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Measurement of damage to scale based on data envelopment analysis: A case study in Saman insurance company

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ABSTRACT
Insurance industry is one of the most important factors for the economic development of the countries. For example, insurance industry can be important for the stability of financial systems mainly because they are large investors in financial markets, because there are growing links between insurers and banks and because insurers are safeguarding the financial stability of households and firms by insuring their risks. Data Envelopment Analysis (DEA) has been used as a powerful tool for the efficiency assessment of the different organizations, such as insurance industries, hospitals, schools and etc. This paper focuses on evaluation the insurance companies and explores a use of DEA to measure the insurers risk in these companies. For this purpose, we use the dataset of the car insurance policies of Saman insurance company during the years 2018-2019 and measure the Returns to Scale (RTS) for desirable outputs and Damages to Scale (DTS) for undesirable outputs.

Keywords: Insurance industry; Data Envelopment Analysis; Returns to scale; Damage to scale; Undesirable output.

1. Introduction

Data Envelopment Analysis (DEA), introduced by Charnes et al. (1978) and further developed by Banker et al. (1984) is a non-parametric technique for assessing the efficiency of a set of Decision Making Units (DMUs) with multiple inputs and multiple outputs. It assigns an efficiency score between 0 and 1 to each unit. The larger the efficiency score, the better performance the unit under evaluation has. Traditional DEA models cannot discriminate among efficient DMUs since they all get the efficiency score equal to 1. In this regard, several ranking methods have been developed in DEA literature. For more studies about ranking methods in DEA, see Adler et al (2002), Soleimani et al. (2020), Jabbari et al. (2019), Sharafi et al. (2019).

Returns to scale (RTS) is one of the most important and highly discussed topics in the DEA literature. Banker (1984) initially proposed a method to identify RTS of DMUs in CCR model. Since then, a series of studies have been done to investigate the different aspects of RTS classification in different types of DEA models. Subsequently, Banker and Thrall (1992) and Zhu and Shen (1995) presented some approaches to estimate RTS in the presence of the multiple optimal solutions of BCC model. Banker et al. (1996) suggested an algorithm to determine RTS in the situation that CCR model has the alternative solutions. Färe et al. (1985, 1994) used the efficiency scores of DMUs to characterize the types of RTS. Golany and Yu (1997) presented an algorithm to determine RTS of the efficient DMUs. Jahanshahloo et al. (2005) extended the method of Golany and Yu (1997).

The classification of RTS in the non-radial models is further complicated, because each inefficient DMU has the multiple projections. Sueyoshi and Sekitani (2007a) applied a non–radial DEA model for finding all the efficient DMUs that belong to the reference set to estimate the RTS of DMUs. Sueyoshi and Sekitani (2007b) considered the situation that the alternative optimal solutions occur in the reference set and extended the method of Sueyoshi and Sekitani (2007a). Fukuyama (2000) proposed some properties about the scale elasticity (SE) of the DMUs. Sueyoshi and Sekitani (2005) suggested a method to specify RTS in dynamic systems in which there are two different types of inputs, variable inputs and quasi-fixed inputs. Førsund et al. (2007) proposed two approaches to identify RTS. The first approach determined the radially projection of DMUs on the frontier and used the efficiency score of DMUs and their dual variables to specify RTS of units. The second approach distinguished RTS of DMUs by applying the intersection of hyperplanes that passes through that unit. Soleimani-damaneh et al. (2009) considered the relationship between the RTS and scale elasticity (SE) in the presence of the alternative optimal solutions. Zarepisheh et al. (2009) used the means of the dual simplex method to estimate RTS on the left and on the right.

Sueyoshi and Goto (2011) considered the environmental assessment by using DEA method in the presence of the desirable and undesirable outputs. They estimated RTS and damages to scale (DTS) by means of desirable and undesirable outputs, respectively. Sueyoshi and Goto (2012) defined the concept of disposability for environmental evaluation and then, they presented non-radial model to distinguish RTS and DTS of units. Sueyoshi and Goto (2013) developed a method to evaluate RTS and DTS of units and they applied their method to the coal-fired power plants.

Witte and Marques (2011), and Soleimani-damaneh and Mostafaee (2009) suggested models in the case of the non-convex production possibility set (PPS). Hatami-Marbini et al. (2017) presented a method to estimate RTS of units in the case of interval data. Miller and Muir (2020) proposed a new perspective on RTS for truckload motor carriers. Qi et al. (2020) used a DEA model to evaluate the efficiency from two perspectives of both operational performance and environment, and to determine the types of RTS and damages to scale (DTS).

This study focuses on the insurers risk in the insurance companies. For this purpose, a dataset of the car insurance policies of Saman insurance company during the years 2018-2019 is used and RTS for desirable outputs and DTS for undesirable outputs are evaluated. The rest of the paper is as follows: Section 2 reviews some basic definitions and preliminaries. Section 3 presents the methodology of our paper. Section 4 used a dataset of the car insurance policies of Saman insurance company during the years 2018-2019 for display the potentiality of the proposed method. Section 5 concludes the paper.

