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Quadratic Finite Element Method for Numerical Solution of Underwater Wave Propagation

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ABSTRACT
In this paper, sound propagation in shallow water environments is studied. A Quadratic Finite Element Method (QFEM) by means of functional technique in conjunction with narrow angle parabolic model of hyperbolic wave equation is applied by solving wave equation in depth dependent and range independent fluid-solid media. The capability of FEM based method for modeling of complex solid-fluid boundaries in bottom and also the random surface is employed for formulating of depth operators in Parabolic Equation (PE) method. After simplifying of some integrals in QFEM, the governing system of equations are obtained. Due to existence of mass matrix in the final system, implicit methods seem to be reliable tools for solving such algebraic system of equations. Hence, Crank-Nicolson approach is used for estimation of solution. Checking of proposed method efficiency is determined by some standard test problems. The obtained results show acceptable agreement with the physical behavior of wave propagation nature but for finding of more accurate and capable methods, it requires to develop this method for wide angle PE in associated with more numerical tricks especially in bottom interfaces.

KEYWORDS: Galerkin Finite Element Method, Parabolic Equation, Underwater Wave Propagation, Shallow Water.

1. INTRODUCTION

Sound propagation in range-independent ocean environments is fully understood, numerically computed solutions are stable and robust, and no surprising new physical effects are expected. However, in range-dependent and shallow water environments, sound interacts with both the sea surface and the ocean bottom. Hence, the effects of sediment and bathymetry have a drastic impact on the transmission loss in the sonar equation [1]. For this reason, the impact of the sediment properties and bathymetry on the transmission loss of a shallow water waveguide must be investigated.

Since ocean experiments are costly, this effect should be investigated with mathematical models. However, for range-dependent environments, it is difficult to provide analytical solutions to the acoustic propagation [2]. Therefore, numerical methods are used. Numerical solution of underwater sound propagation has been investigated by many researchers to overcome difficulties caused by bottom and surface interactions using some well-known methods such as parabolic equation method [3,4], fast field program [5], normal mode method [6,7], ray method [8,9], split step method [10], wave-number spectrum method [11] and direct finite-difference or finite-element method [12].

All of the above methods allow the variation of the ocean environment with ocean depth. Fast field program and normal mode models are used in range-independent environments. Parabolic equation, ray and finite-difference or finite-element are useful to model sound propagation in range-dependent environments.

Among the above-mentioned methods, PE based methods usually provide greater efficiency and can provide accurate solutions when backscatter is weak [3]. One of the most advantages of the PE method is the presentation of the acoustic field solution in range and depth in range dependent environments without additional computational effort [4]. Ref. [13], investigates the consequences of approximating the Helmholtz equation with a parabolic equation by comparing the PE equation solution to a coupled mode solution. In [14] an adiabatic PE method of the wavenumber integrals for weakly range-dependent problems was derived. Wave propagation through

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range-dependent isotropic elastic sediments is investigated by an improved PE method in [15]. Ref. [16] applied Helmholtz and parabolic equation modelers to a two dimensional shallow water seamount problem. Ref. [17] derives a new family of higher order PE approximations based on the Pade series expansion. Ref. [18] applies the high order PE proposed in [19] to underwater propagation. These high order equations can be solved by either finite-difference or finite element techniques.

In this paper, a quadratic finite element method based PE is derived for approximation of the wave equation for depth dependent and range independent fluid solid media. In the computational acoustics, the FEM is powerful numerical equipment for wide angle propagation and severe bottom interacting situation environments. The FEM techniques enable to discretize successfully the partial differential equation (PDE) consisting of both complex boundary conditions and the existing of discontinuity in geometrical shapes as well as coefficient discontinuities.

This paper is organized as follows: The problem is formulated in Sec. 2, and the implementation of the numerical solution given in Sec. 3, with computational examples. To evaluate the accuracy of the proposed method, some standard benchmarks are demonstrated in Sec. 4. Finally, Sec. 5 provides some concluding remarks.

