Metaheuristic Based Multiple Response Process Optimization

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Abstract

The simultaneous optimization of multiple responses is an important problem in the design of industrial processes in order to achieve improved quality. In this paper, we present a new metaheuristic approach including Simulated Annealing and Particle Swarm Optimization to optimize all responses simultaneously. For the purpose of illustration and comparison, the proposed approach is applied to two problems taken from the literature. The results of our study show that the proposed approach outperforms the other approaches and can find better solutions. Finally, in both cases, we present the results of a sensitivity analysis incorporating experimental design.

Keywords: Multiple response optimization, Simulated annealing, Particle swarm optimization, Desirability function

Introduction

Response Surface Methodology (RSM) has extensive applications in industrial settings. It is a collection of techniques for finding the relationship between a response (y) and input variables ($x_1, x_2, ..., x_n$). The purpose of the experimenter often is to find the optimal setting of the input variables to maximize (or minimize) the response. In RSM, the input variables are transformed into coded dimensionless variables.

A standard experimental design in RSM is the Central Composite Design (CCD), used to find the relationship between response and input variables. The various techniques used in RSM are described by Box and Draper (1987) [1], Khuri and Cornell (1996) [2] and Myers and Montgomery (2002) [3].

In some applications there is more than one process or product response. The selection of optimal settings of the input variables with simultaneous consideration of multiple responses is called a Multi Response Surface (MRS) problem. There are typically three stages in the solution of such problems: experimental design and data collection, model building and optimization. After the 1st and 2nd stages we write the model as follows:

$$y_j = f_j(x) + \varepsilon_j, \quad j = 1,2,...,m, \tag{1}$$

where y_j is j^{th} of m responses, $f_j(x)$ is a function relating the j^{th} response to the input variables and ε_j is random error.

This paper presents an approach for simultaneous optimization of all the responses in MRS problems by the use of the two metaheuristics: Simulated Annealing and Particle Swarm Optimization. The paper is organized as follows. The next section reviews current approaches to MRS problems. The third section contains our approach and the new algorithm. In the succeeding section we present two examples solved using our approach and compare our solutions with those obtained from other approaches. Conclusions are in the last section.

Main approaches to MRS problems

Given a model of each response, a basic and simple approach to MRS problems is the

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use of response contour plots, determining the optimal solution by visual inspection. However, unless both the numbers of responses and input variables are small, this method is inefficient and should not be used.

Some approaches to MRS problems aggregate all responses in a single objective form which is then optimized. Examples are the priority based approach [4], desirability functions [5] and the loss function [6].

In the priority based approach, the decision maker selects the most important of the responses as an objective function and uses the desired values of the other responses as constraints; there is no simultaneous optimization of all responses.

In the desirability function approach, all responses are transformed to a scale-free value between 0 and 1 using the desirability function d_j for the j^{th} response. The computed desirability for each response is combined to construct an overall desirability, which is then optimized.

Derringer (1994) proposed a weighted geometric mean for the overall desirability function [7]. Kim and Lin (1998, 2000 and 2002) suggested maximizing the lowest d_j, as overall desirability value of the responses [8, 9, 10]. The loss function approach attempts to minimize the costs associated with the distances of the responses from their targets namely:

$$L(y(x)) = (y(x) - T)' C (y(x) - T), \qquad (2)$$

Here $y(x)$ is the vector of responses, x is the vector of input variables, T is the target vector of the responses and C is the cost matrix containing the relative importance of each response. See Vining (1998) and Ko and Kim (2005) [11, 12].

One of the main objectives in MRS problems is robustness in product or process, reaching the specified mean with minimum variance. Chiao and Hamada (2001) propose a quality measure which is the probability that m component responses are simultaneously meeting their respective specification (S) or the proportion of conformance [13]. They proposed it to incorporate robustness into these problems. The objective function can be written

$$\max \quad p(Y \in S), \qquad (3)$$

where Y is the vector of responses and S is the specification region depending on values l_j, u_j which are the lower and upper limits of the i^{th} response

$$S = \bigcap_{j=1}^{m} (l_j, u_j). \qquad (4)$$

For the optimization stage, Del Castillo and Montgomery (1993) solved the problem by using the generalized reduced gradient (GRG) algorithm, which is available in software packages such as Microsoft Excel [14]. Del Castillo et al. (1996) used a gradient-based optimization approach by modifying the desirability function to be everywhere differentiable [15]. In a latter study Tong and Xu (2002) used a goal programming approach to find the optimal solution [16].

When the number of responses (or objectives) and constraints increase, the probability of finding a local instead of global optimum is increased and, in these cases, metaheuristic approaches can be helpful for finding the global optimum [17].

Ortiz et al. (2004) developed a multiple-response solution technique using a GA in conjunction with an unconstrained desirability function [18]. Some other recent works on multi-response optimization problems are as follows;

Tong et al. (1997) developed a multi-response signal to-noise (MRSN) ratio, which integrates the quality loss for all responses to solve the multi-response problem [19]. Tong et al. (2005) also consider the correlation of responses and use PCA and TOPSIS methods to find the best variable setting [20]. Kun-Lin Hsieh (2006) used neural networks to estimate the relationship between control variables and responses [21]. Tong, et al. (2007) use VIKOR methods in converting Taguchi criteria to single responses and then derive a regression model and the related optimal setting [22]. Kazemzadeh et al., (2008) proposed a general framework for multi-response optimization problems based on goal programming and compared some existing methods [23]. They attempted to aggregate all characteristics into one approach, including the priorities of certain types of decision makers. Bashiri and Hejazi (2009) used Multiple Attribute Decision Making (MADM) methods such as VIKOR, PROMETHEE II, ELECTRE III and TOPSIS in converting multiple responses to a single response in order to analyze data from robust experimental designs [24].

