International Journal of Finance and Managerial Accounting, Vol.8, No.30, Summer 2023

IJFMA

Economic Capital Evaluation Using Two Approaches of Structural Models: Taking Fluctuating Asset Correlations into Account Versus Classical Merton Model

Zahra Eskandari
Ph.D. student in Financial Engineering, Department of Management, Central Tehran Branch, Islamic Azad university, Tehran, Iran
Zahra.eskandari81@gmail.com

Mirfeiz Fallah Shams
Department of Management, Central Tehran Branch, Islamic Azad University, Tehran, Iran
(Corresponding Author)
mir.fallahshams@iauctb.ac.ir

Gholamreza Zomorodian
Department of Management, Central Tehran Branch, Islamic Azad University, Tehran, Iran
gh.zomorodian@gmail.com

Submit: 14/09/2021 **Accept:** 15/01/2022

ABSTRACT

The financial crisis has become one of the most important challenges for financial institutions. To overcome this challenge, financial institutions must have an accurate estimate of the risks involved and maintain adequate capital to protect the bank. In recent years, in the international community, economic capital, as the appropriate capital to cover unexpected loss, has become a more accurate criterion for estimating the required capital to deal with risks. In this paper, we estimate economic capital of a selected bank portfolio which includes publicly traded companies using Monte Carlo simulation with two approaches of structural models. The first approach is to use the random matrix method in order to take fluctuating asset correlations into account and the second one is the classical Merton method which does not take into account the fluctuations of correlations. The results show that the bank 's risk will be significantly underestimated if the classical Merton approach is used.

Keywords:

Merton Model, Loss Distribution, Fluctuations of Covariance Matrix, Economic Capital, Monte-Carlo Simulations

1. Introduction

Significant challenges that financial and credit Institutions faced with during the recent financial crises demonstrate the importance of risk estimation that helps the institutions to manage the crises effectively. Especially, the crisis of 2007-2009 showed that risk underestimation can affect the whole world 's economy seriously. Indeed, these financial crises and bank failures, such as Bankhaus Herstatt in Germany and Franklin National Bank in the United States, were the main reasons of establishing the Basel Committee on Banking Supervision (BIS) which composed of the G10 central banks in 1974. (Baesens & Van Gestel, 2009).

In fact, a financial crisis can be the result of risk spillover as it happened during 2007-2009. Therefore, assessing and managing risk is essential and will remain of interest for preventing crisis effects. Among various risk categories, credit risk is one of the most important risks that a bank might face with and should be controlled in order to avoid significant disasters. Hence, better estimation of credit risk is vital and can guarantee business continuity of financial institution in crises. There are different approaches for assessing credit risk which most of them can be categorized in three classes; structural models, reduced form and fist passage models (Crouhy, Galai, & Mark, 2006; Duffie & Singleton, 2012).

Structural models go back to the Merton's work in 1974 about pricing the corporate debts and the basic concept of Black and Scholes model application. The Merton model assumes a zero-coupon debt. Also in this model, the asset value of the company is modeled by a stochastic process but debt value will remain fixed until maturity. Thus, it can be seen as a call option on the obligor's assets and default can occur when the market asset value falls below the debt or liability value. As a result, probability of default (PD) and loss given default (LGD) and recovery rate are directly determined by market asset value at maturity.

Reduced form models try to model the probability of default and the loss given default by using macroeconomic factors and independent stochastic process.

First passage models were introduced by Black and Cox in 1976. These models are a mixed form of the previous two models. In comparison with Merton model, in first passage models, market asset value of the company is modeled with a stochastic process and default occurs when asset value falls below the threshold at first time.

In the reduced form and first passage models, probability of default and loss given default (also recovery rate) are modeled as independent variables. However, these quantities can be connected strongly especially in crisis time. So this connection may cause underestimation of risk as consequences. In the structural models, these quantities are not separated and both of them are modeled by market asset value at maturity.

In this paper, we are going to use Merton model to calculate risk measures especially economic capital for a portfolio that included publicly- traded companies in a selected bank. To evaluate economic capital, we need the loss distribution which as mentioned before, loss quantity depends on market value of asset in Merton model. However, there are some weaknesses in Merton model. As an example, it assumes the fixed covariance matrix for asset variable during time horizon. This assumption cannot be true especially in situation that the institute suffers financial problems. To overcome this issue and to have a better estimate of credit risk measures, in this paper, we use random matrix theory to take fluctuations of covariance matrix into account and employ the result to estimate economic capital of selected portfolio. For a better view, we compare the result of Merton model with covariance matrix fluctuations and classical Merton.