2. PRELIMINARIES

Parabolic equation methods use an approximation to the Helmholtz equation to dramatically improve computational efficiency. There are a variety of approximation techniques available which affect the accuracy of the parabolic equation acoustic modelers.
The parabolic equation approximation methods start with the Helmholtz equation for an isotropic-density medium, in cylindrical coordinates (r,θ,z) given by equation (1) [4]:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial p(z,r)}{\partial r}\right)+\rho\frac{\partial}{\partial z}\left(\frac{1}{\rho}\frac{\partial p(z,r)}{\partial r}\right)+k_0^2 n^2 p(z,r)=0 \tag{1}$$

where, p is the acoustic pressure, ρ is the density, z is the receiver depth, r is the receiver range, and k_0 is the wave number and $n(z,r) = c_0/c(z,r)$ is the index of refraction. c_0 is the speed of sound at the source position and c is the speed of sound. The cylindrical angle θ, is ignored by assumption of azimuthal symmetry in velocity. Meanwhile, $\psi(z,r)$ is a transformation parameter from which the acoustic pressure is determined as [6]:

$$p(z,r) = \psi(z,r)H_0^{(1)}(k_0 r) \tag{2}$$

Here $H_0^{(1)}$ represents zero order of Hankel Function of first kind. Applying the asymptotic approximation for far field, $k_0 r \gg 1$, given by equation (3) [6]:

$$H_0^{(1)}(k_0 r)\sim\sqrt{\frac{2}{\pi k_0 r}}\,\mathrm{e}^{i\left(k_0 r-\frac{\pi}{4}\right)} \tag{3}$$

where, i is the notation for the imaginary numbers. In the range independent environment, by using above equation, the wave equation (1) can be factorized as two terms outgoing and incoming parabolic wave equations. In this paper, by solving outgoing one, the effects of scattering that is modeled by incoming parabolic wave equation, is ignored. Thus, equation (1) is written as equation (4):

$$\frac{\partial\psi}{\partial r}+ik_0(1-Q)\psi=0 \tag{4}$$

where, Q is the pseudo differential operator and is defined by $Q=\sqrt{1+q}$. In this equation we have [11]:

$$q = k_0^{-2}\partial_{zz}+(n^2(z)-1). \tag{5}$$

The parabolic equation in standard form is yielded by first order Taylor expansion for the square root Q as equation (6).

$$Q \cong 1+\frac{1}{2}q \tag{6}$$

As the propagation angle of interest be small, the PE in standard form is very good for long range propagation. However, Claerbout, for problems involving large angle, introduced more accurate approximation of Q by applying Pade series expansion of the square root operator. The (1,1)-Pade approximation of Q is defined by [17]:

$$Q \cong \frac{1+a\,q}{1+bq} \tag{7}$$

where, $a = 0.75$ and $b = 0.25$. For more details, readers refereed to [15]. In numerical implementation the starter field for the source located at the zero range is considered as Gaussian starter by the following definition [8]:

$$\psi(z,0) = \sqrt{k_0}\,\mathrm{e}^{-\frac{k_0^2}{2}(z-z_s)^2} \tag{8}$$

3. QUADRATIC FINITE ELEMENT METHOD

The finite element method (FEM) is a numerical technique for providing approximate solutions to boundary-value problems. This method has been adapted to many problems in engineering such as

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underwater scattering problems [16] and study transmission loss and reverberation from a rough seabed in a shallow water waveguide [17]. Recently, wavelet Galerkin finite element based methods have gained widespread use in the field of computational acoustics. A full derivation of this method can be found in [18].

Since the function ψ is the true solution of equation (4), the right hand side of this equation is definitely zero. Because, this true solution cannot actually be known, we obtain only an approximation function ψ. By replacing true solution ψ in (4) by the approximate solution $\bar{\psi}$, the right hand side of equation (4) does not be zero but generates an error residual R as follows [15]:

$$R = \frac{\partial \bar{\psi}}{\partial r} + ik_0(1-Q)\bar{\psi} \qquad (9)$$

While $R \to 0$, the difference between ψ and $\bar{\psi}$ tend averagely to zero. Introducing weight function ν, we get:

$$\int_{\Omega} \nu R d\Omega = 0 \qquad (10)$$

where, Ω is the depth region of interesting, $\Omega = [0, z_{max}]$. In FEM, the region Ω is divided into many element. Let, $z_j = j\Delta z, j = 0,\dots,N$, be the vertical grid points and let $r_0, r_1, \dots, r_M$ be the range grid points where N and M are the number of nodes in depth and range respectively. In the FEM with quadratic basis functions each depth element consists three nodes as $\Omega^e = [z_{j-1}, z_{j+1}], j = 1,\dots,N-1$. For simplicity in approximating of obtained integrals, in arbitrary depth z, local coordinate is defined using parameter ξ, which takes a value between -1 and 1, and $z = z_j + \frac{1}{2}(z_{j+1} - z_{j-1})\xi$. The interpolation function on element Ω^e is expanded as [9]:

$$\overline{\psi^e}(\xi, r) = \sum_{j=1}^{3} u_j^e(r)\varphi_j(\xi), \qquad (11)$$

where, the basis functions $u_j^e(r)$ and $\varphi_j(\xi)$, $j = 1,\dots,3$ are called shape functions. These shape functions for Lagrange quadratic basis functions in local coordinate are expressed by the following quadratic polynomials [17].

$$\varphi_j(\xi) = \begin{cases} -\frac{1}{2}\xi(1-\xi) & j=1 \\ (1+\xi)(1-\xi) & j=2 \\ \frac{1}{2}\xi(1+\xi) & j=3 \end{cases} \qquad (12)$$

The weighted residual method, in which both the approximate wave function ψ and exact solution ψ are expanded by the same basis functions, is called Galerkin method. By substituting equation (11) and $\nu = \psi_j, j = 1,2,3$ on each element into equation (9), it is obtained that:

$$\sum_{j=1}^{3} \frac{\partial u_j^e}{\partial r} \int_{-1}^{1} \varphi_j(\xi)\varphi_k(\xi) d\xi + \frac{ik_0}{2} \sum_{j=1}^{3} u_j^e \int_{-1}^{1} (1 - Q)\varphi_j(\xi)\varphi_k(\xi) d\xi = 0 \qquad (13)$$
$$k = 1,2,3$$

Substituting equation (6) for environment depth operator Q, the integrals in above equation is split to following expression on each element [18]:

$$\sum_{j=1}^{3} \frac{\partial u_j^e}{\partial r} \int_{-1}^{1} \varphi_j(\xi)\varphi_k(\xi) d\xi - \frac{i}{2k_0h^2} \sum_{j=1}^{3} u_j^e \int_{-1}^{1} \frac{\partial \varphi_j(\xi)}{\partial \xi}\frac{\partial \varphi_k(\xi)}{\partial \xi} d\xi + ik_0 \sum_{j=1}^{3} \bar{n}^e u_j^e \int_{-1}^{1} \varphi_j(\xi)\varphi_k(\xi) d\xi \, d\xi = 0 \qquad (14)$$

For the values of refraction coefficients n_1, n_2 in the end points of each element, we have:

$$\bar{n}^e = \frac{(\bar{n}_1^2 - 1) + (\bar{n}_2^2 - 1)}{4} \qquad (15)$$

The above equations can be expressed and the following matrix form.

$$A^e \frac{\partial u^e}{\partial r} + [\mu B^e + \nu C^e]u^e = 0 \qquad (16)$$

where, $\mu = \frac{i}{2k_0h^2}$ and $\nu = \frac{ik_0}{h}$, and,

$$A_{jk}^e = \int_{-1}^{1} \varphi_j(\xi)\varphi_k(\xi) d\xi = \frac{h^e}{30}\begin{pmatrix} 4 & 2 & -1 \\ 2 & 16 & 2 \\ -1 & 2 & 4 \end{pmatrix} \qquad (17)$$

$$B_{jk}^e = \int_{-1}^{1} \acute{\varphi}_j(\xi)\acute{\varphi}_k(\xi) d\xi = \frac{1}{6h^e}\begin{pmatrix} 14 & -16 & 2 \\ -16 & 32 & -16 \\ 2 & -16 & 14 \end{pmatrix} \qquad (18)$$

$$C^e = \bar{n} \int_{-1}^{1} \varphi_j(\xi)\varphi_k(\xi) d\xi = \bar{n}^e A_{jk}^e \qquad (19)$$