Particle Swarm Optimization

Particle Swarm Optimization (PSO) is a significant member of swarm intelligence techniques. It was proposed by Eberhart and Kennedy (1995) as an optimization method [25]. PSO is a population based search algorithm founded on the simulation of the social behavior of bees, birds or a school of fishes. Each individual within the swarm is represented by a vector in multidimensional search space. This vector has one assigned vector that determines the next movement of the particle and is called the velocity vector. The PSO algorithm determines how to update the velocity of a particle. Each particle updates its velocity based on the current velocity, the best position (*p_best*) it has explored so far and on the global best position (*g_best*) explored by the swarm [26, 27, 28]. Movement of each particle is shown in Figure 1, and it is based on equations (5), (6). Equation (5) shows that the velocity vector is updated by the global best position, personal best position and current position of each particle. Equation (6) shows that each particle moves with its own velocity.

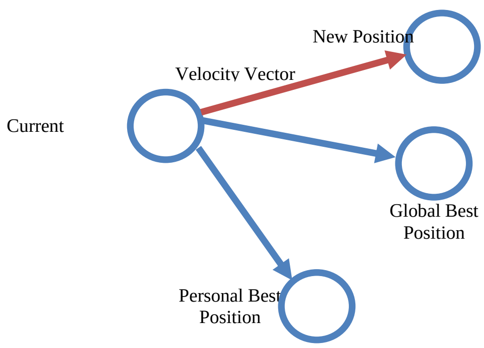

Figure 1: Individual particle movement

$$v_i(t+1) = w.v_i(t) + b_1.rand(p\_best - x_i(t)) + b_2.rand(g\_best - x_i(t)) \quad (5)$$

$$x_i(t+1) = x_i(t) + v_i(t) \quad (6)$$

Here i indexes the particle, t is the index of iteration, v_i is the vector of velocity, x_i is the position, w is the weight of the current velocity, b_1 is the weight of the difference between personal best and current position, b_2 is the weight of the difference between global best and current position and $rand$ is used for randomization.

This approach is widely used in recent research such as that of Wang and Liu (2010) who applied a chaotic PSO to assembly sequence planning [29]. Yang and Lin (2010) took advantage of PSO to solve a serial multi-echelon inventory model [30]. Lin et al. (2010) base an efficient job-shop scheduling algorithm on particle swarm optimization [31]. Our proposed metaheuristic approach for MRS problems, based on PSO, is described in the following section.

M.R.S.A & M.R.P.S.O approach

After fitting *m* response functions to the experimental data, the estimated responses are,

$$\hat{y}_j = f_j(x_1, x_2, ..., x_n), \quad j = 1, ..., m \quad . \quad (7)$$

It is necessary to aggregate all responses into a single function. In the proposed method we use a desirability function-based approach for this aggregation. To find both an optimal setting of the input variables and a robust design, we need a second criterion in our optimization process. By assuming that the response vector $Y_{(m\times1)}$ has a multivariate normal distribution with mean and variance Γ, Σ respectively, the joint probability density function of $Y_{(m\times1)}$ is:

$$f(Y;\Gamma,\Sigma) = (2\pi)^{-m/2} |\Sigma|^{-1/2} \times \exp\left[-\tfrac{1}{2}(Y-\Gamma)^T \Sigma^{-1} (Y-\Gamma)\right] . \quad (8)$$

Here $\Gamma = (\mu_1, \mu_2, ..., \mu_n)$ is the mean vector of responses and Σ is their variance-covariance matrix. Now we can compute the criterion $p(Y \in S)$ for each setting of the input variables). The larger the value of this criterion, the more robust is the design.

After computing the criterion for each setting of the input variables, we use the criterion value as a second response y_{pr}. The fitted model for this response is

$$\hat{y}_{pr} = f_{pr}(x_1, x_2, ..., x_n) . \tag{9}$$

Finally the overall function to aggregate all responses will be:

$$\min \quad ND = \left\{ \sum_{j=1}^{m+1} \gamma_j \left[(1 - d_j) \right]^p \right\}^{1/p} . \qquad x \in \Omega \ , \tag{10}$$

In (10) γ_j is the weight of the j^{th} response to be defined by the decision maker (DM) and d_j is the desirability value which can be computed from the functions below for STB (smaller the better), NTB (nominally the best) and LTB (larger the better) types of responses.