2. Preliminaries and basic definitions

Consider a system of n DMUs, denoted by $DMU_j, j = 1, \dots, n$, where each unit consumes m different inputs to generate s different good outputs and h different bad outputs. The i^{th} input, the r^{th} good output and the k^{th} bad output for DMU_j are denoted by x_{ij}, g_{rj} and b_{fj}, respectively, for $i = 1, \dots, m$, $r = 1, \dots, s$ and $f = 1, \dots, h$. Sueyoshi and Goto (2011) studied the environmental assessment by applying the DEA techniques. They proposed model (1a) and (1b) to identify the type of RTS in the case of desirable outputs:

Table 1. The models of Sueyoshi and Goto (2011) to classify of RTS

The lower bound	The upper bound
$\underline{\sigma}^* = \min \sigma \quad (1a)$	$\bar{\sigma}^* = \max \sigma \quad (1b)$
$s.t.$	$s.t.$
$\sum_{j=1}^{n} \lambda_j x_{ij} + d_i^x = x_{io}, \quad \forall i,$	$\sum_{j=1}^{n} \lambda_j x_{ij} + d_i^x = x_{io}, \quad \forall i,$
$\sum_{j=1}^{n} \lambda_j g_{rj} - d_r^g = g_{ro}, \quad \forall r,$	$\sum_{j=1}^{n} \lambda_j g_{rj} - d_r^g = g_{ro}, \quad \forall r,$
$\sum_{j=1}^{n} \lambda_j = 1,$	$\sum_{j=1}^{n} \lambda_j = 1,$
$\sum_{i=1}^{m} v_i x_{ij} - \sum_{r=1}^{s} u_r g_{rj} + \sigma \geq 0, \quad \forall j,$	$\sum_{i=1}^{m} v_i x_{ij} - \sum_{r=1}^{s} u_r g_{rj} + \sigma \geq 0, \quad \forall j,$
$v_i \geq R_i^x, \quad \forall i,$	$v_i \geq R_i^x, \quad \forall i,$
$u_r \geq R_r^g, \quad \forall r,$	$u_r \geq R_r^g, \quad \forall r,$
$\sum_{i=1}^{m} R_i^x d_i^x + \sum_{r=1}^{s} R_r^g d_r^g = \sum_{i=1}^{m} v_i x_{io} - \sum_{r=1}^{s} u_r g_{ro} + \sigma,$	$\sum_{i=1}^{m} R_i^x d_i^x + \sum_{r=1}^{s} R_r^g d_r^g = \sum_{i=1}^{m} v_i x_{io} - \sum_{r=1}^{s} u_r g_{ro} + \sigma,$
$d_i^x, d_r^g \geq 0, \quad \forall i, r,$	$d_i^x, d_r^g \geq 0, \quad \forall i, r,$
$\sigma: URS,$	$\sigma: URS,$

where, $d_i^x (i = 1, \dots, m)$ and $d_r^g (r = 1, \dots, s)$ are all slack variables related to inputs and desirable (good) outputs, respectively. In models (1a) and (1b), $v_i (i = 1, \dots, m)$ and $u_r (r = 1, \dots, s)$ are the input weights and the good output weights, respectively. The symbol (URS) shows that σ is unrestricted variable. In models (1a) and (1b), $R_i^x (i = 1, \dots, m)$ and $R_r^g (r = 1, \dots, s)$ are defined as follows:

$$R_i^x = \frac{1}{(m+s)(\bar{x}_i - \underline{x}_i)}; \bar{x}_i = \max_j\{x_{ij}\} \;\&\; \underline{x}_i = \min_j\{x_{ij}\}$$

$$R_r^g = \frac{1}{(m+s)(\bar{g}_r - \underline{g}_r)}; \bar{g}_r = \max_j\{g_{rj}\} \;\&\; \underline{g}_r = \min_j\{g_{rj}\}$$

Sueyoshi and Goto (2011) used the lower and upper bounds of σ^*, and classified the RTS of DMUs in the case of good outputs, as follows:

(a) Increasing RTS $\leftrightarrow 0 > \bar{\sigma}^* \geq \underline{\sigma}^*$,
(b) Constant RTS $\leftrightarrow \bar{\sigma}^* \geq 0 \geq \underline{\sigma}^*$,
(c) Decreasing RTS $\leftrightarrow \bar{\sigma}^* \geq \underline{\sigma}^* > 0$.

Also, they presented models (2a) and (2b) to identify the type of DTS in the case of undesirable (bad) outputs.

In models (2a) and (2b), $d_i^x (i = 1, \dots, m)$ and $d_f^b (f = 1, \dots, h)$ are all slack variables related to inputs and undesirable (bad) outputs, respectively. In these models, $v_i (i = 1, \dots, m)$ and $w_f (f = 1, \dots, h)$ are the input weights and the bad output weights, respectively. The symbol (URS) shows that σ is unrestricted variable. . In models (2a) and (2b), $R_i^x (i = 1, \dots, 5)$ and $R_f^b (f = 1, \dots, h)$ are defined as follows:

$$R_i^x = \frac{1}{(m+h)(\bar{x}_i - \underline{x}_i)}; \bar{x}_i = \max_j\{x_{ij}\} \;\&\; \underline{x}_i = \min_j\{x_{ij}\}$$

$$R_f^b = \frac{1}{(m+h)(\bar{b}_f - \underline{b}_f)}; \bar{b}_f = \max_j\{b_{1j}\} \;\&\; \underline{b}_f = \min_j\{b_{1j}\}$$