To obtain the system equations including all nodes, we assemble these element matrices as the following form:

$$A \frac{\partial u}{\partial r} + [\mu B + \nu C]u = 0 \qquad (20)$$

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Substituting Crank-Nicolson approach $u = (u^{n+1} + u^n)/2$ and forward finite difference $u_r = (u^{n+1} - u^n)/\Delta r$ into equation (11) leads to $N \times N$ tridiagonal matrix system:

$$\left[A + \frac{\Delta r}{2}(\mu B + \nu C)\right] u^{n+1} = \left[A - \frac{\Delta r}{2}(\mu B + \nu C)\right] u^n \qquad (21)$$

where, Δr is the range step and $n = 1, \ldots, M - 1$. By doing all of above steps Galerkin finite element parabolic equation method is derived.

Fig. 1 shows a chart representation of the flow of the QFEM based PE. The purpose of this flowchart is to show the parameters that the user has the power to choose in order to gain either accuracy or computation time. From the top: the user will input the known quantities based on the propagation environment. These values (Known Inputs) will not change. The user then chooses the step values in the second segment based on the frequency. They will also choose the sound speed profile. The QFEM based PE then steps into the loop over range step (Δr) and computes the sound field based on the sound field at the previous step and the user's choice of parameters.

4. NUMERICAL RESULTS

In this section we apply the model developed in the previous section to various propagation problems. The first numerical example concern with shallow water environment consisting of following parameters: The environment involves two layers (water-bottom). The sound speed velocity in water column with the final depth $150m$ is $1500m/s$. Bottom velocity, density and attenuation are $1700\,m/s$, $1.5g/cm^3$ and $0.2dB/km$ respectively. Harmonic source is located at depth $50m$ and range of propagation is $400m$. Severe discontinuity on the solid-fluid interface where is observed especially on shallow water, is important for numerical stability of problem. Some methods such as split step Fourier technique, in such situations, miss their efficiency. Although, these kinds of difficulties are modified by using finer mesh grid for approximation of depth.

Now, we want to use QFEM for overcoming against these difficulties. As can be seen in Fig. 2, the effect of this discontinuity changes the nature of propagation both in the bottom layer and in water column. Transmission loss (TL) in receiver depth $75m$ is depicted in Fig. 3. The first example was limited on the straight bottom. As mentioned before, FEM automatically enables to handle the complex bottom interface. This is what that we want to study in the second example for source depth $90m$ and source frequency $50Hz$. Final implementation range is $10km$. Bottom velocity, density and attenuation are $1600m/s$, $1.5g/cm3$ and $0.2dB/km$ respectively.

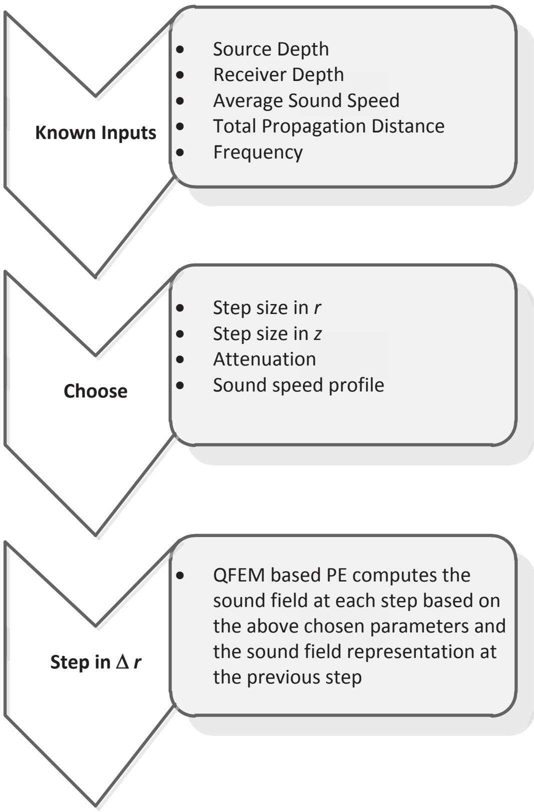

Fig. 1. QFEM based PE Flowchart.