2. Literature Review

Economic Capital (EC) is an important measure that can play essential role to protect banks against financial crisis. In June 2004, Basel Committee on Banking Supervision published Basel II accord included three pillars. The concept of Pillar 1 is capital requirements that a bank must hold to be protected against the financial and operational risks. This capital is also known as regulatory capital. In Pillar 2 of that accord, internal capital adequacy assessment process (ICAAP) was introduced which consider economic capital (Basel Comittee, 2004). Indeed, contrary to regulatory capital, EC and ICAAP frameworks can recognize concentration risks and diversification benefits. Furthermore, EC considers both probability of default and up and downgrades (Elizalde & Repullo, 2007).

Ong (1999) investigated EC and provided the result in a book named "Internal Credit Risk Models:

Capital Allocation and Performance Measurement". He emphasized that EC is the answer to the question "What level of capital is necessary for the bank to remain solvent in the event of such catastrophic or extreme losses". In his book estimating loss distribution was considered (Ong, 1999).

According to Ericsson and Renault, credit risk turned to an intensive research area since Merton introduced structural models in 1974 (Ericsson & Renault, 2006) and many new models were created based on the original method developed by Merton to relax some of its assumptions and eliminate its weaknesses (Altman, Brady, Resti, & Sironi, 2005). Even estimating portfolio credit risk based on conditional PDs in Basel IRB method is derived from an adaptation of Merton's (1974) single asset model (Basel Comittee, 2005).

Altman et al. (2005) showed that risk measures, introduced as IRB[1] in Basel II accord, could be applied adequately for capital requirements. Vasicek showed that under certain conditions, Merton's model can naturally be extended to a specific ASRF[2] credit portfolio model. With a view on Merton's and Vasicek's ground work, the Basel Committee decided to adopt the assumptions of a normal distribution for the systematic and idiosyncratic risk factors (Basel Comittee, 2005).

In fact, Merton model uses the principles of option pricing. The basic idea comes from fundamental accounting equation or the asset value of company, which is the sum of equity and time-independent liabilities ($V = F + E$). Altman et al. (2005) demonstrated under Merton model, market assets value follows Brownian motion process and default occurs when a company's asset value is less than its liability value. Indeed, default events happen at maturity and the equity of the company can be assumed as a European call option on the company's asset value which strike price is liability. In this model, all credit risk elements such as probability of default and recovery rate are function of the structural characteristics of the company. In other words, credit risk elements estimation depends on two factors, asset volatility and leverage which are named as "business risk" and "financial risk" measures, respectively (Altman et al., 2005).

In March 2009 Basel Committee published a document on "range of practices and issues in economic capital framework" and discussed using a variety of risk measures for economic capital purposes. In risk measures, Appropriate capital allocation depends heavily on the modeling of dependencies and also it is important for supervisors when they examine a bank's ICAAP under Pillar 2, since these dependency structures are not considered in capital requirements under pillar 1. (Basel Committee,2009)

Black and Cox (1976) and Leland (1994) discussed some weaknesses of Merton model and extended the framework of Merton in many aspects. Against Merton model, Black and Cox represented a model that allows default to happen in uncertain time when asset value falls below a certain threshold, which is not necessarily liability value. Thus, the equity could not be considered as a European call option on the asset value. Leland made another significant extension of Merton by introducing taxes and bankruptcy costs (Sundaresan, 2013).

Duan (1994) tried to develop a general methodology that uses maximum likelihood to compute parameters for an unobserved asset value process by using observed prices (Duan, 1994). Eom et al. in 2004 showed that asset value can be estimated by the observable equity values (Eom, Helwege, & Huang, 2004).

In the framework of structural models, arising from modeling equity as a call on the asset value V, the time series is assumed to have a constant volatility. Christie (1982) examined the relation between the variance of equity returns and other explanatory variables and also their non-stationary characteristics. Ronn and Verma (1986) extended his estimation to the cases of non-stationary equity variance and stochastic interest rates in structural models (Ronn &Verma,1986).