$$d_j^{STB} = \begin{cases} 0 & \hat{y}_j(\mathbf{x}) \ge u_j \\ \left[\dfrac{u_j - \hat{y}_j(\mathbf{x})}{u_j - l_j} \right] & l_j \le \hat{y}_j(\mathbf{x}) \le u_j \\ 1 & \hat{y}_j(\mathbf{x}) \le l_j \end{cases} \tag{11}$$

$$d_j^{LTB} = \begin{cases} 0 & \hat{y}_j(\mathbf{x}) \le l_j \\ \left[\dfrac{\hat{y}_j(\mathbf{x}) - l_j}{u_j - l_j} \right] & l_j \le \hat{y}_j(\mathbf{x}) \le u_j \\ 1 & \hat{y}_j(\mathbf{x}) \ge u_j \end{cases} \tag{12}$$

$$d_j^{NTB} = \begin{cases} 0 & \hat{y}_j(\mathbf{x}) \le l_j \ \ or \ \ \hat{y}_j(\mathbf{x}) \ge u_j \\ 1 - \dfrac{T_j - \hat{y}_j(\mathbf{x})}{T_j - l_j} & l_j \le \hat{y}_j(\mathbf{x}) \le T_j \\ 1 - \dfrac{\hat{y}_j(\mathbf{x}) - T_j}{u_j - T_j} & T_j \le \hat{y}_j(\mathbf{x}) \le u_j \end{cases} \tag{13}$$

Here T_j is the target value of the j^{th} response which is of NTB type, to be defined by the producer according to production requirements. Equation (10) can be written as:

$$\min \quad ND = \left\{ \sum_{j=1}^{m} \gamma_j \left[(1 - d_j) \right]^p + \gamma_{pr} \left[(1 - \hat{y}_{pr}) \right]^p \right\}^{1/p} , \qquad . \ x \in \Omega \tag{14}$$

In (14) $\hat{y}_{pr}$ is the response that is the probability of conforming to the specification region defined in (9). It is of LTB type response ($\hat{y}_{pr} = d_{pr}$), with γ_{pr} the weight of this response.

At this stage, we need to optimize (14) and find the optimal value. Our proposed approach is illustrated in Figure 2. We used *MATLAB* for the calculations.

Some characteristics of the proposed approach are:

1) It is based on the desirability function;

2) It pays attention to product or process robustness;

3) MRSA (Multiple Response Simulated Annealing) and MRPSO (Multiple Response Particle Swarm Optimization) do not guarantee that we find the optimal setting of the input variables. However, but the probability of getting trapped at a local optimum is very small;

4) The proposed approach has no restriction on the number of input or response variables. Further, additional quality criteria can be added at the model building stage.

In the next section we present the results of using MRSA & MRPSO in two examples from the literature and compare our results with other methods used for the same examples.

Comparison of Proposed Metaheuristic approach with other approaches

1. Example of GMA welding process

This example concerns Gas Metal Arc Welding (GMAW). Correia et al. (2004) study optimization of a GMA welding process [32]. They used RSM to find the optimal setting of the process combined with a Genetic Algorithm (GA) for numerical optimization. They make comparisons with two other methods.

The GMA welding process establishes an electric arc between a continuous metal filler electrode and the weld pool. The process variables are reference voltage (*T*), wire feed speed (*F*) and welding speed (*S*), which we called x_1, x_2 and x_3 in our general formulation. The response variables (*y*) are deposition efficiency (d_{exp}), penetration (p_{exp}), width (W_{exp}) and reinforcement (R_{exp}). Target values at the optimum for these responses are in Table 3.