Sound speed profile is plotted on Fig. 4. Fig. 5 shows the bottom geography and TL vs. range and depth. The least velocity in Fig. 4, occurs at depth $150m$, thus we expect a sound channel in this depth. The TL at receiver depth $75m$ is plotted on Fig. 6. Fig. 7 represents the result of wave propagation for constant bottom depth $250m$. Other parameter is set similar to pervious example.

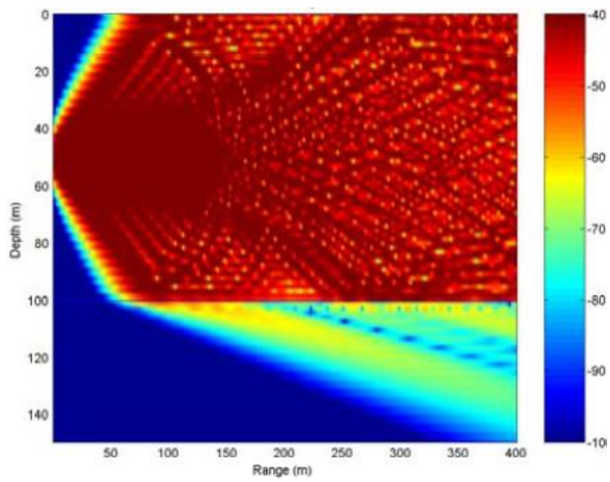

Fig. 2. Wave propagation implementation for shallow water with short propagating range.

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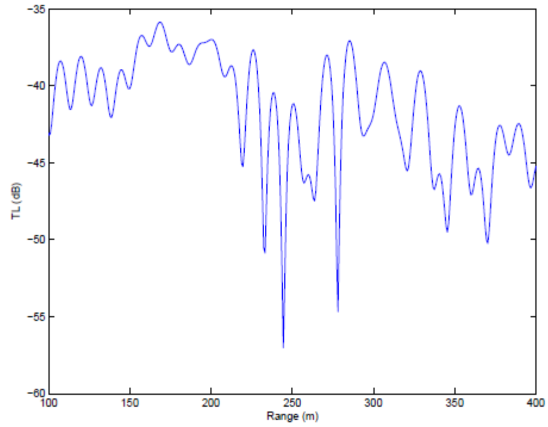

Fig. 3. TL in receiver depth $75m$ and source depth $50m$.

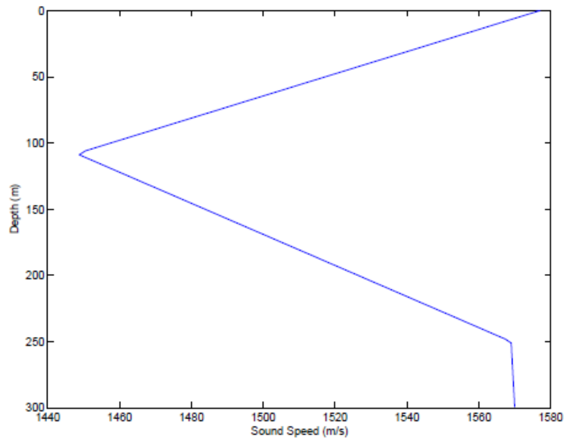

Fig. 4. Sound speed profile

Another example includes a shallow water environment with final depth $100m$ and final range $10km$ that has a shallow sound channel in $10m$. We locate the harmonic source at depth of sound channel. Source frequency is $100Hz$. Bottom boundary has a soft slop that ranged from $65m$ to $70m$ with the genus of clay. The propagation behavior of sound wave in such environment is demonstrated in Fig. 8. The TL at receiver depth $50m$ is also plotted in Fig. 9.