Some processes such as the (generalized) autoregressive conditional heteroscedasticity models treat the variance as a stochastic variable as well and can consider the non-stationary in time series, but their parameters lack a clear economic interpretation (Schmitt, Chetalova, Schäfer, & Guhr, 2013).

Recently some researchers have worked on Merton model in correlated markets, which can be viewed particularly in crisis period. As a consequence, Zhang et al. (2011) and Sandoval and Franca (2012) showed that the covariance and correlation matrix of asset

[1] - Internal Rating -Based approach
[2] - Asymptotic Single Factor Risk Model

values as a risk measure changes during the time. Schmitt et al. (2013) and Chetalova et al. (2015) constructed a correlation averaged multivariate distribution to describe the stock market returns distribution and address non-stationarity in correlated financial time series. They showed that financial markets are highly non–stationary systems. On the other hand, in financial markets, variances and correlation coefficients depend on the time windows in which they are estimated in. Fluctuations of the correlations leads to heavy tails distribution. They used random matrix model to take into account fluctuations of the correlations and established the correlation averaged multivariate distribution (Mühlbacher & Guhr, 2018; Schmitt, 2014; Schmitt, Chetalova, Schäfer, & Guhr, 2013).

Schmitt (2014) examined the usage of the correlation averaged multivariate distribution in Merton model to calculate credit risk measures, such as VaR or Expected Tail Loss in his dissertation and demonstrated its validity (Schmitt, 2014).

Mühlbacher & Guhr confirmed the findings of Schmitt et al. 2014 and reused the same approach to model concurrent credit portfolio losses and evaluate VaR or Expected Tail Loss for different portfolio (Mühlbacher & Guhr, 2018).

Chamizo et al. (2019) analyzed whether the credit market anticipated the financial crisis before the regulators using a methodology that combines the Merton model for the determination of economic capital with Vasicek's factor model for asset correlation (Chamizo, Fonollosa, & Novales, 2019).

Krebs and Nippel (2020) compared the traditional calculation of economic capital for credit default losses with a more comprehensive one, based on the bank's (net) profit from credit business as accounted for in the bank's P&L statement (Krebs & Nippel, 2021).

Omar and Prasanna (2021) studied some weaknesses of Merton model and extended the application of the Merton model in six emerging Asian markets to estimate corporate default risk (Omar & Prasanna, 2021).

To the best of our knowledge, no research has been done in the area of using random matrix model to consider non-stationary of time series and estimating risk measures in Iran.

3. Methodology

The Economic Capital (EC) can be interpreted as the appropriate capital to cover unexpected loss in given time horizon. The EC at a given confidence level $(1-\alpha)$ is defined as the difference between the Value-at-Risk and the expected loss:

$$EC = VaR(\alpha) - EL, \qquad (1)$$

where VaR of a portfolio with loss distribution L_p at a given time horizon and a $(1-\alpha)$ confidence level is given by $VaR(\alpha) = \min\{L|P(L_p > L) \leq (1-\alpha)\}$ and EL is the expected loss or the average loss of the portfolio (Baesens & Van Gestel, 2009).Thus to assess EC for credit risk, we need credit portfolio loss distribution.

We develop Merton model to measure EC for a portfolio that includes publicly traded companies which in this paper, we named it "credit portfolio". As discussed before, based on structural models, default occurs when asset value at maturity drops below the liability value. This model employs asset value at maturity to predict probability of default. In this case, the normalized loss is given by:

$$\mathrm{L_k} = \frac{F_k - V_k}{F_k}\,\Theta\big(F_k - V_k(T)\big), \qquad (2)$$

where F_k and V_k are the firm k liability and asset value, respectively and $\Theta(x)$ is the unit step function:

$$\Theta(x) = \begin{cases} 0 & x < 0 \\ 1 & x \geq 0 \end{cases} \qquad (3)$$

Unit step function is unity only if F_k is larger than V_k or when default occurs. The portfolio loss is the weighted sum of all firms' loss as below:

$$L = \sum_{k=1}^{K} f_k L_k \qquad f_k = \frac{F_k}{\sum_{k=1}^{K} F_k}, \qquad (4)$$