Table 2. The models of Sueyoshi and Goto (2011) to classify of DTS

The lower bound	The upper bound
$\underline{\sigma}^* = \min \sigma$ (2a)	$\bar{\sigma}^* = \max \sigma$ (2b)
$s.t.$	$s.t.$
$\sum_{j=1}^{n} \lambda_j x_{ij} - d_i^x = x_{io}, \quad \forall i,$	$\sum_{j=1}^{n} \lambda_j x_{ij} - d_i^x = x_{io}, \quad \forall i,$
$\sum_{j=1}^{n} \lambda_j b_{fj} + d_f^b = b_{fo}, \quad \forall f,$	$\sum_{j=1}^{n} \lambda_j b_{fj} + d_f^b = b_{fo}, \quad \forall f,$
$\sum_{j=1}^{n} \lambda_j = 1,$	$\sum_{j=1}^{n} \lambda_j = 1,$
$-\sum_{i=1}^{m} v_i x_{ij} - \sum_{f=1}^{h} w_f b_{fj} + \sigma \geq 0, \quad \forall j,$	$-\sum_{i=1}^{m} v_i x_{ij} - \sum_{f=1}^{h} w_f b_{fj} + \sigma \geq 0, \quad \forall j,$
$v_i \geq R_i^x, \quad \forall i,$	$v_i \geq R_i^x, \quad \forall i,$
$u_r \geq R_f^b, \quad \forall f,$	$u_r \geq R_f^b, \quad \forall f,$
$\sum_{i=1}^{m} R_i^x d_i^x + \sum_{f=1}^{h} R_f^b d_f^b = -\sum_{i=1}^{m} v_i x_{io} + \sum_{f=1}^{h} w_f b_{fo} + \sigma,$	$\sum_{i=1}^{m} R_i^x d_i^x + \sum_{f=1}^{h} R_f^b d_f^b = -\sum_{i=1}^{m} v_i x_{io} + \sum_{f=1}^{h} w_f b_{fo} + \sigma,$
$d_i^x, d_f^b \geq 0, \quad \forall i, f,$	$d_i^x, d_f^b \geq 0, \quad \forall i, f,$
$\sigma : URS,$	$\sigma : URS,$

Sueyoshi and Goto (2011) used the lower and upper bounds of σ^*, and classified the DTS of DMUs in the case of bad outputs, as follows:

(a) Increasing RTS $\leftrightarrow \bar{\sigma}^* \geq \underline{\sigma}^* > 0$,

(b) Constant RTS $\leftrightarrow \bar{\sigma}^* \geq 0 \geq \underline{\sigma}^*$,

(c) Decreasing RTS $\leftrightarrow 0 > \bar{\sigma}^* \geq \underline{\sigma}^*$.

3. Evaluating RTS and DTS for the car insurance policies

In this section, we develop the method of Sueyoshi and Goto (2011) for the insurance industry. For this purpose, we consider the car insurance policies. This data set has five inputs (the Number of years of car operation, the price, the driver gender, the driver age, the province of driver's residence) and two outputs (the number of years without damages, the damage ratio). The damage ratio is calculated as the ratio of the amount of damage cost to the amount of premium paid by the insurer which is considered as undesirable output. According to the type of inputs and outputs in the car insurance policies, the decision maker decides to determine the relative importance of inputs and outputs via the restrictions on the input and output weights. So, we aim to develop the method of Sueyoshi and Goto (2011) in the presence of the weight restrictions.

According to the previous section, define $R_i^x (i = 1, \dots, 5)$ as follows:

$$R_i^x = \frac{1}{6(\bar{x}_i - \underline{x}_i)},$$

in which, $\bar{x}_i = \max_j\{x_{ij}\}$ and $\underline{x}_i = \min_j\{x_{ij}\}$.

Also, define R_1^g as follows:

$$R_1^g = \frac{1}{6(\bar{g}_1 - \underline{g}_1)},$$

where, $\bar{g}_1 = \max_j\{g_{1j}\}$ and $\underline{g}_1 = \min_j\{g_{1j}\}$.

Next, we propose model (3a) and (3b), reported in Table 3, to identify the type of RTS of the insurer in the case of desirable output (the number of years without damages). It should be noted that, in the example of the car insurance industry, the third input is the driver gender in which the number 1 is attributed to the female gender and the number 2 to the male gender. Given that the target of this type of input cannot take values other than 1 and 2, so we divide the inputs into two categories, T_1 and T_2. T_1 contains the driver gender input and T_2 contains the other inputs.

In models (3a) and (3b), $d_i^x (i = 1, \dots, m)$ and d_1^g are all slack variables related to inputs and desirable (good) output, respectively. In these models, $v_i(i=1,\dots,m)$ and u_1 are the input weights and the good output weight, respectively. The symbol (URS) shows that σ is unrestricted variable. These models have some weight restrictions, i.e. $v_2 \geq v_1, v_1 \geq 2v_4, v_5 \geq v_4$ and $v_5 \geq 2v_3$ on the weight of inputs.