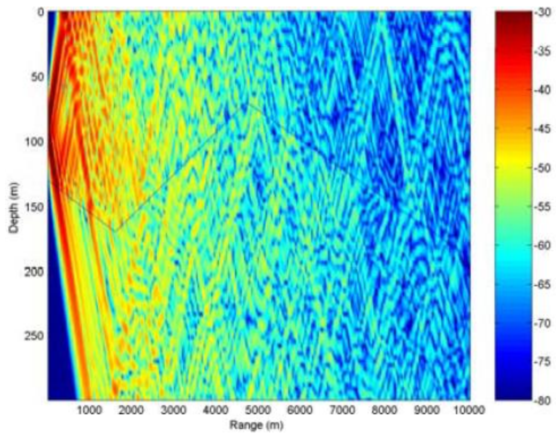

Fig. 5. TL in dB vs. range and depth

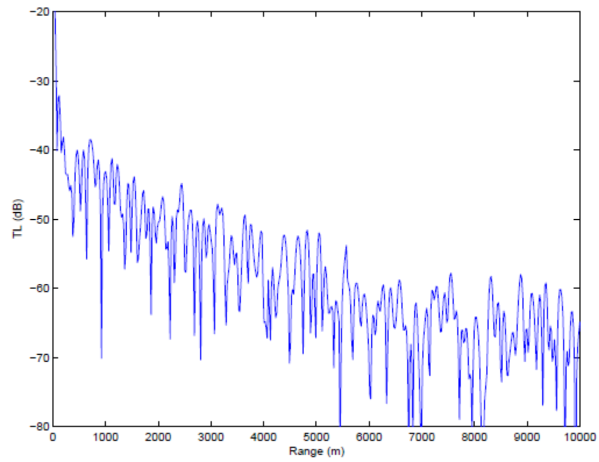

Fig. 6. TL in receiver depth $75m$ and source depth $50\ m$

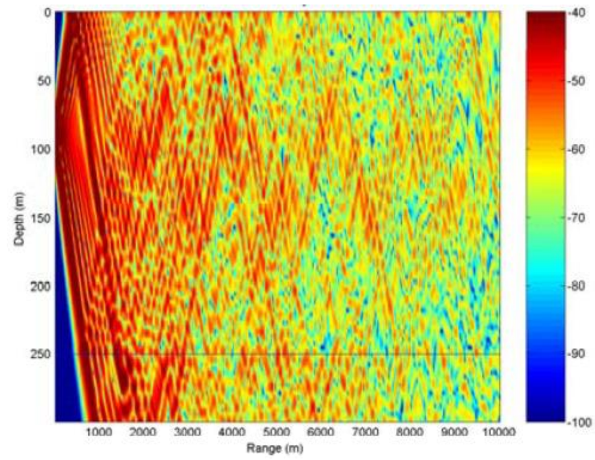

Fig. 7. TL in dB vs. range and depth

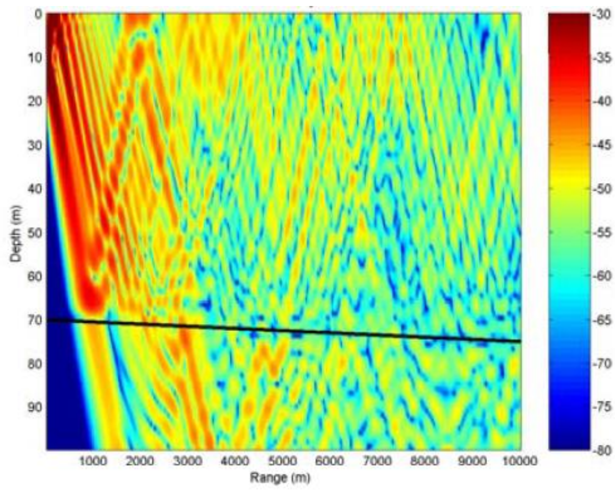

Fig. 8. TL in dB vs. range and depth

In the last scenario, the water depth, is equal to $600m$ over the first $2km$, and decreases beyond that range with a slope of 3 and up to the range of $10km$ where the water depth is $100m$. The sound speed in the water column is $1500m/s$, and the density is

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$1\ g/cm3$. The bottom parameters, namely the sound speed and attenuation in the bottom is $1700m/s$ and $0.1dB/km$ respectively. A single source is located at depth $100m$ and receiver is located at a depth, $25m$. Figs 10 and 11 show the comparison of TL at two frequencies 50 and $100Hz$ resulted from PECan and proposed method. PECan, the Canadian Parabolic Equation (PE) model, is a fully modern underwater acoustic propagation modeling tool capable of computing efficient acoustic predictions in oceanic environments [19]. As can be seen from these figures, the agreement between these two approaches is sufficiently good.