In the Merton model, it is assumed that the liability is fixed during time horizon and the value of firm V_k can be describe by a geometric Brownian motion:

$$d\mathrm{V_k}\,(\mathrm{t}) = \mu_k \mathrm{V_k}\,(\mathrm{t})\mathrm{dt} + \sigma_k \mathrm{V_k}\,(\mathrm{t})\mathrm{dW}(\mathrm{t}), \qquad (5)$$

where $\mathrm{dW}(\mathrm{t})$ is a Wiener process, μ_k and σ_k are the drift and the volatility of the asset value of firm k, respectively. So in order to achieve the loss distribution shown in expression (6) we must find the jointed distribution of the asset values $g(V|\Sigma)$ at maturity T with $V=(V_1(T),\ldots,V_{K(T)})$

$$P(L)=\int_{[0,\infty)k} d[V]g(V|\Sigma)\delta\left(L-\sum_{k=1}^{K} f_k L_k\right), \quad (6)$$

But the time series of the asset value is non-stationary and covariance matrix Σ changes during time. This fact can affect the asset distribution. To take into account non-stationarity and covariance matrices changes, we use random matrix approach and replace the covariance matrix with a random matrix:

$$\Sigma_t \rightarrow \sigma WW'\sigma, \quad (7)$$

where the elements of a $K\times N$ random matrix W is drawn from a multivariate normal distribution and then WW' follows Wishart distribution:

$$\widetilde{w}((WW'|C,N) = \sqrt{\frac{N}{2}}^{KN} \frac{\sqrt{\det WW'}^{N-K-1}}{\Gamma_K\left(\frac{N}{2}\right)\sqrt{\det C}^{N}} \exp\left(-\frac{N}{2}\mathrm{tr}W'C^{-1}W\right), \quad (8)$$

which

$$var(WW')_{kl}=\frac{C_{kl}^2+1}{N}, \quad (9)$$

where C is the average correlation matrix and N shows the strength of the fluctuations. Smaller N causes more fluctuations in covariance matrix and larger N can lead to stationary in covariance matrix. Schmitt et al (2013) showed that multivariate Gaussian distribution is a good approximation for returns distribution if the covariance matrix is fixed. So following the structure of random matrix distribution (6) and returns distribution, one can construct a correlation-averaged multivariate distribution that is taking into account fluctuations of correlations:

$$g(r|\Sigma_0,N) = \frac{1}{2^{\frac{N}{2}+1}\Gamma(\frac{N}{2})\sqrt{\det(2\pi\Sigma_0/N)}} \frac{\mathcal{K}_{\frac{K-N}{2}}(\sqrt{Nr'\Sigma_0 r})}{\sqrt{Nr'\Sigma_0 r}^{\frac{K-N}{2}}}, \quad (10)$$

where $\mathcal{K}_\nu$ is the modified Bessel function of the second kind of order ν. After performing a change of variable and using Ito' lemma

$$r_k \rightarrow \ln\left(\frac{V_k(T)}{V_0}\right)-(\mu_k-\frac{\rho_k^2}{2})T, \quad (11)$$

where μ and ρ are asset value drift and volatility respectively. Now we can achieve the portfolio loss distribution which takes into account the fluctuating correlation:

$$\langle p\rangle(L|c,N) = \frac{1}{\sqrt{2\pi}2^{N/2}\Gamma(N/2)}\int_0^\infty dz\, z^{N/2-1}e^{-z/2}\sqrt{\frac{N}{2\pi}} \times\int_{-\infty}^{+\infty} du\, \exp(-\frac{N}{2}u^2)\frac{1}{\sqrt{M_2(z,u)}}\exp(-\frac{(L-M_1(z,u))^2}{2M_2(z,u)}), \quad (12)$$

where

$$M_1(z,u)=\sum_{k=1}^{K} f_k m_{1k}(z,u), \quad (13)$$

$$M_2(z,u)=\sum_{k=1}^{K} f_k^2(m_{2k}(z,u)-m_{1k}^2(z,u)), \quad (14)$$

which m_{jk} is j-th moment.