Therefore, using the lower and upper bounds of σ^*, the type of RTS of DMUs in the case of good output is as follows:

(a) Increasing RTS $\leftrightarrow 0 > \bar{\sigma}^* \geq \underline{\sigma}^*$,

(b) Constant RTS $\leftrightarrow \bar{\sigma}^* \geq 0 \geq \underline{\sigma}^*$,

(c) Decreasing RTS $\leftrightarrow \bar{\sigma}^* \geq \underline{\sigma}^* > 0$.

Table 3. The proposed models to classify of RTS

The lower bound	The upper bound
$$\begin{aligned} &\underline{\sigma}^* = \min \sigma && (3a)\\ &s.t.\\ &\sum_{j=1}^{n} \lambda_j x_{ij} + d_i^x = x_{io}, && \forall i,\\ &x_{io} - s_i^- = z_i + 1, && i \in T_1,\\ &\sum_{j=1}^{n} \lambda_j g_{rj} - d_1^g = g_{1o}, && \forall r,\\ &\sum_{j=1}^{n} \lambda_j = 1,\\ &\sum_{i=1}^{5} v_i x_{ij} - u_1 g_{1j} + \sigma \geq 0, && \forall j,\\ &\sum_{i=1}^{5} R_i^x d_i^x + R_1^g d_1^g = \sum_{i=1}^{5} v_i x_{io} - u_1 g_{1o} + \sigma,\\ &v_i \geq R_i^x, && \forall i,\\ &u_1 \geq R_1^g,\\ &v_2 \geq v_1,\\ &v_1 \geq 2v_4,\\ &v_5 \geq v_4,\\ &v_5 \geq 2v_3,\\ &d_i^x, d_1^g \geq 0, && \forall i,\\ &\sigma: URS,\\ &s_i^- \geq 0, && i \in T_1\\ &z_i \in \{0,1\}, && i \in T_1. \end{aligned}$$	$$\begin{aligned} &\bar{\sigma}^* = \max \sigma && (3b)\\ &s.t.\\ &\sum_{j=1}^{n} \lambda_j x_{ij} + d_i^x = x_{io}, && \forall i,\\ &x_{io} - s_i^- = z_i + 1, && i \in T_1,\\ &\sum_{j=1}^{n} \lambda_j g_{rj} - d_1^g = g_{1o}, && \forall r,\\ &\sum_{j=1}^{n} \lambda_j = 1,\\ &\sum_{i=1}^{5} v_i x_{ij} - u_1 g_{1j} + \sigma \geq 0, && \forall j,\\ &\sum_{i=1}^{5} R_i^x d_i^x + R_1^g d_1^g = \sum_{i=1}^{5} v_i x_{io} - u_1 g_{1o} + \sigma,\\ &v_i \geq R_i^x, && \forall i,\\ &u_1 \geq R_1^g,\\ &v_2 \geq v_1,\\ &v_1 \geq 2v_4,\\ &v_5 \geq v_4,\\ &v_5 \geq 2v_3,\\ &d_i^x, d_1^g \geq 0, && \forall i,\\ &\sigma: URS,\\ &s_i^- \geq 0, && i \in T_1,\\ &z_i \in \{0,1\}, && i \in T_1. \end{aligned}$$

Also, we propose model (4a) and (4b) to identify the type of DTS of the insurer in the case of undesirable output (the damage ratio):

Table 4. The proposed models to classify of DTS

The lower bound	The upper bound

$$
\begin{aligned}
&\underline{\delta}^* = \min \delta && (4a)\\
&s.t.\\
&\sum_{j=1}^{n} \lambda_j x_{ij} - d_i^x = x_{io}, && \forall i,\\
&x_{io} - s_i^- = z_i + 1, && i \in T_1,\\
&\sum_{j=1}^{n} \lambda_j b_{1j} + d_1^b = b_{1o},\\
&\sum_{j=1}^{n} \lambda_j = 1,\\
&-\sum_{i=1}^{m} v_i x_{ij} + w_1 b_{1j} + \delta \geq 0, && \forall j,\\
&\sum_{i=1}^{m} R_i^x d_i^x + R_1^b d_1^b = -\sum_{i=1}^{m} v_i x_{io} + w_1 b_{1o} + \delta,\\
&v_i \geq R_i^x, && \forall i,\\
&u_r \geq R_f^b, && \forall f,\\
&v_2 \geq v_1,\\
&v_1 \geq 2v_4,\\
&v_5 \geq v_4,\\
&v_5 \geq 2v_3\\
&d_i^x, d_1^b \geq 0, && \forall i,\\
&\delta : URS,\\
&s_i^- \geq 0, && i \in T_1,\\
&z_i \in \{0,1\}, && i \in T_1.
\end{aligned}
$$

