

# A New Cooperative Approach for Cognitive Radio Networks with Correlated Wireless Channels

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Received: 16/Jul/2013

Accepted: 31/Aug/2013

## Abstract

An effective cooperative cognitive radio system is proposed, when the wireless channels are highly correlated. The system model consists of two multi-antenna secondary users (SU TX and SU RX), constituting the desired link and some single-antenna primary and secondary users. The objective is the maximization of the data rates of the desired SU link subject to the interference constraints on the primary users. An effective system, exploiting Transmit Beamforming (TB) at SU TX, cooperation of some single-antenna SUs and Cooperative Beamforming (CB) at them and the antenna selection at SU RX to reduce the costs associated with RF-chains at the radio front end at SU RX, is proposed. Due to the issue of MIMO channels with correlated fading, some problems arise such as inapplicability of the well-known Grassmanian Beamforming as TB scheme at SU TX. We then propose a method to overcome this problem. After formulating the problem, a novel iterative scheme is proposed to find the best TB weight vector in SU TX and best subset of antennas at SU RX, considering the correlated channel.

**Keywords:** Cognitive Radio Networks, Cooperative Communications, MIMO Systems, Correlated Channels.

## 1. Introduction

Cognitive radio (CR) and multi-input multi-output (MIMO) communications are among the most promising solutions to improve spectrum utilization and efficiency. Dynamic and opportunistic spectrum access allows secondary users (SUs) to communicate on temporarily idle or underutilized frequencies. MIMO systems boost spectral efficiency by having a multi-antenna node simultaneously transmit multiple data streams. Newly emerging systems and standards (e.g., WiMAX, 4G Advanced-LTE, IEEE 802.16e) adopt MIMO communications as a core feature. TV white bands have also been approved by the FCC for opportunistic, secondary use [1]. A timely issue is to embrace recent innovations of the two technologies into a single system.

Transmit beamforming with receive combining is one of the simplest approaches to achieving full diversity. Compared with traditional space-time codes, beamforming and combining systems provide the same diversity order as well as significantly more array gain [2] at the expense of requiring channel state information (CSI) at the transmitter. The issue of transmit beamforming in cognitive radio

networks (CRN) has been investigated from various points of view in [3-6]. In [3], transmit beamforming (TB) is designed for MIMO cognitive radio networks with a single primary user-single secondary user network, to minimize the transmit power of the SU, while limiting the interference temperature to PU. The joint problems of TB and power control in CRN were considered in [4], [5], where the objective was to optimize the sum rate of SUs. The joint problems of TB in transmitter and antenna sub-set selection at receiver of a secondary network was considered in [6], where TB was recruited at multi-antenna secondary transmitter to maximize the data rates in SU link, meanwhile the interference on PUs was minimized.

The cooperative beamforming (CB) issue in Cooperative Cognitive Radio Networks (CCRN) was discussed in a few papers such as [7-9]. In [7], with the objective of maximizing the worst SINR of the destinations, a number of relays in a dual-hop amplify-and-forward cooperative scheme in CCRN were recruited. The bursty traffic case in CRN was considered in [8], where CB was exploited to access busy time slots or spatial spectrum holes. A cooperative beamforming aided incremental relaying scheme in CRN was presented in [9], in which, the

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source and relays can exploit CB to activate packet retransmission in busy time slots without inducing interference to primary users.

To the best knowledge of authors, the application of joint TB and CB in CCRN has not been investigated, yet. In addition to the complexity of the CCRN with joint TB and CB, another reason of not taking advantage of joint TB and CB can be attributed to the unavailability of CSI at the transmitter and relay side. In [10], based on Grassmannian Line Packing technic (GLP), a method was presented which does not need CSI at transmitter for TB and works when a limited feedback is available from receiver to transmitter. The beamforming codebook is generated using GLP technic. The transmitter and the receiver preserve the same codebook, which contains, for example,  $M$  weight vectors for the Grassmannian beamforming (GB). In the GB, the index for the optimal beamforming weight vector, not the vector itself, is fed back from the receiver to the transmitter. Thus, the amount of feedback information can be reduced to  $\lceil \log_2 M \rceil$  bits.