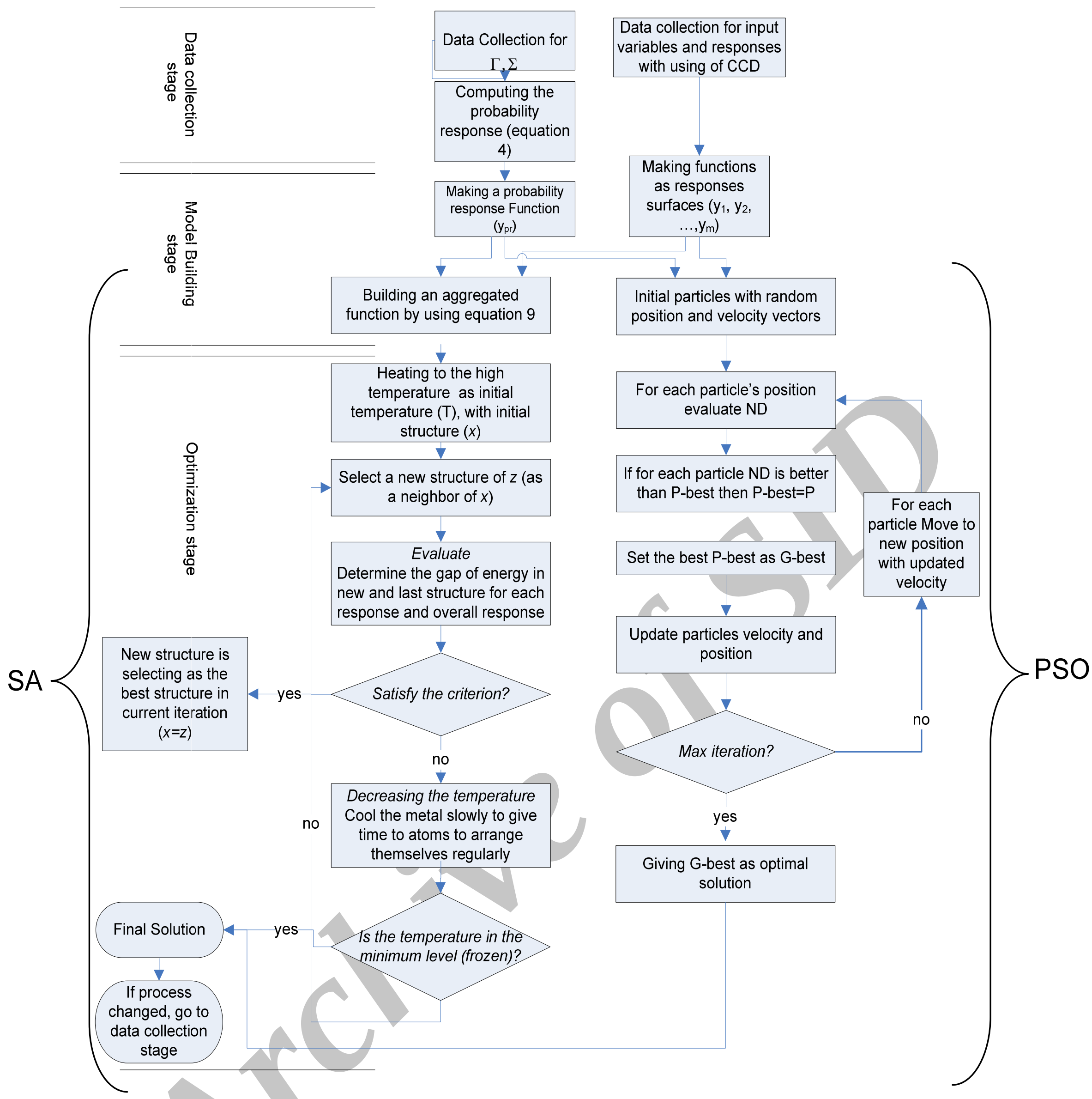

Figure 2: Overview of MRSA & MRPSO proposed algorithm in multiple response optimization

Correia et al. 2004 used a CCD (Central Composite Design), the values of the four responses from which are in Table 2 [32]. The function they used for aggregating the responses is;

$$Of\,(i) = cp\,\frac{\left(P_t - p_{\exp}(i)\right)^2}{P_t} + cd\,\frac{\left(D_t - d_{\exp}(i)\right)^2}{D_t} + cw\,\frac{\left(W_t - w_{\exp}(i)\right)^2}{W_t} + cr\,\frac{\left(R_t - r_{\exp}(i)\right)^2}{R_t},$$

where *Of(i)* is the aggregated function value for the *i*th experiment in their RSM and GA method, cp, cd, cw and cr are weights of the responses (γ_1, γ_2, γ_3 and γ_4 in our MRSA method) and P_t, D_t, W_t, R_t are target values for penetration, deposition efficiency, width and reinforcement respectively ($T_j, j = 1,\ldots,4$ in our proposed method). Also $p_{\exp}(i)$, $d_{\exp}(i)$, $w_{\exp}(i)$ and $r_{\exp}(i)$ are the predicted values for each response ($\hat{y}_j(x)$ in our method). The ranges of settings for input variables are given in Table 1. The experimental settings, as well as the values of each response, are in Table 2.

Table 1: Range of setting for input variables in welding process example

Parameter	Range
Reference Voltage ,T (V)	36.0-39.1
Wire feed speed, F (m/min)	3.9-9.7
Welding speed, S (cm/min)	50-70

Table 2: Experiments and results in welding process example

Factor			Response			
T	**F**	**S**	**Pe**	**De**	**We**	**Re**
36.8	5.4	55.0	3.5	81.8	5.1	1.2
38.4	5.4	55.0	3.8	79.6	4.7	1.3
36.8	8.3	55.0	5.1	91.3	9.1	1.8
38.4	8.3	55.0	5.3	90.4	9.1	1.6
36.8	5.4	65.0	3.0	79.4	4.8	1.0
38.4	5.4	65.0	3.1	79.6	4.6	1.5
36.8	8.3	65.0	4.5	70.9	7.0	1.3
38.4	8.3	65.0	4.5	69.6	6.8	1.1
36.0	6.9	60.0	3.5	84.2	6.7	2.0
39.1	6.9	60.0	3.5	88.3	6.2	1.8
37.7	3.9	60.0	3.1	79.7	3.8	1.0
37.7	9.7	60.0	5.5	90.8	7.8	2.0
37.7	6.9	51.6	4.3	88.0	6.1	1.0
37.7	6.9	68.4	3.8	69.0	4.1	0.8
37.7	6.9	60.0	4.0	74.3	5.5	1.3
37.7	6.9	60.0	3.9	78.0	5.6	1.3
37.7	6.9	60.0	4.0	72.1	6.2	1.2
37.7	6.9	60.0	4.0	73.3	6.3	1.2
37.7	6.9	60.0	3.8	73.5	6.2	1.3
37.7	6.9	60.0	3.4	76.3	5.9	1.5

The fitted response functions with coded variables are:

$$\hat{y}_{pe} = 3.81 + 1.35F - 0.49S - 0.26T^2 + 0.57F^2 + 0.43S^2$$

$$\hat{y}_{de} = 74.32 + 2.79F - 10.65S + 11.047T^2 + 10.10F^2 - 19.4FS$$

$$\hat{y}_{we} = 5.93 + 2.59F - 1.15S + 0.77T^2 - 1.99FS$$

$$\hat{y}_{re} = 1.28 + 0.37F - 0.196S + 0.59T^2 + 0.2F^2 - 0.58S^2 - 0.45TF - 0.49FS$$

In this study the preferences for all responses are not equal but the priorities are: 0.5, 0.3, 0.1 and 0.1 for penetration, deposition, width and reinforcement respectively. The target values and maximum and minimum values (l_j, u_j) for each response are reported in Table 3.