To calculate EC, we need to estimate VaR and mean of loss distribution or expected loss. In this regard, we use Monte-Carlo simulations by using the result of random matrix. For each step of Monte-Carlo simulations we must calculate the value of k dimensional stochastic process V as asset value at maturity T:

$$V(T)=V_0\exp\left(\frac{\sqrt{T}}{\sqrt{N}}\sigma(U^{-1}\Lambda\mathcal{N})n+\mu T -\frac{1}{2}\sigma^2 eT\right), \quad (15)$$

where

- $\mathcal{N}$ is $K\times N$ dimensional matrix and its elements are drawn from a standard normal distribution.
- σ is the volatility matrix which its diagonal elements are $\sigma=diag(\sigma_1,\ldots,\sigma_k)$ and other elements are zero.
- e is a k dimensional vector and contains 1.

- Matrix U is the eigenvectors of the average correlation matrix C, and Λ is a matrix that contains the eigenvalues of the correlation matrix in its diagonal elements.
- n is N dimensional vector and its elements are drawn from a standard normal distribution.
- V_0 is initial price of asset value for each firm, T is maturity, and N is the parameter that controls strength of fluctuations and can be determined by fitting the distribution to the data by Cramer von Misses test.

For each step of simulation, we have to calculate loss of portfolio:

$$L^{MC} = \sum_{k=1}^{K} f_k \frac{F_k - V_k(T)}{F_k} \Theta\big(F_k - V_k(T)\big), \qquad (16)$$

which

$$f_k = \frac{f'_k}{\sum_{k=1}^{K} f'_k} \qquad (17)$$

which f'_k is the remaining loan balance of firm k in the chosen bank.

4. Results

We select a bank[3] as a sample that its portfolio includes publicly traded companies. Our aim is to calculate the economic capital of the bank for the year of 1398. We extract names of the companies from the bank's database and explore the amount of their debts to the bank. Also we aggregate other financial information of these companies from Tehran Stock Exchange (TSE) in the following steps:

- Extracting daily-adjusted prices from TSE for the time between 1395 and 1397. For some stocks on some days, there is no price so we simulate them with the Monte Carlo simulation method.
- Extracting financial information including asset value (V_k) and liability value (L_k) of each companies (this information is gathered from financial reports of companies on Esfand 29, 1397)

Now we need to calculate returns. According to Merton model $V_k = F_k + E_k$ and V_k is a stochastic process that represents the unobservable assets value. So we calculate asset returns as bellow:

$$r_k = \frac{V_k(t+\Delta t) - V_k(t)}{V_k(t)},$$

which Δt is time to maturity or one year. Following Basel II accord guideline, for calculating credit risk economic capital, we should estimate Value at Risk (VaR) for 99.9 percent confidence level and 1-year time horizon.

As mentioned in the methodology, we run 5 million Monte Carlo simulations using expression (15) which takes into account the fluctuations of correlations and then we sort the result into histogram, which yields loss distribution. We can estimate VaR and EL using obtained loss distribution.

All the parameters, which are defined above can be directly calculated from data[4] except N. The parameter N is determined by fitting data to formula (10) and confirming by the Crammer von Mises test. As first step, we examine the distribution of data. Using a least squares fit, N will be around 12. The theoretical and the empirical distributions for the normalized yearly returns are shown in Figure 1:

In the Figure 1 the empirical distribution is shown in solid black line while the theoretical result is shown in dotted red line (both of them on a logarithmic scale). Also the small box is in linear scale. For yearly returns, value around 12 is needed for the parameter N to describe the empirical data. We test the result for accuracy by using Crammer von Mises test. The result of the test is shown in table 1:

According to p-value, around 0.16, the null hypothesis of no significant difference between the observed and the theoretical distribution is not rejected. Put differently, the observed values are completely consistent with the theoretical distribution. We test the other values of N and the result are provided in Figure 2:

Different values of N against Crammer von Mises statistics and Crammer von Mises p-values are shown in the left and right panel of Figure 2, respectively. Red marks indicate the best value. Both figures verify that the best value of N for fitting the empirical data is around 12.

Further, to calculate risk measures such as VaR and ETL[5], we run Monte Carlo simulation for different levels of confidence based on the calculated

[3]. We refuse to disclosure the name of the bank due to confidentiality.

[4]. The parameters include σ, U and … as defined in the expression (15).