$$
\begin{aligned}
&\bar{\delta}^* = \max \delta && (4a)\\
&s.t.\\
&\sum_{j=1}^{n} \lambda_j x_{ij} - d_i^x = x_{io}, && \forall i,\\
&x_{io} - s_i^- = z_i + 1, && i \in T_1,\\
&\sum_{j=1}^{n} \lambda_j b_{1j} + d_1^b = b_{1o},\\
&\sum_{j=1}^{n} \lambda_j = 1,\\
&-\sum_{i=1}^{m} v_i x_{ij} + w_1 b_{1j} + \delta \geq 0, && \forall j,\\
&\sum_{i=1}^{m} R_i^x d_i^x + R_1^b d_1^b = -\sum_{i=1}^{m} v_i x_{io} + w_1 b_{1o} + \delta,\\
&v_i \geq R_i^x, && \forall i,\\
&u_r \geq R_f^b, && \forall f,\\
&v_2 \geq v_1,\\
&v_1 \geq 2v_4,\\
&v_5 \geq v_4,\\
&v_5 \geq 2v_3\\
&d_i^x, d_1^b \geq 0, && \forall i,\\
&\delta : URS,\\
&s_i^- \geq 0, && i \in T_1,\\
&z_i \in \{0,1\}, && i \in T_1.
\end{aligned}
$$

where, $d_i^x (i = 1, \dots, m)$ and d_1^b are all slack variables related to inputs and undesirable (bad) output, respectively. In models (4a) and (4b), $v_i (i = 1, \dots, m)$ and w_1 are the input weights and the bad output weight, respectively. The symbol (URS) shows that δ is unrestricted variable. In models (4a) and (4b), $R_i^x (i = 1, \dots, 5)$ and R_1^b are defined as follows:

$$R_i^x = \frac{1}{6(\bar{x}_i - \underline{x}_i)}; \bar{x}_i = \max_j \{x_{ij}\} \ \& \ \underline{x}_i = \min_j \{x_{ij}\}$$
$$R_1^b = \frac{1}{6(\bar{b}_1 - \underline{b}_1)}; \bar{b}_1 = \max_j \{b_{1j}\} \ \& \ \underline{b}_1 = \min_j \{b_{1j}\}$$

These models have some weight restrictions, i.e. $v_2 \geq v_1, v_1 \geq 2v_4, v_5 \geq v_4$ and $v_5 \geq 2v_3$ on the weight of inputs.

Therefore, using the lower and upper bounds of σ^*, the type of DTS of DMUs in the case of bad output is as follows:

(a) Increasing DTS $\leftrightarrow \bar{\delta}^* \geq \underline{\delta}^* > 0$,

(b) Constant DTS $\leftrightarrow \bar{\delta}^* \geq 0 \geq \underline{\delta}^*$,

(c) Decreasing DTS $\leftrightarrow 0 > \bar{\delta}^* \geq \underline{\delta}^*$.

4. Numerical example

In this example, the results of applying our proposed approach to the data set which includes 201 insurers who have purchased insurance policies from Saman Insurance company during the years 2018-2019, are reported. Each insurer is considered as a decision making unit with five inputs (the Number of years of car operation (x_1), the price (x_2), the driver gender (x_3), the driver age (x_4), the province of driver's residence (x_5)) and two outputs (the number of years without damages (y_1), the damage ratio (y_2)). Table 5 shows the information of units. For the second input (x_2), an integer number is assigned to the price of each insured car, so that the smaller integer number is assigned to the more expensive car. For the third input (x_3), the number 1 is attributed to the female gender and the number 2 to the male gender. For the fourth input (x_4), the numbers 1, 2 and 3 are assigned to the insurers age, so that the smaller number is assigned to the older insurer. According to the manager's view point, an integer number is assigned to each province of drivers's residence. The damage ratio (y_2) is calculated as the ratio of the amount of damage cost to the amount of premium paid by the insurer which is considered as undesirable output.

Table 5. The information of units.

Inputs and Outputs	Mean	Median	Mode	Variance	Minimum	Maximum
x_1	5.4975	4	1	18.2712	1	21
x_2	10.8905	13	13	6.5180	1	13
x_3	1.6716	2	2	0.2144	1	2
x_4	2.1045	2	2	0.5440	1	3
x_5	1.5572	1.5	1	0.6980	1	3
y_1	2.0498	2	0	4.3875	0	8
y_2	53.8632	47	0	19433.18	0	1272.95