5. CONCLUSION

Wave propagation in shallow water is affected by the elastic parameters of the bottom, which must be taken into consideration for accurate prediction of low frequency sound propagation. The quadratic finite element method based on parabolic equation has been derived and solved numerically for shallow water environment. In this approach, depth opera tor of PE model is discretized by using QFEM. Using quadratic Lagrange basis functions increase the accuracy of method and calculation of integrals in the means of these bases does not have any complexity. After assembling of element matrices boundary condition of bottom interface is imposed to the system.

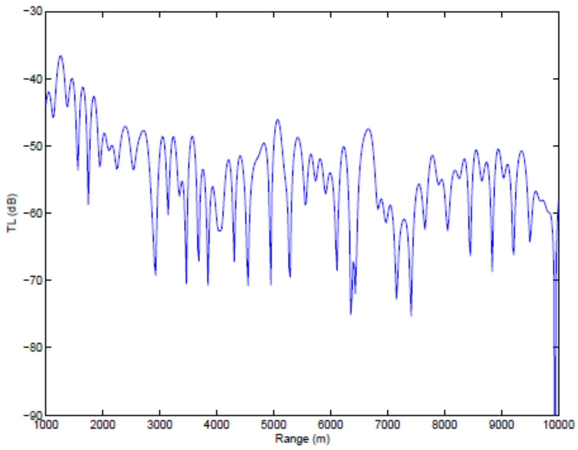

Fig. 9. TL in receiver depth $50m$ and source depth $50m$.

The obtained ordinary differential equation with respect to range is solved by Crank-Nicolson implicit method. Computer code of numerical results is done by MATLAB software. By observing of obtained numerical results in pervious section, capability of method is approved. But for developing this method to operational case, some changes need to be imposed to the structure of method such as development of method to the wide angle situation for decreasing of phase error. Obtained numerical solutions show good agreement in comparison with physically behavior acoustic wave propagation.

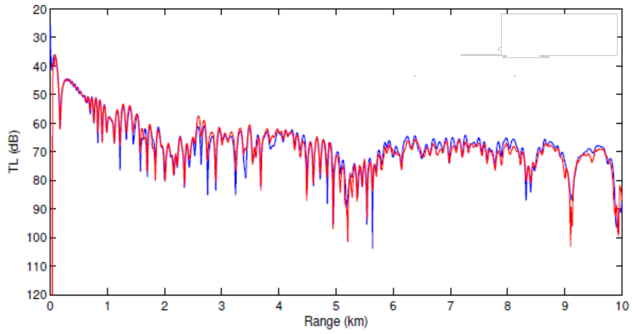

Fig. 10. Comparison TL at 50Hz. Blue line: PECan, Red line: Proposed method.

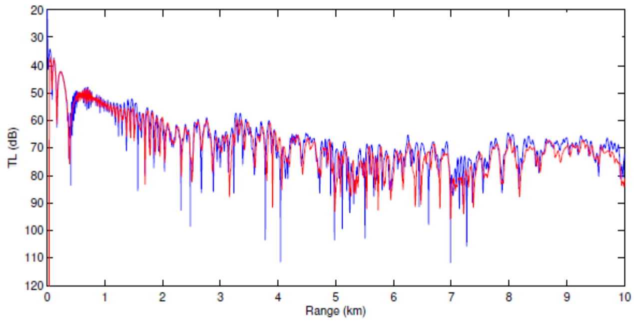

Fig. 11. Comparison TL at 100Hz. Blue line: PECan, Red line: Proposed method.

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