[5]. Expected Tail Loss

Archive of SID.ir

parameters and formula (16). We aggregate the loss values (L^{MC}) in a histogram in order to find loss distribution. The result for loss distribution density is represented in the Figure 3:

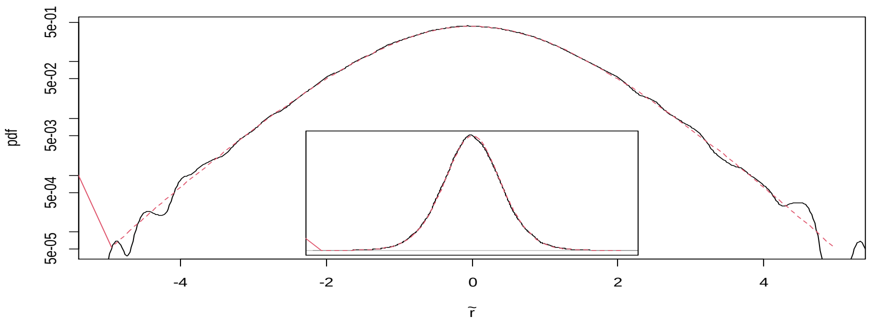

Figure 1: Theoretical and the Empirical Distributions for the Normalized Yearly Returns

Table1. Cramer-Von Mises Test of Goodness-of-Fit

	estimate	Cramer-von Mises statistic	p.value
N	12	0.278	0.156

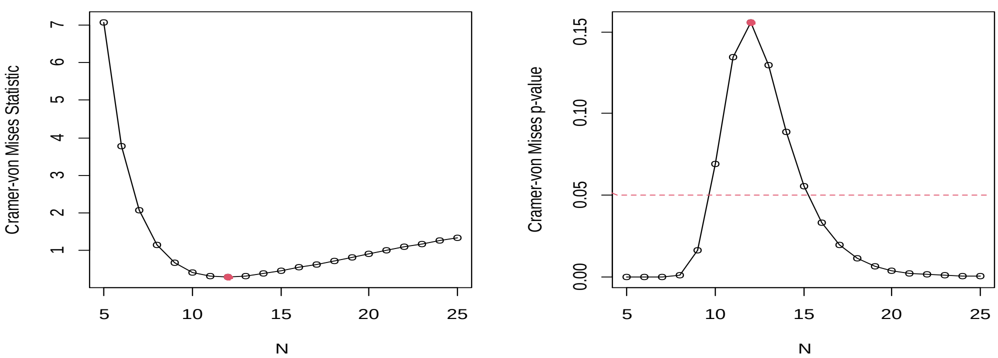

Figure 2: Test Results for Different Values of N

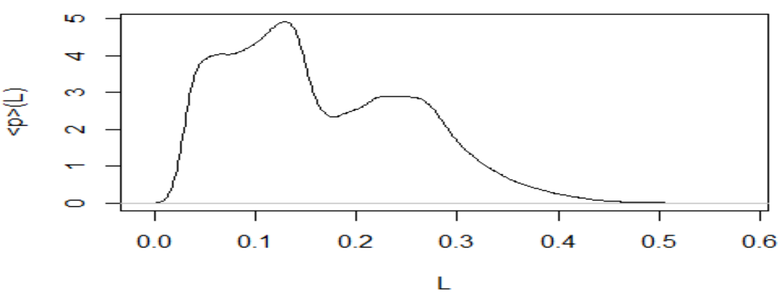

Figure 3: Loss Distribution Density

Archive of SID.ir

Now, risk measures can be easily computed based on the loss distribution. We compute Value at Risk (VaR), Expected Tail Loss (ETL), mean of distribution or expected loss (Mean) and Economic Capital (EC). VaR, Mean and EC are described in the methodology section. Expected Tail Loss quantifies the amount of tail risk that a portfolio has. On the other hand, ETL represents the expected loss if VaR (the worst-case loss) is ever crossed. The result is presented in Table 2:

We round the numbers to three decimal places in table 2. The table reveals that in the next year (1398), with 99%, 99.5% and 99.9% levels of confidence, the maximum loss or worst-case will not exceed 29%,30% and 33%, respectively. Also as can be seen, if the loss exceeds the worst-case threshold; the expected loss in 99%, 99.5% and 99.9% levels of confidence will be 30%, 31% and 34%, respectively. Additionally, the amount of reserve that bank must hold to protect the expected loss (Mean) is about 17% of its credit portfolio.