A promising way of capturing a large portion of the capacity in MIMO systems at reduced hardware costs and computational complexity is to select a small number of the best antennas from the larger set of antennas available. The performance of systems with antenna selection was shown to be significantly higher than that of the systems using the same number of antennas without any sub-set selection [11]. However, the number of computations required for such optimal selection grows exponentially with the total number of the antennas available. In [6], an alternative approach for receive antenna selection was presented that offers near optimal performance at a complexity significantly lower than the schemes in [12]. In this paper a similar method is employed.

As shown in [13], realistic channel models show a temporal correlation as opposed to the conventional Rayleigh fading channel model. Whereas, the solution introduced in [10], is not able to exploit these correlations to further improve the accuracy of the feedback. To address this issue, new methodologies were proposed [14] [15]; however, their codebooks are fixed codebooks in the sense that once they are designed for a specific transmitter, they cannot adaptively change as the channel changes. In [16], the successive beamforming was proposed which uses an adaptive codebook that was designed for a specific channel model, and does not address more generic spatio-temporally correlated channel models.

The motivation of this work is to determine the optimum TB weight vector at the transmitter side, the optimum CB weight vector at relay side and antenna sub-set selection in the receiver side of a CCRN. Moreover, we introduce a novel adaptive algorithm that utilizes the temporal correlation of the channel to change the TB codebook in a real-time fashion. Initially, the codebook is set to one of the conventional beamforming codebooks in [10], [14], [15]. The codebook is then updated with new feedback information that the transmitter receives. The simulation results show that this adaptive updating technique significantly improves the BER performance compared to the case where the codebook remains fixed.

The contributions of this work can be summarized as

- Proposing a low complexity method to determine the optimum TB and CB weight vectors and the best set of antennas in CCRN.
- Determining the optimal CB weight vector in the virtual array comprised of cooperating relays to remove the interference on PUs.
- Proposing an algorithm to adapt the TB fixed codebook to the correlation of the channels.

The remainder of the paper is organized as follows: in section 2, the system model is described and the problem is formulated. Section 3 proposes a solution for the problem. In section 4, simulation results are presented and section 5 concludes the paper.

**Notation:** Boldface uppercase is used for matrices and boldface lowercase for vectors.  $\det(\cdot)$ ,  $\text{Tr}(\cdot)$  and  $(\cdot)^H$  denote the determinant, trace and the conjugate transpose operators respectively.  $I_M$  denotes an  $M \times M$  identity matrix.  $\text{CN}(0, I)$  represents the distribution of a zero mean circularly symmetric complex Gaussian (ZMCSCG) vector with covariance matrix  $I$ .

## 2. System Model and Problem Formulation

### 2.1 System Description

The system model is depicted in Fig. 1. It is assumed that there are  $N_{SU} + 2$  SUs, two multi-antenna and  $N_{SU}$  single-antenna SUs and  $N_{PU}$  single-antenna PUs in the system. All users use the same frequency band. Two multi-antenna secondary users (SU TX and SU RX) constitute the desired SU link. The core aim of this work is to maximize the data rates of the desired SU link, using TB at SU TX and CB at cooperating SUs. The strategy of cooperation of single-antenna SUs with the desired multi-antenna SU link is

decode-and-forward. Other cooperation strategies can be analyzed in a similar fashion. It is assumed that the CSI is not available at SU TX.