We solved the welding process problem by MRSA (α=0.99, T=10, T0=0.001 and p=1) and MRPSO (number of particles=40, W=4.5, B1=1.5, B2=2.5). The methods and results are shown in Table 4, which also includes a comparison with the GA and RSM methods.

If we take weights of 0.5, 0.3, 0.1 and 0.1 for the response preferences the ND (non desirability) values for RSM, GA, MRPSO and MRSA are 0.65, 0.71, 0.12 and 0.32 respectively; the MRSA and MRPSO methods outperform GA and RSM in the GMAW problem**.** Note that there is no attention to Robustness in the RSM and GA methods so that in calculating the MRSA and MRPSO for this comparison it has been assumed that $\gamma_{pr} = 0$.

Table 3 : Target values, limits and types of responses in the welding process example

	Penetration	*Deposition*	*Width*	*Reinforcement*
Target value	5.3	100%	8.5	1.5
Maximum	5.5	100	9.1	2
Minimum	3	65	4.1	0.8
Type	NTB	LTB	NTB	NTB

As described above, interest in the MRSA and MRPSO methods, is in minimizing the overall non-desirability by using of equation (10). The final results for the controllable factors (T, F and S) in MRSA are -0.2884, 0.4289 and -0.9098 and for MRPSO -0.3194, 0.6002, and -0.0648 respectively. We can see that MRSA has the better result when compared to the MRPSO and the other approaches in this example.

For the proposed MRSA and MRPSO approaches, a sensitivity analysis was made using a factorial design. Figure 3 illustrates the results of the factorial design analysis of the MRSA approach as main effects plots. It shows that the best settings for the parameters are Alpha=0.99, T=10 and T0=0.

The main effects plots from the same analysis for the MRPSO are in Figure 4. The results show that the best parameter settings for this algorithm are: number of particles=40, W=4.5, B1=1.5, B2=2.5.

Table 4: Computed optimal values for each response with four methods in welding process example

response	Target	RSM		GA		MRSA		MRPSO	
		Final value	Difference (%)	Final value	Difference (%)	Final value	Difference(%)	Final value	Difference(%)
Penetration	5.3	5.5	3.8	5.5	3.8	5.27	0.56	4.83	8.86
Deposition	100	93.8	6.2	92.2	7.8	95.56	4.44	82.21	17.79
Width	8.5	8.3	2.3	6.5	23.5	8.93	7.59	7.71	9.30
Reinforcement	1.5	2.0	33.3	2.2	46.7	1.47	2	1.75	16.66
Solution Time(s)	-	-		-		0.175130		0.250514	
ND value	0	0.6574		0.7094		0.1193		0.3208	

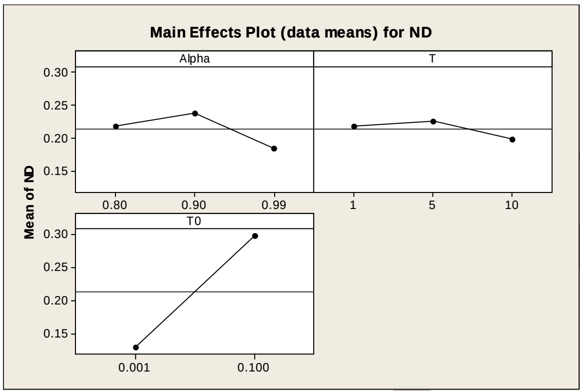

Figure 3: Main Effects plot of general Factorial Design of ND response according to the MRSA parameter settings for Example 1

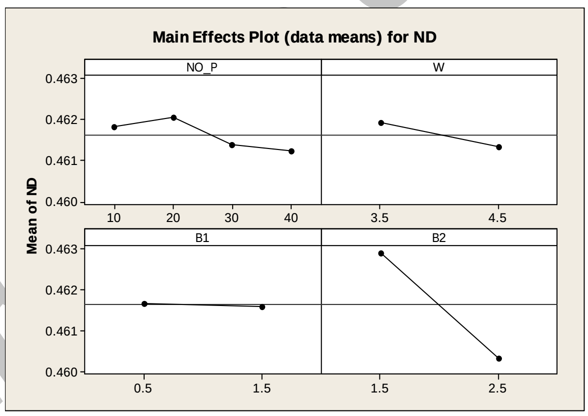

Figure 4: Main Effects plot of general Factorial Design of ND response according to the MRPSO parameter settings for Example 1

Table 5: Experimental results with 5 replications for wheel cover component example

run	x_1	x_2	x_3	x_4	x_5	x_6	x_7	replicate 1		replicate 2		replicate 3		replicate 4		replicate 5	
								y_1	y_2	y_1	y_2	y_1	y_2	y_1	y_2	y_1	y_2
1	-1	-1	-1	-1	-1	-1	-1	711.9	0.59	713.4	0.59	712.3	0.47	712.4	0.71	711.9	0.63
2	-1	-1	-1	1	1	1	1	725	0.7	720.1	0.91	711.8	1.13	723.9	0.79	720.9	0.78
3	-1	1	1	-1	-1	1	1	711.6	0.56	711.7	0.44	711.3	0.46	712.1	0.53	711.7	0.46
4	-1	1	1	1	1	-1	-1	733.7	1.5	724.1	1.55	732	1.38	732.7	1.45	733.3	1.45
5	1	-1	1	-1	1	-1	1	725.4	1.25	721.6	1.36	722.6	1.51	723.1	1.22	721.1	1.25
6	1	-1	1	1	-1	1	-1	728.1	1.17	721.1	0.97	722.9	0.98	723	0.97	719.7	0.73
7	1	1	-1	-1	1	1	-1	726.6	1.52	731.4	1.58	731.4	1.61	729.6	1.4	731.3	1.57
8	1	1	-1	1	-1	-1	1	714.3	0.57	714.4	0.51	713.6	0.44	716.3	0.44	714.6	0.56