Economic capital for each level is shown in the Table 2 as EC. So based on this model, the bank must hold 12% of its portfolio's credit exposure as an Economic capital to be protected against crisis with 99% confidence level. But the Basel II Capital Accord offers the concept of a 99.9% credit risk VaR. Therefore, the bank needs to hold at least 15.8% of its credit exposure to support itself against the risks of its portfolio.

To test this model with the classical Merton model, we estimated VaR for Classical Merton, which doesn't consider the fluctuations of the correlation matrix. As discussed before, if $N \to \infty$ then the correlation matrix will be fixed. Using this point, another Monte Carlo simulation run and the result for classical Merton model is shown in the Table 3:

We round the numbers to three decimal places. As is obvious from the table, EC is about 14%. So according to classical Merton model, in 99.9% confidence level, the bank needs to hold at least 14% of its credit exposure to support itself against the risks of its portfolio. The amount of EC in this model is significantly less than the previous model. The benefits of random matrix approach can be seen by comparing its results with classical Merton model 's results.

Now, in order to test the impact of considering the fluctuation correlation in the model we run model for different N and calculate the relative deviation of the VaR as $N \to \infty$ or classical Merton model. The result is shown in Figure 4

Table 2. Risk Measures at Different Confidence Level

Measure	Confidence Level		
	99%	99.50%	99.90%
VaR	0.29	0.30	0.326
ETL	0.303	0.315	0.335
Mean	0.169	0.169	0.169
EC	0.121	0.131	0.158

Table 3: Risk measures in classical Merton model

Measure	Confidence Level		
	99%	99.50%	99.90%
VaR	0.228	0.242	0.265
ETL	0.239	0.254	0.272
Mean	0.121	0.121	0.121
EC	0.107	0.121	0.144

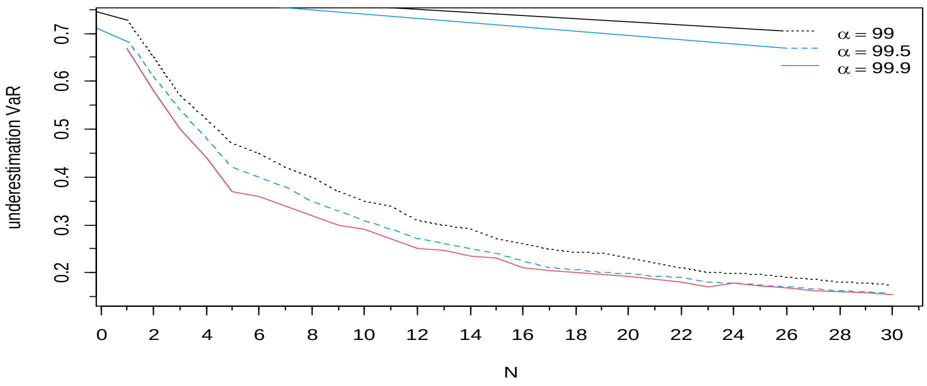

Figure 4: Underestimation of the VaR

As mentioned above, for the bank portfolio, N is 12 and Figure 1 show that for this N the VaR is underestimated between 25% and 35%.

According to the Figure 4 there is an exponential decay between the underestimation of the VaR and N Values. So to test the impact of N on the underestimation VaR, we fit an exponential regression. The summary of the hypothesis test including the parameters estimation as well as p-values are shown in Tables 5, 6 and 7 for 99%,99.5% and 99.9% confidence levels, respectively

In the above tables (Tables 4, 5 and 6), the coefficient a is the initial value and the coefficient b is the growth rate. According to Tables 4, 5 and 6, p-values for each parameter in the whole tables are quite smaller than 0.05 that suggests the null hypothesis is rejected at any significance level. It means that using N in the model as a measure of correlations fluctuations is quite important which if not considered, can result in a significant underestimation of the VaR and EC. Furthermore, we can see an exponential decay in Figure 4 that suggests the larger the N, the smaller the value of underestimation. On the other hand, as N becomes larger, the difference between two method gets smaller.