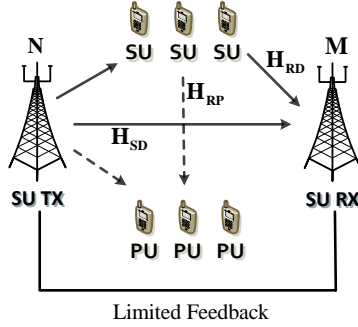


Fig. 1 System Model

Thus, we assume that a kind of quantized beamforming, i.e., Grassmannian beamforming is utilized as transmit beamforming scheme at SU TX. As previously stated, the SU TX and the SU RX preserve the same codebook, which contains a number of beamforming weight vectors for the Grassmannian beamforming (GB). The beamforming codebook is generated using Grassmannian Line Packing technique (GLP) [10]. The GB method works when a limited feedback is available from SU RX to SU TX. Therefore, the optimum TB weight vector is determined at SU RX. Then the index for the optimal TB weight vector is fed back, using a few bits of limited feedback, from SU RX to SU TX. SU RX is equipped with  $M$  antennas. At SU RX  $m$  out of  $M$  antennas are selected. Also, the optimum TB weight vector is the one which maximizes the achievable rates, on condition that interference on PUs does not exceed the threshold. Moreover, zero-forcing beamforming is utilized in the virtual MIMO of cooperating SUs, in order not to disturb the existing PUs as a result of cooperation. The channels between all nodes are assumed to experience frequency at Rayleigh fading. The array manifold, defined as the set of steering vectors is known.

## 2.2 Problem Formulation

The received signal at SU RX can be written as

$$y = H_{SD}w_{TB}x + H_{RD}w_{ZFBF}\hat{x} + i + z \quad (1)$$

where  $w_{ZFBF}$  denotes the zero-forcing beamforming (ZFBF) weight vector at the cooperating SUs;  $H_{SD} \in \mathbb{C}^{M \times N}$  and  $H_{RD} \in \mathbb{C}^{M \times N_{SU}}$  represent the channel coefficients matrix with ZMCSCG entries from SU TX to SU RX and from single-antenna SUs (virtual MIMO) to SU RX, respectively;  $i$  is interference due to primary users and  $z$  represents the white noise (where  $z \sim \text{CN}(0, I_M)$ ). Note that all channel matrices must be considered for each subcarrier, to be more accurate. However, as we aim to find the optimal set of antennas at SU RX, the

optimum transmit and cooperative beamformer weight vectors are applied to all of the subcarriers regardless of different channel characteristics of different subcarriers, the dependence of all system parameters on the subcarrier index can be dropped. For simplicity we also assume that the detection process at cooperating SUs is error-free and as a result (1) can be rewritten as

$$y = (H_{SD}w_{TB} + H_{RD}w_{ZFBF})x + i + z \quad (2)$$

The transmit covariance matrix at the SU TX and at the cooperating SUs are denoted by  $Q$  and  $Q'$ , respectively. We assume that the total transmit power of the SU TX is limited to  $P_{T,1}$ , i.e.,

$$Q = E\{w_{TB}w_{TB}^H x x^*\} = w_{TB}w_{TB}^H P_x \leq P_{T,1} \quad (3)$$

We further assume that the transmit power of the cooperating SUs is constrained to  $P_{T,2}$ :

$$Q' = E\{w_{ZFBF}w_{ZFBF}^H x x^*\} = w_{ZFBF}w_{ZFBF}^H P_x \leq P_{T,2} \quad (4)$$

The covariance matrix of the noise and interference at SU RX is given by

$$U = E\{ii^H + zz^H\} = \sigma_0^2 I_M + H_{PS}H_{PS}^H \quad (5)$$

where  $H_{PS}$  denotes the channel matrix from PUs to SU RX and  $\sigma_0^2$  is the noise level at the SU RX. For satisfactory operation of the incumbent PUs in the presence of the SU TX, interference seen at the PU RX should not exceed a particular threshold,  $P_j$  ( $j = 1, \dots, N_{PU}$ ):

$$h_{SP,j}Qh_{SP,j}^H \leq P_j, \quad j = 1, \dots, N_{PU} \quad (6)$$

where  $h_{SP,j}$  denotes the channel vector from the SU TX to PU  $j$ . The achievable rates of the desired SU link at the output of the maximum ratio combiner at SU RX, using the cooperation of single-antenna SUs and also all available antennas at SU RX can be written as [17]

$$R = \frac{1}{2} \log_2 \det(I_M + H_{SD}QH_{SD}^H U^{-1} + H_{RD}Q'H_{RD}^H U^{-1}) \quad (7)$$