2. Wheel Cover Component Experiment

Harper, Kosbe, and Peyton (1987) report an experiment for finding the optimum combination of injection molding parameters to minimize the imbalance of a plastic wheel cover component [33]. The purpose of the experiment was to determine the effects of seven injection molding parameters (denoted as $x_1, x_2, ..., x_7$) on the quality characteristics of the component. These are measured by the total weight (y_1 in grams) and the balance (y_2 in inch-ounces). Both y_1 and y_2 are NTB-type responses with limits of (710, 715) and (0.3, 0.4) as the specification region, respectively. The results of the experiment are in Table5.

The fitted response functions for y_1 and y_2 are:

$$\hat{y}_1 = 720.763 + 1.873\,x_1 + 5.318\,x_5 - 3.408\,x_7$$
$$\hat{y}_2 = 0.967 + 0.113\,x_1 + 0.328\,x_5 - 0.174\,x_7$$

If we assume that Y has a normal distribution with mean $\Gamma = (\mu_1, \mu_2)$ and variance-covariance matrix Σ, we can then find the probability of all responses being in their specification region for each run $p(Y_R \in S)$ where Y_R is the m dimensional response vector in the R^{th} run and S is the defined specification region.

The covariance matrix and mean responses for each run were computed from the results of Table 5 and are given in Table 6.

Table 6: computed Γ, Σ in each run for wheel cover component example.

Run	Variance-covariance	Mean
1	$\begin{pmatrix} 0.377 & -0.0018 \\ -0.0018 & 0.00752 \end{pmatrix}$	$\begin{pmatrix} 712.38 \\ 0.598 \end{pmatrix}$
2	$\begin{pmatrix} 26.923 & -0.83935 \\ -0.83935 & 0.028 \end{pmatrix}$	$\begin{pmatrix} 720.34 \\ 0.862 \end{pmatrix}$
3	$\begin{pmatrix} 0.082 & 0.00525 \\ 0.00525 & 0.0027 \end{pmatrix}$	$\begin{pmatrix} 711.68 \\ 0.49 \end{pmatrix}$
4	$\begin{pmatrix} 15.988 & -0.15945 \\ -0.15945 & 0.004030 \end{pmatrix}$	$\begin{pmatrix} 731.16 \\ 1.466 \end{pmatrix}$
5	$\begin{pmatrix} 2.803 & -0.04485 \\ -0.04485 & 0.01437 \end{pmatrix}$	$\begin{pmatrix} 722.76 \\ 1.318 \end{pmatrix}$
6	$\begin{pmatrix} 11.742 & 0.48335 \\ 0.48335 & 0.02438 \end{pmatrix}$	$\begin{pmatrix} 723.08 \\ 0.964 \end{pmatrix}$
7	$\begin{pmatrix} 4.328 & 0.07955 \\ 0.07955 & 0.00683 \end{pmatrix}$	$\begin{pmatrix} 730.06 \\ 1.536 \end{pmatrix}$
8	$\begin{pmatrix} 1.003 & -0.01645 \\ -0.01645 & 0.00393 \end{pmatrix}$	$\begin{pmatrix} 714.64 \\ 0.5040 \end{pmatrix}$

$p(Y_R \in S)$ is computed from (15). And the results are reported in Table7.

$$p(Y_R \in S) = \int_{710}^{715}\int_{0.3}^{0.4} (2\pi)^{-1} |\Sigma_R|^{-1/2} \times e^{-1/2(Y_R-\Gamma_R)'\Sigma_R(Y_R-\Gamma_R)} dy_1 dy_2 \tag{15}$$

Table 7: Computed $\hat{y}_{pr}$ for each run of the wheel cover component example.

Run	x_1	x_2	x_3	x_4	x_5	x_6	x_7	$p(Y \in S)$
1	-1	-1	-1	-1	-1	-1	-1	0.0109
2	-1	-1	-1	1	1	1	1	2.38E-51
3	-1	1	1	-1	-1	1	1	0.0415
4	-1	1	1	1	1	-1	-1	8.86E-142
5	1	-1	1	-1	1	-1	1	2.97E-25
6	1	-1	1	1	-1	1	-1	1.11E-04
7	1	1	-1	-1	1	1	-1	4.60E-44
8	1	1	-1	1	-1	-1	1	0.0205

Now $p(Y_R \in S)$ is again treated as another response variable with its value depending on the settings of the factors. So we can fit a response surface to our new probability response. For the data of Table7 the fitted response function is:

$$y_{pr} = 0.0091 - 0.0040\,x_1 + 0.0064\,x_2 + 0.0013\,x_3$$
$$- 0.004\,x_4 - 0.0091\,x_5 + 0.0013\,x_6 + 0.0064\,x_7$$

In the MRSA & MRPSO approach we have used the data to find the probability and fitted a surface to predict the probability for any setting of the control variables. Chiao and Hamada (2001), on the other hand, fitted response surfaces to the mean, standard deviation and correlation of the responses in order to compute the probability [13]. Also in our problem, finding the probability response is not our main objective; we need to optimize the probability response simultaneously with the other responses.