Table 4: Exponential Regression Results for 99% Confidence Level

	Estimate	Std. Error	t-value	p-value
a	0.6662	0.01881	35.41	<2e-16
b	-0.05509	0.0026	-21.19	<2e-16
Formula	underestimation VaR99 ~.666 * exp(-.055* N)			

Table 5: Exponential Regression Results for 99.5% Confidence Level

	Estimate	Std. Error	t-value	p-value
a	0.624611	0.021424	29.16	<2e-16
b	-0.059493	0.003294	-18.06	<2e-16
Formula	underestimation VaR99.5 ~ .625 * exp(-.059 * N)			

Table 6: Exponential Regression Results for 99.9% Confidence Level

	Estimate	Std. Error	t-value	p-value
a	0.575815	0.024247	23.75	<2.00E-16
b	-0.057652	0.003973	-14.51	1.49E-14
Formula	underestimation VaR99.9 ~0.576* exp(-.058 * N)			

Discussion and Conclusion

The main purpose of this research was to determine EC of a bank credit portfolio including publicly traded companies. But the market feature can change loss distribution of the portfolio and causes underestimation of risk. In this paper we took into account correlation matrix fluctuations using random matrix theory and applied this feature in the Merton model. The result was compared with the classical Merton model. The comparison showed that if we don't consider correlation changes, we can face significant underestimation in risk measures especially VaR and EC. Hence, to avoid the underestimation of risk and as a consequence to be protected against crises, the fluctuations of correlations must be accounted for by financial institutions.

References

1) Altman, E. I., Brady, B., Resti, A., & Sironi, A. (2005). The link between default and recovery rates: Theory, empirical evidence, and implications. *The Journal of Business, 78*(6), 2203-2228.
2) Baesens, B., & Van Gestel, T. (2009). Credit risk management: Basic concepts.
3) Basel Comittee. (2004). International Convergence of Capital Measurement and Capital Standards.
4) Basel Comittee. (2005). An Explanatory Note on the Basel II IRB Risk Weight Functions , July 2005.
5) Basel Comittee. (2009).Range of practices and issues in economic capital frameworks,March 2009.
6) Chamizo, Á., Fonollosa, A., & Novales, A. (2019). Forward-looking asset correlations in the estimation of economic capital. *Journal of International Financial Markets, Institutions and Money, 61*, 264-288. doi:https://doi.org/10.1016/j.intfin.2019.04.001
7) Crouhy, M., Galai, D., & Mark, R. (2006). *The essentials of risk management* (Vol. 1): McGraw-Hill New York.
8) Duan, J. C. (1994). Maximum likelihood estimation using price data of the derivative contract. *Mathematical Finance, 4*(2), 155-167.
9) Duffie, D., & Singleton, K. J. (2012). *Credit risk: pricing, measurement, and management*: Princeton university press.
10) Elizalde, A., & Repullo, R. (2007). Economic and Regulatory Capital in Banking: What Is the Difference? *International Journal of Central Banking, 3*, 87-117.
11) Eom, Y. H., Helwege, J., & Huang, J.-z. (2004). Structural models of corporate bond pricing: An empirical analysis. *The review of financial studies, 17*(2), 499-544.
12) Ericsson, J., & Renault, O. (2006). Liquidity and credit risk. *The journal of finance, 61*(5), 2219-2250.
13) Mühlbacher, A., & Guhr, T. (2018). Credit Risk Meets Random Matrices: Coping with Non-Stationary Asset Correlations. *Risks, 6*(2), 42.
14) Omar, A., & Prasanna, P. K. (2021). Asymmetric effects of noise in Merton default risk model: Evidence from emerging Asia. *Pacific-Basin Finance Journal, 65*, 101497. doi:https://doi.org/10.1016/j.pacfin.2021.101497
15) Ong, M. K. (1999). *Internal credit risk models: Capital allocation and performance measurement*: Risk publications.
16) Ronn, E. I., and Verma, A. K., 1986, "Pricing Risk-Adjusted Deposit Insurance: An Option Based Model," Journal of Finance 41, 871-895.
17) Schmitt, T. A. (2014). Non-stationarity as a central aspect of financial markets. (doctorate). Retrieved from https://duepublico2.unidue.de/receive/duepublico_mods_00036759
18) Schmitt, T. A., Chetalova, D., Schäfer, R., & Guhr, T. (2013). Non-stationarity in financial time series: Generic features and tail behavior. *EPL (Europhysics Letters), 103*(5), 58003. https://doi:10.1209/0295-5075/103/58003
19) Sundaresan, S. (2013). A review of Merton's model of the firm's capital structure with its wide applications. *Annu. Rev. Financ. Econ., 5*(1), 21-41.