The coefficient  $1/2$  is due to the fact that cooperative transmission only uses half of resources (e.g., time slots, frequency bands, etc.). Similar to the method presented in [6], we define a diagonal matrix  $S$  (where  $S \in \mathbb{R}^{M \times M}$ ):

$$(S)_{ii} = \begin{cases} 1 & i\text{-th receive antenna selected} \\ 0 & \text{otherwise} \end{cases} \quad (8)$$

In the SU RX, the antennas that maximize the achievable data rates are selected. The diagonal matrix provides us with the index of selected antennas in the SU RX. Hence, if  $m$  antennas are chosen ( $m \leq M$ ), new channel matrices,  $\hat{H}_{SD}$  and  $\hat{H}_{RD}$ , with the same dimension as  $H_{SD}$  and  $H_{RD}$ , will have  $M - m$  all-zero rows. Thus, the rate expression at SU RX can be expressed as

$$R = \frac{1}{2} \log_2 \det(I_m + \hat{H}_{SD}Q\hat{H}_{SD}^H + \hat{H}_{RD}Q'\hat{H}_{RD}^H) \quad (9)$$

where  $\hat{H}_{SD} = \hat{U}^{-1/2}SH_{SD}$ ,  $\hat{H}_{RD} = \hat{U}^{-1/2}SH_{RD}$  and  $\hat{U}$  is defined as follows. With the selected receive antennas we have reduced  $m \times 1$  interference and noise vectors which give a new

interference and noise covariance matrix,  $U_{\text{reduced}}$ , of dimension  $m \times m$ . This matrix is inflated to form  $\hat{U}$ , an  $M \times M$  matrix, by adding rows and columns of zeros corresponding to the receive antennas not selected.

The distributed zero-forcing (ZF) beamforming can be realized by a virtual antenna array, which can be created by a set of relays in cooperative relaying networks [18]. With this notation, the problem of joint transmit beamforming, cooperative beamforming and antenna selection can be mathematically explained by

$$\begin{aligned} P: \max_{Q, Q', S} & \frac{1}{2} \log_2 \det(I_M + \hat{H}_{SD} Q \hat{H}_{SD}^H + \hat{H}_{RD} Q' \hat{H}_{RD}^H) \\ \text{s. t.} & \quad (C1): (S)_{ii} \in \{0,1\}, i = 1, \dots, M \\ & \quad (C2): \text{Tr}(Q) \leq P_{T,1} \\ & \quad (C3): \text{Tr}(Q') \leq P_{T,2} \\ & \quad (C4): \text{Tr}(S) = m \\ & \quad (C5): h_{SP,j} Q h_{SP,j}^H \leq P_j, j = 1, \dots, N_{PU} \\ & \quad (C6): |h_{RP,j} w_{ZFBF}| = 0, j = 1, \dots, N_{PU} \end{aligned} \quad (10)$$

where  $h_{RP,j}$ ,  $j = 1, \dots, N_{PU}$ , denotes the channel coefficients from cooperating SUs to single-antenna PUs.