Now, our proposed approaches are implemented for identifying the type of RTS and DTS in the case of desirable outputs and undesirable outputs, respectively. For this purpose, we solve models (3a) and (3b) to assess RTS of units and then, we solve models (4a) and (4b) to recognize DTS of DMUs and the results are summarized in Table 6. The columns 2 and 3 show the lower and upper bounds of σ^*, respectively. Column 4 determines the type of RTS in the case of desirable output. Regarding the concept of RTS, for the units with increasing RTS, if the inputs are increased, then the outputs are increased and the percentage increase in outputs is greater than the percentage increase in inputs. Also, if the inputs are decreased, then the outputs are decreased and the percentage decrease in outputs is greater than the percentage decrease in inputs. Therefore, the decision maker prefers to increase the size of the DMU.
On the other hand, for the units with decreasing RTS, if the inputs are increased, then the outputs are increased and the percentage increase in outputs is lower than the percentage increase in inputs. Also, if the inputs are decreased, then the outputs are decreased and the percentage decrease in outputs is lower than the percentage decrease in inputs. Therefore, the decision maker prefers to decrease the size of the DMU.
Columns 6 and 7 report the lower and upper bounds of δ^*, respectively. Column 8 shows the type of DTS in the case of undesirable output. Regarding the concept of DTS, for the units with increasing RTS, if the inputs are increased, then the undesirable outputs are increased and the percentage increase in undesirable outputs is greater than the percentage increase in inputs. Also, if the inputs are decreased, then the undesirable outputs are decreased and the percentage decrease in undesirable outputs is greater than the percentage decrease in inputs. Therefore, the decision maker prefers to decrease the size of the DMU.
On the other hand, for the units with decreasing RTS, if the inputs are increased, then the undesirable outputs are increased and the percentage increase in undesirable outputs is lower than the percentage increase in inputs. Also, if the inputs are decreased, then the undesirable outputs are decreased and the percentage decrease in undesirable outputs is lower than the percentage decrease in inputs. Therefore, the decision maker prefers to increase the size of the DMU. Given the above discussion, the decision maker can easily decide whether to increase or decrease the size of the DMUs.

Table 6. The type of RTS and DTS.

DMU	$\bar{\sigma}^*$	$\underline{\sigma}^*$	RTS	DMU	$\bar{\delta}^*$	$\underline{\delta}^*$	DTS

DMU	$\overline{\sigma}^*$	$\underline{\sigma}^*$	RTS	DMU	$\overline{\delta}^*$	$\underline{\delta}^*$	DTS
1	0.124	0.085	D	1	0.217	0.131	I
2	0.163	0.125	D	2	0.197	0.122	I
3	0.043	0.043	D	3	0.117	0.295	I
4	0.038	0.038	D	4	0.418	0.034	I
5	0.109	0.109	D	5	0.030	0.044	I
6	0.077	0.077	D	6	0.139	0.195	I
7	0.079	0.079	D	7	0.033	0.033	I
8	0.079	0.079	D	8	0.026	0.026	I
9	0.079	0.079	D	9	0.118	0.096	I
10	6.313	-4.000	C	10	0.062	0.062	I
11	-2.325	-2.325	I	11	0.038	0.038	I
12	0.079	0.079	D	12	0.022	0.022	I
13	0.078	0.135	D	13	0.055	0.055	I
14	0.078	0.078	D	14	0.093	0.069	I
15	0.192	0.142	D	15	4.357	3.097	I
16	0.142	0.142	D	16	0.005	0.005	I
17	0.082	0.082	D	17	0.036	0.036	I
18	0.079	0.079	D	18	4.378	2.217	I
19	0.378	0.142	D	19	0.079	0.079	I
20	0.110	0.110	D	20	0.062	0.189	I
21	0.112	0.063	D	21	0.127	0.113	I
22	0.108	0.108	D	22	0.069	0.078	I
23	0.110	0.110	D	23	6.428	6.310	I
24	0.213	0.179	D	24	0.559	0.042	I
25	0.102	0.076	D	25	0.083	0.055	I

DMU	$\overline{\sigma}^*$	$\underline{\sigma}^*$	RTS	DMU	$\overline{\delta}^*$	$\underline{\delta}^*$	DTS
26	0.043	0.019	D	26	0.139	0.139	I
27	0.109	0.109	D	27	0.030	0.045	I
28	0.397	0.218	D	28	0.091	0.091	I
29	0.142	0.142	D	29	0.077	0.014	I
30	0.297	0.125	D	30	0.121	0.064	I
31	0.064	0.064	D	31	0.217	0.217	I
32	0.419	0.256	D	32	0.145	0.145	I
33	0.179	0.076	D	33	0.039	0.034	I
34	0.079	0.079	D	34	0.226	0.149	I
35	0.214	0.181	D	35	0.051	0.051	I
36	0.178	0.178	D	36	0.090	0.079	I
37	0.185	0.078	D	37	0.113	0.095	I
38	0.173	0.094	D	38	0.069	0.069	I
39	0.077	0.077	D	39	0.195	0.139	I
40	0.079	0.079	D	40	0.458	0.050	I
41	0.197	0.139	D	41	0.079	0.079	I
42	0.076	0.076	D	42	0.139	0.139	I
43	0.078	0.078	D	43	0.749	0.204	I
44	0.239	0.118	D	44	0.079	0.065	I
45	0.078	0.078	D	45	0.110	0.110	I
46	0.076	0.076	D	46	0.139	0.139	I
47	0.076	0.076	D	47	0.363	0.195	I
48	0.078	0.078	D	48	0.109	0.109	I
49	0.077	0.077	D	49	0.139	0.197	I
50	0.139	0.139	D	50	0.079	0.079	I

DMU	$\overline{\sigma}^*$	$\underline{\sigma}^*$	RTS	DMU	$\overline{\delta}^*$	$\underline{\delta}^*$	DTS
51	0.141	0.141	D	51	0.079	0.079	I
52	0.189	0.189	D	52	0.085	0.085	I
53	0.223	0.194	D	53	0.139	0.084	I
54	0.197	0.139	D	54	0.007	0.079	I
55	0.078	0.078	D	55	0.204	0.140	I
56	0.218	0.139	D	56	0.559	0.042	I
57	0.079	0.079	D	57	5.078	4.359	I
58	0.142	0.142	D	58	0.216	0.195	I
59	0.092	0.092	D	59	0.125	0.064	I
60	0.197	0.139	D	60	0.069	0.054	I
61	0.093	0.093	D	61	0.789	0.204	I
62	0.112	0.098	D	62	0.139	0.195	I
63	0.164	0.164	D	63	0.195	0.139	I
64	0.009	0.009	D	64	0.042	0.042	I
65	0.077	0.077	D	65	0.139	0.197	I
66	0.079	0.079	D	66	0.053	0.053	I
67	0.108	0.108	D	67	0.069	0.078	I
68	0.079	0.079	D	68	0.073	0.073	I
69	0.074	0.063	D	69	0.110	0.110	I
70	0.205	0.154	D	70	0.206	0.108	I
71	0.129	0.118	D	71	0.017	0.079	I
72	0.078	0.192	D	72	0.043	0.043	I
73	0.076	0.076	D	73	0.053	0.053	I
74	0.079	0.079	D	74	0.059	0.059	I
75	0.354	0.230	D	75	0.110	0.086	I

DMU	$\overline{\sigma}^*$	$\underline{\sigma}^*$	RTS	DMU	$\overline{\delta}^*$	$\underline{\delta}^*$	DTS
76	0.194	0.053	D	76	0.179	0.089	I
77	0.320	0.165	D	77	5.937	5.937	I
78	1.054	0.087	D	78	0.139	0.139	I
79	0.279	0.164	D	79	0.073	0.073	I
80	0.076	0.076	D	80	0.151	0.051	I
81	0.140	0.140	D	81	0.142	0.079	I
82	0.140	0.140	D	82	0.079	0.079	I
83	0.398	0.214	D	83	0.195	0.094	I
84	0.263	0.175	D	84	0.213	0.157	I
85	0.196	0.139	D	85	0.046	0.079	I
86	0.078	0.078	D	86	0.080	0.080	I
87	0.110	0.110	D	87	0.110	0.064	I
88	0.079	0.079	D	88	0.036	0.036	I
89	0.079	0.079	D	89	0.735	0.523	I
90	0.194	0.140	D	90	0.079	0.078	I
91	0.078	0.078	D	91	0.197	0.195	I
92	0.078	0.078	D	92	0.165	0.165	I
93	0.076	0.076	D	93	0.031	0.031	I
94	0.314	0.279	D	94	0.057	0.057	I
95	0.079	0.079	D	95	0.022	0.022	I
96	6.957	-2.314	C	96	1.537	0.195	I
97	0.206	0.110	D	97	0.010	0.010	I
98	0.139	0.139	D	98	0.079	0.079	I
99	0.078	0.078	D	99	0.139	0.139	I
100	0.079	0.078	D	100	0.021	0.021	I

DMU	$\overline{\sigma}^*$	$\underline{\sigma}^*$	RTS	DMU	$\overline{\delta}^*$	$\underline{\delta}^*$	DTS
101	0.094	0.094	D	101	0.044	0.044	I
102	0.076	0.076	D	102	0.139	0.194	I
103	0.077	0.077	D	103	0.341	0.139	I
104	0.079	0.078	D	104	0.068	0.068	I
105	0.076	0.076	D	105	0.195	0.139	I
106	0.078	0.078	D	106	0.061	0.061	I
107	0.079	0.079	D	107	0.002	0.002	I
108	0.164	0.092	D	108	0.113	0.095	I
109	0.110	0.110	D	109	0.186	0.108	I
110	0.078	0.078	D	110	0.196	0.139	I
111	0.160	0.160	D	111	0.073	0.073	I
112	0.077	0.077	D	112	0.195	0.139	I
113	0.079	0.079	D	113	0.049	0.049	I
114	0.094	0.094	D	114	0.064	0.064	I
115	0.079	0.079	D	115	0.041	0.041	I
116	0.077	0.077	D	116	0.139	0.195	I
117	0.079	0.079	D	117	0.069	0.069	I
118	0.140	0.140	D	118	0.071	0.079	I
119	0.078	0.078	D	119	0.142	0.139	I
120	0.078	0.078	D	120	0.139	0.195	I
121	0.078	0.078	D	121	0.041	0.041	I
122	0.162	0.162	D	122	0.042	0.042	I
123	0.095	0.095	D	123	0.249	0.197	I
124	0.076	0.076	D	124	0.361	0.195	I
125	0.079	0.079	D	125	0.035	0.035	I

DMU	$\overline{\sigma}^*$	$\underline{\sigma}^*$	RTS	DMU	$\overline{\delta}^*$	$\underline{\delta}^*$	DTS
126	0.076	0.076	D	126	0.110	0.110	I
127	0.078	0.164	D	127	0.078	0.078	I
128	0.191	0.142	D	128	0.063	0.063	I
129	0.078	0.212	D	129	0.077	0.077	I
130	0.079	0.079	D	130	0.061	0.061	I
131	0.078	0.152	D	131	0.073	0.073	I
132	0.077	0.039	D	132	0.025	0.025	I
133	0.079	0.079	D	133	0.075	0.075	I
134	0.078	0.078	D	134	0.030	0.030	I
135	0.141	0.141	D	135	0.089	0.078	I
136	0.170	0.108	D	136	0.035	0.035	I
137	0.426	0.312	D	137	3.997	1.113	I
138	0.077	0.077	D	138	0.037	0.037	I
139	0.077	0.077	D	139	0.042	0.042	I
140	0.109	0.109	D	140	0.031	0.031	I
141	0.079	0.079	D	141	0.011	0.011	I
142	0.166	0.166	D	142	0.079	0.078	I
143	0.078	0.078	D	143	0.058	0.058	I
144	0.077	0.077	D	144	0.110	0.110	I
145	0.214	0.119	D	145	0.115	0.046	I
146	0.109	0.109	D	146	0.031	0.031	I
147	0.108	0.108	D	147	0.039	0.039	I
148	0.201	0.201	D	148	0.055	0.055	I
149	0.077	0.077	D	149	0.110	0.110	I
150	0.276	0.109	D	150	0.049	0.049	I

DMU	$\overline{\sigma}^*$	$\underline{\sigma}^*$	RTS	DMU	$\overline{\delta}^*$	$\underline{\delta}^*$	DTS
151	0.079	0.079	D	151	0.067	0.067	I
152	0.140	0.140	D	152	0.022	0.079	I
153	0.079	0.079	D	153	0.020	0.020	I
154	0.079	0.079	D	154	0.022	0.022	I
155	0.140	0.140	D	155	0.079	0.079	I
156	0.079	0.079	D	156	0.029	0.029	I
157	0.141	0.141	D	157	0.129	0.064	I
158	0.139	0.139	D	158	0.310	0.154	I
159	0.076	0.076	D	159	0.082	0.082	I
160	0.077	0.077	D	160	0.140	0.140	I
161	0.079	0.079	D	161	0.012	0.012	I
162	0.077	0.077	D	162	0.031	0.031	I
163	0.078	0.078	D	163	0.037	0.037	I
164	0.079	0.079	D	164	0.010	0.010	I
165	0.110	0.110	D	165	0.052	0.052	I
166	0.218	0.079	D	166	0.049	0.049	I
167	0.202	0.142	D	167	0.004	0.038	I
168	0.079	0.079	D	168	0.101	0.086	I
169	0.076	0.109	D	169	0.026	0.026	I
170	0.076	0.076	D	170	0.028	0.028	I
171	0.139	0.139	D	171	0.063	0.019	I
172	0.031	0.078	D	172	0.086	0.086	I
173	0.160	0.093	D	173	0.027	0.027	I
174	0.142	0.142	D	174	0.149	0.074	I
175	0.079	0.149	D	175	0.069	0.025	I

DMU	$\overline{\sigma}^*$	$\underline{\sigma}^*$	RTS	DMU	$\overline{\delta}^*$	$\underline{\delta}^*$	DTS
176	3.417	-1.250	C	176	0.020	0.020	I
177	0.079	0.079	D	177	0.23	0.110	I
178	0.079	0.079	D	178	0.037	0.037	I
179	0.197	0.139	D	179	0.113	0.079	I
180	0.076	0.076	D	180	0.245	0.167	I
181	-3.639	-4.178	I	181	0.110	0.110	I
182	0.079	0.078	D	182	0.014	0.014	I
183	0.076	0.076	D	183	0.026	0.026	I
184	0.077	0.077	D	184	0.021	0.021	I
185	0.076	0.076	D	185	0.042	0.042	I
186	0.078	0.078	D	186	0.040	0.040	I
187	0.069	0.069	D	187	0.046	0.046	I
188	22.250	22.250	D	188	0.038	0.038	I
189	0.079	0.079	D	189	0.036	0.036	I
190	0.115	0.095	D	190	1.785	1.314	I
191	0.206	0.110	D	191	0.003	0.010	I
192	0.112	0.169	D	192	0.035	0.035	I
193	0.176	0.045	D	193	0.213	0.079	I
194	1.214	0.463	D	194	0.040	0.079	I
195	0.087	0.008	D	195	0.051	0.051	I
196	0.197	0.065	D	196	0.022	0.022	I
197	0.079	0.079	D	197	0.012	0.012	I
198	0.078	0.078	D	198	0.084	0.084	I
199	0.213	0.213	D	199	0.159	0.018	I
200	0.198	0.145	D	200	0.034	0.034	I

DMU	$\bar{\sigma}^*$	$\underline{\sigma}^*$	RTS	DMU	$\bar{\delta}^*$	$\underline{\delta}^*$	DTS
201	0.177	0.054	D	201	0.113	0.065	I

5. Conclusion

This study focused on the efficiency evaluation of the insurance industry. For this purpose, we used the dataset of the car insurance policies of Saman Insurance Company during the years 2018-2019 and implemented an extended method to recognize the returns to scale in the case of desirable outputs and the damage to scale in the case of undesirable output. This study can be used in the future policies of the insurance company. For example, the insurance companies can use the results of this paper to adjust the premiums received from different insurers and increase the satisfaction for insurers and their profitability by creating a rating system based on the insurers 'risk.

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