In this example we had two responses with separate functions to which we added another response. It is necessary to optimize all three responses simultaneously.

We used the MRSA with $\alpha = 0.99$ as a cooling ratio, T=10, T0=0.001 and the MRPSO algorithm with the number of particles=40, W=4.5, B1=0.5, B2=1.5 and p=1. For both responses $\gamma_j = 1$. The computed optimal values for input and response variables are given in Table 8. In order to find the best parameter settings for both algorithms we again used a factorial design with the parameters selected according to the results of this analysis. The main effects plots for the parameters of the algorithm are in Figures 5 and 6.

Table 8: Comparison of MRSA and Chiao&Hamada (C&H) method for wheel cover component example

	C&H method*	MRSA method	MRPSO method
ND	1.8970	1.0803	1.0002
Improvement of overall result (%)	0%	42.05%	47.27%
x_1^*	-1	-0.8479	-0.6093
x_2^*	+1	0.2079	-0.7194
x_3^*	0	-0.3373	0.8492
x_4^*	-1	-0.0764	0.0213
x_5^*	-1	-0.9976	-0.9392
x_6^*	0	0.8607	-0.2725
x_7^*	+1	0.3746	0.6242
y_1^*	710.1640	712.5929	712.4996
y_2^*	0.3520	0.4788	0.4815
y_{pr}^*	0.0390	0.0263	0.0201
Solution Time(s)	-	0.192279	0.310988

*- Chiao and Hamada, (2001)

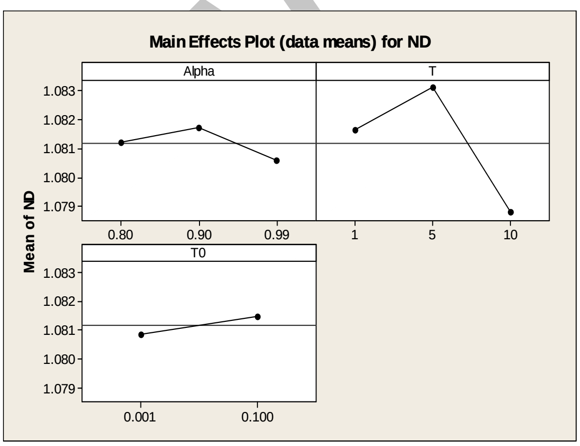

Figure 5: Effects plot of general Factorial Design of ND response according to the MRSA parameter settings in Wheel Cover Component Experiment

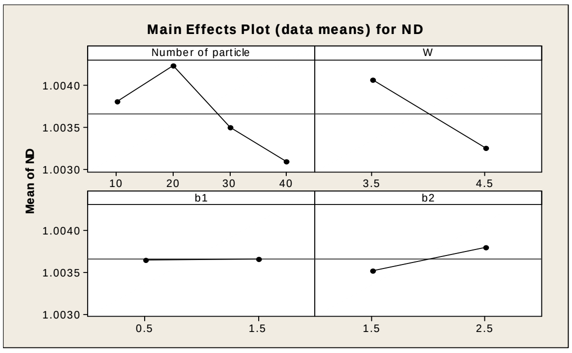

Figure 6: Effects plot of general Factorial Design of ND response according to the MRPSO parameter settings in Wheel Cover Component Experiment

From Table 8 it can be seen that the aggregated function (*ND*) of the MRSA and MRPSO methods is lower than that for the C&H method. Also, for this example, the MRPSO method has better results than the MRSA approach. The computational time for both algorithms is reasonable, with that for MRSA lower than that for MRPSO.

Conclusion

In this paper, we have proposed an approach to improve quality and to achieve robust quality by the use of the statistical design of experiments with multiple responses and metaheuristic optimization methods that we have called MRSA & MRPSO. The proposed approach uses the desirability value of all responses and also their proportion of conformance in meeting their specifications. The MRSA & MRPSO approaches we have developed are flexible in the number of input variables, number of response variables and their respective weights. The methods can easily be applied to different optimal process setting problems. The results show that the proposed algorithms have an improved objective function value and so outperform previous ones.

Future research will investigate the combination of other methods, such as the loss function approach, with the proposed method as well as the solution of MRS problems by other metaheuristic algorithms. We also hope to report in future on the use of these methods in practical examples.

References:

1- Box, G. and Draper, N. (1987) "Empirical model building and response surface." Wiley, New York.

2- Khuri, A, and Cornell, J. (1996) "Response surfaces: design and analyses." Dekker, New York.

3- Myers, R.H. and Montgomery, D.C. (2002) "Response surface methodology: process and product optimization using designed experiments." John Wiley.

4- Myers, R. and Carter, W. Jr. (1973) "Response surface techniques for dual response systems." *Technometrics*, Vol.15, PP. 301-317.

5- Derringer, G. and Suich, R. (1980) "Simultaneous optimization of several response variables." *Journal of Quality Technology*, Vol.12, PP. 214-219.

6- Pignatiello, J. (1993) "Strategies for robust multiresponse quality engineering." *IIE Transactions*, Vol.25, PP. 5-15.

7- Derringer,G. (1994) "A balancing act: optimizing a product's properties." *Quality Progress*, Vol.27 , PP. 51-57

8- Kim, K. and Lin, D. (1998) "Dual response surface optimization: a fuzzy modeling approach." *Journal of Quality Technology*, Vol. 30, PP. 1–10.

9- Kim, K. and Lin, D. (2000) "Simultaneous optimization of mechanical properties of steel by maximizing exponential desirability functions." *Journal of Royal Statistical Society-Series C*, Vol. 49, PP. 311–325.

10- Kim, K. and Lin, D. (2002) "Optimization of multiple responses considering both location and dispersion effects." Working paper, POSTECH.

11- Vining, G. (1998) "A compromise approach to multiresponse optimization." *Journal of Quality Technology*, Vol. 30, PP. 309-313.

12- Ko, Y.H., Kim, K.J. and Jun, C.H. (2005) " A new loss function-based method for multiresponse optimization." *Journal of Quality Technology*, Vol.37, PP. 50-59.

13- Chiao, C. and Hamada, M. (2001) "Analyzing experiments with correlated multiple responses." *Journal of Quality Technology*, Vol. 33, PP. 451-465.

14- Del Castillo, E. and Montgomery, D. (1993) "A nonlinear programming solution to the dual response problem." *Journal of Quality Technology*, Vol. 25, PP. 199-204.

15- Del Castillo, E., Montgomery, D., and McCarville, D. (1996) "Modified desirability functions for multiple response optimization." *Journal of Quality Technology*, Vol. 28, PP. 337-345.

16- Tong, L. C. and Xu, K. (2002) "A unified approach for dual response surface optimization." *Journal of Quality Technology*, Vol. 34, PP. 437-447.

17- Collette, Y. and Siarry, P. (2003) "Multiobjective optimization: principles and case studies." Springer.

18- Ortiz, F.,Simpson J., Pignatiello J. and Langer A. (2004) " A genetic algorithm approach to multiple-response optimization." Vol.36, PP. 432-450.

19- Tong, L-I, Su, C-T and Wang, C-H. (1997) "The optimization of multiresponse problems in Taguchi method." *Int J Qual Reliab Manage*, 14(4): PP. 367–380.

20- Tong L-I et al. (2005) "Optimization of multiple responses using principal component analysis and technique for order preference by similarity to ideal solution." *International Journal of Advanced Manufacturing Technology*, Vol. 27, PP. 407–414.

21- Kun-Lin, Hsieh (2006) "Parameter optimization of a multi-response process for lead frame manufacturing by employing artificial neural networks." *International Journal of Advanced Manufacturing Technology*, Vol.28, PP. 584–591.

22- Tong, L., Chen, C., and Wang, C. (2007) "Optimization of multi-response processes using the VIKOR method." *International Journal of Advanced Manufacturing Technology*, Vol. 31, PP. 1049-1057.

23- Kazemzadeh, R.B., Bashiri, M., Atkinson A.C. and Noorossana R. (2008) "A general framework for multiresponse optimization problems based on goal programming." *European Journal of Operational Research*, Vol. 189, PP. 421–429.

24- Bashiri, M. and Hejazi, T.H. (2009) “An extension of Multiresponse optimization in MADM view.” *Jornal of Applied Sciences*. Vol. 9, PP. 1695- 1702.

25- Eberhart R.C. and J. Kennedy. (1995) "A new optimizer using particle swarm theory." Proceedings of the Sixth International Symposium on Micromachine and Human Science, Nagoya, Japan, PP. 39–43.

26- Engelbrecht, A. P. (2002), Computational Intelligence, John Wiley and Sons, Pampara.

27- Engelbrecht A. P. (2005), Fundamentals of Computational Swarm Intelligence, Wiley, New York.

28- Sadri, J., and Ching and Y. Suen. (2006) "A genetic binary particle swarm optimization model." IEEE Congress on Evolutionary Computation, Vancouver, BC, Canada.

29- Wang, Y., and Liu, J. (2010) "Chaotic particle swarm optimization for assembly sequence planning." Robotics and Computer-Integrated Manufacturing, Vol.26, PP. 212-222.

30- Yang, M. F. and Lin, Y. (2010) "Applying the linear particle swarm optimization to a serial multi-echelon inventory model." *Expert Systems with Applications*, Vol.37 , PP. 2599-2608.

31- Lin, T. L., Horng, S. J., Kao, T. W., Chen, Y. H., Run, R. S., Chen, R. J., et al. (2010) "An efficient job-shop scheduling algorithm based on particle swarm optimization." *Expert Systems with Applications*, Vo. 37, PP. 2629-2636.

32- Correia, D.S., Goncalves, C. V., Cunha, S. and Ferraresi, V. A. (2004) "Comparison between genetic algorithms and response surface methodology in GMAW welding optimization." *Journal of the Brazilian Society of Mechanical Sciences and Engineering*, Vol.26, PP. 28-32.

33- Harper, D., Kosbe, M., and Peyton, L. (1987) "Optimization of ford taurus wheel cover balance (by design of experiments-Taguchi's method)." Fifth Symposium on Taguchi Methods, Romulus, MI: American Supplier Institute, PP. 527-539.