### 2.3 Correlated Channels

It is time to consider the issue of correlated channels. It is noteworthy that the effect of correlated channels must only be considered in determining the TB weight vector, where the Grassmanian codebook matrix has to be adapted to the correlation of the channel. A correlated channel between SU TX and SU RX can be modeled as [19]

$$H_{SD, \text{corr}} = A_T^{1/2} H_{SD, \text{iid}} A_R^{1/2} \quad (11)$$

where  $A_T$  and  $A_R$  are the correlation matrices for the transmitter and receiver antennas, respectively.  $H_{SD, \text{iid}}$  is a matrix with zero-mean circularly symmetric complex Gaussian entries (ZMCSCG). It must be mentioned that for practical applications, the correlation matrices change at a rate much slower than  $H_{SD, \text{iid}}$ . At first, we consider the simple MISO case, i.e.  $M = 1$  and propose an algorithm to adapt the codebook of GB to the correlated channel case. Then, the proposed method is extended to the general case of MIMO systems. If  $A_T$  is known in both the SU TX and the SU RX, a codebook adapted to the correlation of the channel can be computed in both the SU TX and the SU RX:

$$K_{\text{corr}} = A_T^{1/2} K_{\text{iid}} \quad (12)$$

where  $K_{\text{iid}}$  is the codebook matrix with its columns corresponding to the codewords of the codebook, e.g. taken from the Grassmanian codebook and only applicable to uncorrelated channels. Note that by performing (12), the modified codebook for the correlated channel,  $K_{\text{corr}}$ , would have code vectors more shaped in

the directions close to the direction of the channel. Clearly, to compute (12),  $A_T$  must be estimated in both the SU TX and the SU RX. If both the SU TX and the SU RX knew the exact channel realization, they could both calculate  $A_T$  as [19]

$$A_T = E\{H_{SD, \text{corr}}^H H_{SD, \text{corr}}\} \quad (13)$$

It is also possible to estimate the expected value using the average of the last  $P$  channel realizations:

$$A_T \cong \frac{1}{P} \sum_{i=1}^P H_{SD, \text{corr}, i}^H H_{SD, \text{corr}, i} \quad (14)$$

where  $H_{SD, \text{corr}, i}$  is the channel estimate at the receiver at time  $i$ . Based on (14),  $A_T$  can be estimated only if channel realizations are completely known in both the SU TX and SU RX, which is not a practical assumption for SU TX. In order to compute  $A_T$  only based on the feedback link, we assume that at time  $i$ , the SU RX and SU TX use the updated  $K_i$  as the codebook, which is known to both of them. Based on the channel estimate at the receiver, the index of the best codeword,  $w_{ij}$  ( $j$ -th column of  $K_i$ ) is sent to the SU TX through the feedback link. After  $P$  iterations, the correlation matrix in both sides can be approximated by

$$A_T \cong \frac{1}{P} \sum_{i=1}^P w_{i,j} w_{i,j}^H \quad (15)$$

This approximation relies on the fact that for a sufficiently large number of feedback bits, the selected codeword,  $w_{i,j}$  converges to  $H_{SD, \text{corr}, i}^H$ .

In order to extend the adaptive codebook design to MIMO case, it is enough to perform (15) for both SU TX and SU RX. We will postpone extending the results achieved for MISO desired SU link to after proposing an iterative solution for problem P. The target of the next section is to provide an applicable solution for problem P.

### 3. A Sub-optimal Method to Determine the Optimum TB and CB weight vectors and best antennas in Correlated Channels

In problem P, four unknown variables must be determined jointly. Due to excessive complexity, in this section we propose a suboptimum solution to solve P. In this way, the problem P is decomposed into two problems, P1 and P2, according to the following:

$$P1: \max_{w_{TB}, S, P_x} \frac{1}{2} \log_2 \det(I_M + P_x \hat{H}_{SD} w_{TB} w_{TB}^H \hat{H}_{SD}^H)$$

$$\text{s. t.} \quad (C1): (S)_{ii} \in \{0,1\}, i = 1, \dots, M$$

$$(C2): \text{Tr}(w_{TB} w_{TB}^H) \leq \frac{P_{T,1}}{P_x}$$

$$(C3): \text{Tr}(S) = m$$

$$(C4): h_{SP,j} w_{TB} w_{ZFBF}^H h_{SP,j}^H \leq \frac{P_j}{P_x}, j = 1, \dots, N_{PU} \quad (16)$$

and:

$$P2: \max_{w_{ZFBF}} \|H_{RD} w_{ZFBF}\|^2$$

$$\text{s. t. (C1): } \text{Tr}(w_{ZFBF} w_{ZFBF}^H) \leq \frac{P_{T,2}}{P_x}$$

$$(C2): |h_{RP,j} w_{ZFBF}| = 0, j = 1, \dots, N_{PU} \quad (17)$$

Note that prior to solving the problem P2, the problem P1 has to be solved to determine the optimum value of  $P_x$  and  $S$ . Moreover, in problem P2, the received signal power at SU RX due to cooperation of single-antenna SUs is aimed to be maximized, instead of achievable data rates at SU RX. This facilitates the finding the optimal ZF beamforming weight vector, as will be discussed soon.

### 3.1 Solving P1

A straightforward way to solve P1 is to perform an exhaustive search (ES) over all possible combinations of antenna elements and TB weight vectors and optimize over  $P_x$ . Hence, ES amounts to optimizing  $P_x$ ,  $\binom{M}{m} \times \binom{K}{1}$  times subject to interference and power constraints, where  $K$  denotes the number of codewords in the TB codebook matrix. Each optimization of  $P_x$  can be considered as a convex problem. However, the need to iterate through all possible combinations gives a complexity which explodes for higher dimensional systems.

The problem P1 is highly non-convex and can be classified as an example of an integer programming problem, since matrix  $S$  has only binary elements [20]. The non-convexity of the problem arises due to the nature of the objective function, interference and binary constraints. Further, the binary variable renders the problem NP-hard problem. In order to obtain a more computationally efficient approach, we modify the problem in the following way. The binary structure of  $S$  can be relaxed so that the antenna selection variable takes on values in the interval 0 to 1. Finally we note that in this approach the effect of matrix  $U$  cannot be included. This limitation is discussed below. With these changes, P1 can be written as

$$P3: \max_{w_{TB}, S, P_x} \log_2 \det(I_M + P_x S H_{SD} w_{TB} w_{TB}^H H_{SD}^H S^H)$$

$$\text{s. t. (C1): } 0 \leq (S)_{ii} \leq 1, i = 1, \dots, M$$

$$(C2), (C3) \text{ and } (C4) \text{ as in P1} \quad (18)$$

Note that the problem P3 is still non-convex, due to non-concavity of the objective function. Thus, we seek a convex approximation (CA) to this problem.

Proposition 1: With two of the three variables known, the utility function in the problem P3 is concave in the third one and this renders the problem convex in this variable.

Proof: Three different cases must be investigated and for each case it is straightforward to prove concavity of the utility function in (20). Details are omitted for brevity.

Thus, to solve P3, we initialize  $P_x$ ,  $w_{TB}$  and optimize over  $S$ . Then using the optimum  $S$  and the initial value for  $P_x$ , optimum TB weight vector is chosen and ultimately, with  $S$  and  $w_{TB}$  known, optimum value for  $P_x$  is obtained. Since elements of  $S$  are non-binary, the index of the chosen antennas are the  $K$ -largest diagonal elements of  $S$ . In Table 1, the proposed procedure is summarized.

A comment on the convergence of the proposed iterative algorithm is in order here. During the  $(k+1)$ -th iteration  $S^{(k+1)} = \text{argmax}_S P3(S, w^{(k)}, P_x^{(k)})$  is calculated and we obtain achievable data rate  $r_1$ . Then  $w^{(k+1)} = \text{argmax}_{w_{TB}} P3(w, P_x^{(k)}, S^{(k+1)})$  is calculated, giving rate  $r_2$ . Finally  $P_x^{(k+1)} = \text{argmax}_{P_x} P3(P_x, S^{(k+1)}, w^{(k+1)})$  is evaluated and the corresponding achievable data rate  $r_3$ . Since  $r_1 \leq r_2 \leq r_3$  forms a monotonically increasing sequence which is bounded above, we conclude that the sequence of achievable data rates converges to a limit. It is time to incorporate the issue of correlated channels into the proposed iterative algorithm. As mentioned, for each channel realization, the codebook matrix needs to be updated and the best TB weight vector is utilized in estimating the correlation matrices in SU TX and SU RX. The details are described in Table I. Our simulations indicate that iterating 6 times is almost sufficient to attain an optimum value of problem P3.

### 3.2 Solving P2

The cooperating SUs are assumed cognitive in the sense that they can obtain the channel state information (CSI) on the channels from themselves to PUs. The objective function of P2 can be written as

$$\|H_{RD} w_{ZFBF}\|^2 = \sum_{i=1}^m |h_{RD,i} w_{ZFBF}|^2 \quad (19)$$

where  $h_{RD,i}$  denotes the  $i$ -th row of  $H_{RD}$ .

Theorem 1: The optimal zero-forcing beamforming weight vector which maximizes  $\|H_{RD} w_{ZFBF}\|^2$  and satisfies the constraints of the problem P2 is one of the orthogonal projection of rows of  $H_{RD}$  onto the orthogonal complementary  $\gamma^\perp$  of the subspace  $\gamma = \text{span}\{h_{CP}^{(2)}, \dots, h_{CP}^{(N_{PU})}\}$

which maximizes  $\sum_{i=1}^m |h_{RD,i} w_{ZFBF}|^2$ . To satisfy the cooperative transmit power constraint, the elements of the optimum ZFBF weight vector must be multiplied to  $\sqrt{\frac{P_{T,1}}{P_x \text{Tr}(w_{ZFBF} w_{ZFBF}^H)}}$ .

Proof: The channel vector  $h_{RD,i}$ ,  $i = 1, \dots, m$ , can be written as  $h_{RD,i} = a_1^{(i)} e_1 + \dots + a_{N_{SU}}^{(i)} e_{N_{SU}}$ , using the  $N_{SU}$  basis vectors  $\{e_1, \dots, e_{N_{SU}}\}$  (where  $a_k^{(i)} \sim \text{CN}(0,1)$ ,  $k = 1, \dots, N_{SU}$ ,  $i = 1, \dots, m$ ) [21]. Hence, the matrix  $E = [e_1, \dots, e_{N_{SU}}] = I_{N_{SU}}$ , is  $N_{SU} \times N_{SU}$  identity matrix. Then, we consider the set of basis vectors  $\{e'_1, \dots, e'_{N_{PU}}\}$ , which is the orthogonal basis for the subspace  $\gamma = \text{span}\{h_{RP,1}, \dots, h_{RP,N_{PU}}\}$ . Actually,  $\gamma$  is a  $N_{PU}$  dimensional subspace, for the reason that the probability of the realizations of the independent and continuous random vectors  $h_{RP,1}, \dots, h_{RP,N_{PU}}$  being interrelated is very much small and thus can be ignored. If  $\{e'_{N_{PU}+1}, \dots, e'_{N_{SU}}\}$ , is an orthogonal basis for  $\gamma^\perp$ , the orthogonal set  $\{e'_1, \dots, e'_{N_{PU}}\} \cup \{e'_{N_{PU}+1}, \dots, e'_{N_{SU}}\}$ , is another orthogonal basis for  $\mathbb{C}^{1 \times N_{SU}}$ . Similarly,  $h_{RD,i}$  can be represented by  $h_{RD,i} = b_1^{(i)} e'_1 + \dots + b_{N_{SU}}^{(i)} e'_{N_{SU}}$ . Clearly,  $L = [e'_1, \dots, e'_{N_{SU}}]$  is also a unitary matrix. Moreover, by matrix manipulation we have

$$[b_1^{(i)}, \dots, b_{N_{SU}}^{(i)}]^T = L^H [a_1^{(i)}, \dots, a_{N_{SU}}^{(i)}]^T \quad (20)$$

Since the random matrix  $L^H$  is unitary and independent with  $h_{RP,i}$ ,  $[a_1^{(i)}, \dots, a_{N_{SU}}^{(i)}]$  has the same distribution as  $[b_1^{(i)}, \dots, b_{N_{SU}}^{(i)}]$ , from (A.22) in [22]. As a result,  $b_k^{(i)} \sim \text{CN}(0,1)$ ,  $k = 1, \dots, N_{SU}$ ,  $i = 1, \dots, m$ . The ZF beamforming weight vector  $w_{ZFBF}$  is orthogonal to each  $h_{RP,i}$  ( $i = 1, \dots, m$ ). Hence, it is perpendicular to each vector in  $\gamma$ , and belongs to  $\gamma^\perp$ . In order to maximize  $\sum_{i=1}^m |h_{RD,i} w_{ZFBF}|^2$ , we need to find the vector  $w_{ZFBF}^{(i)} \in \gamma^\perp$  which is closest to  $h_{RD,i}$  ( $i = 1, \dots, N_{PU}$ ). From the Closest Point Theorem,  $w_{ZFBF}^{(i)}$  is the orthogonal projection of  $h_{RD,i}$  onto the subspace  $\gamma^\perp$ . The optimum ZFBF weight vector is the one which maximizes  $\sum_{i=1}^m |h_{RD,i} w_{ZFBF}|^2$ :

$$w_{ZFBF, \text{opt}} = \max_{w_{ZFBF}^{(k)}} \sum_{i=1}^m |h_{RD,i} w_{ZFBF}^{(k)}|^2, \quad k = 1, \dots, m \quad (21)$$

Table 1. The proposed algorithm for solving P1 (joint TB and antenna selection in CCRN with correlated channels)

<p><b>Step1-Initializations</b></p> <ul style="list-style-type: none"> <li>• Select an initial value for <math>P_x</math> and <math>P_{T,1}</math></li> <li>• Set <math>K_{\text{corr}} = K_{\text{iid}}</math>.</li> <li>• Set <math>\tilde{A}_T = I_M</math> and <math>\tilde{A}_R = I_N</math></li> <li>• (Estimates of channel correlation matrices)</li> <li>• Select an Initial value for <math>w</math> (one of columns of codebook matrix <math>K_{\text{iid}}</math> which satisfies <math>\text{tr}(w_{\text{TB}} w_{\text{TB}}^H) \leq P_{T,1}/P_x</math>)</li> </ul>
<p><b>Step 2-Calculation:</b> repeat the following for each new channel realization</p> <p>In SU RX do the following</p> <ul style="list-style-type: none"> <li>• Find correlated channel realization, <math>H_{\text{SD}}</math>.</li> <li>• Solve the convex optimization problem and find <math>S</math></li> <li>• Change the codebook to <math>K_{\text{corr}} = \tilde{A}_R^{1/2} K_{\text{iid}}</math>.</li> <li>• Find the optimum <math>w_{\text{TB},i}</math> from <math>K_{\text{corr}}</math> that maximizes achievable rates.</li> <li>• Update <math>\tilde{A}_R = (1 - \beta)\tilde{A}_R + \beta w_{\text{TB},i} w_{\text{TB},i}^H</math>.</li> <li>• Send <math>i</math>, the index of the codeword, to SU TX using limited feedback bits.</li> <li>• Solve the resultant convex optimization problem, knowing <math>S</math> and <math>w_i</math>, to calculate the optimum <math>P_x</math>.</li> </ul> <p>In SU TX do the following</p> <ul style="list-style-type: none"> <li>• Receive the codeword index, <math>i</math>.</li> <li>• Look up <math>w_i</math> from the codebook.</li> <li>• Change the codebook to <math>K_{\text{corr}} = \tilde{A}_T^{1/2} K_{\text{iid}}</math>.</li> <li>• Update <math>\tilde{A}_T = (1 - \beta)\tilde{A}_T + \beta w_i w_i^H</math>.</li> <li>• