these parameters, the required energy for crack expansion remains constant. Therefore, track surface remains stable by changing these three parameters. Thus, in-

crease in the length leads to the corresponding decrease in the height and vice versa.

There is also decrease in operation time by increase in the injection rate. According to the figures, geometry changes highly with increase in viscosity, leading to a subsequent increase in the width and height of the fracture and decrease in its length at the same time. This result is consistent with that reported in [28]. Fig. 6 illustrates that fluid leak-off increase leads to decrease in the length and increase in the height and width of the fracture (Figs. 7 and 8). This behavior could be useful for choosing the suitable fluid. It is shown that more permeability results in a slight decrease in fracture height. As shown in Figs. 6 and 7, fracture length and width increase by decrease in permeability, suggesting the effect of fluid on increase in the fracture width.
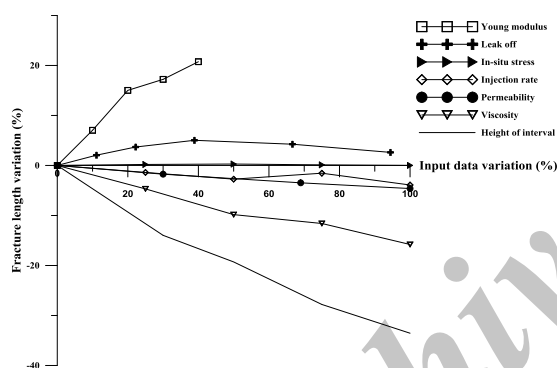

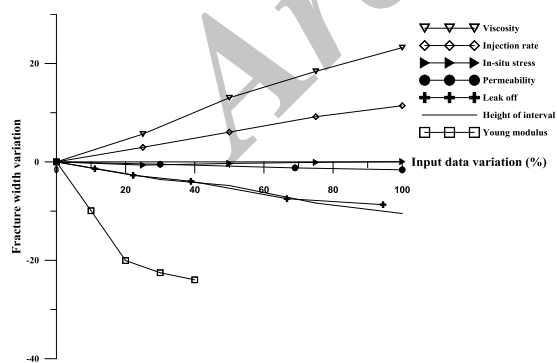**Figure 6.** Effect of various parameters on the fracture length

Figure 7. Effect of various parameters on the fracture width

6. Discussion and Conclusion

A simulation was carried out in this study in layers from Ilam, Lafan, Sarvak, and Kajdomy for-

mations in Bangestan group, which were mostly limestone; they were divided in accordance with the rock type. It was seen that there was the possibility of proppant fracturing, considering low permeability and Brinell hardness. Hence, these reservoirs were detected to be suitable for HF treatment, guaranteeing that this operation could be performed by a proper plan.
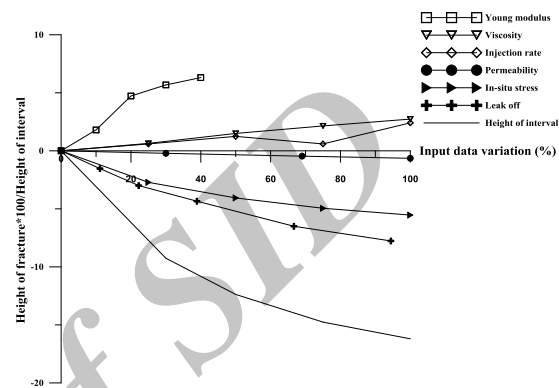

Figure 8. Effect of various parameters on the ratio of fracture height to interval height

For modeling, at first, a suitable oil zone was chosen from depth 2760 to 2913 m by considering oil content, lithology, geomechanical properties, and in-situ stress. The P3D modeling showed that this zone was not safe due to the low thickness at depth 2751 to 2760 m while in-situ stress was high. Thus, the selected pay zone was limited only to the depth 2760 down to 2800 m. FDM and P3D models had much better precision. When the height of the selected pay zone was not fixed, the height would change according to the layers and in-situ stress. Both PKN and KGD would not be useful for design in such a complex in-situ stress and layer condition. The HF simulated in PKN, P3D, KGD, and FDM models revealed that FDM model predicted higher values for pressure and width than other methods did. In addition, FDM, KGD, P3D, and PKN models overestimated width and underestimated length. PKN and P3D models showed almost the same widths and lengths for the fracture since they had common bases. Moreover, a higher pressure was estimated by the model, which overestimated the fracture width.

Subsequently, a sensitivity study of input variables was carried out to examine the effect of different field conditions. The effects of changes in reservoir geomechanical, in-situ stress, and oper-

ational parameters on the model response were evaluated accordingly. Increase in the in-situ stress and rock elastic modulus resulted in consequent rise of the required hydraulic fracture initiation pressure. Compared with the minimum in-situ stress, elasticity had very low effect on fracture initiation pressure. As other parameters increased, no considerable change was recognized in the pressure. As the injection rate and the fluid viscosity increased, a substantial increase in the fracture width was noticed. Furthermore, at higher injection rates, fracture needed less operation time. Leak-off indicated an inverse effect on these two parameters and an important effect on surface pressure. The results indicated the significance of operational parameters for optimizing HF operation. When Young's modulus increased, fracture geometry would be wider. The height of interval was another important parameter in fracture geometry. In-situ stress had very low effects on fracture geometry, probably due to the fact that in-situ stress ratio remained constant. The findings suggested that in-situ stress condition and geomechanical properties should be determined before designing HF treatment. For the future study, the next step is to model the interaction between the HF and natural fractures.

Acknowledgement

Nomenclature

E	Young's modulus (Gpa)
H	Fracture half height (m)
L	Fracture half length (m)
P	Pressure (MPa)
P_f	Total fluid pressure in the fracture (MPa)
P_p	Pore pressure (MPa)
Q	Total injection flow rate (m³/min)
T_0	Tensile strength (MPa)
V	Fracture volume (m³)
w	Fracture width (m)
Δp	Net fracture pressure (MPa)
n'	Flow behavior index
k'	Slurry consistency index

σ_H	Maximum horizontal stresse (MPa)
σ_h	Minimum horizontal stresse (MPa)
α	Biots factor
υ	Poisson's ratio

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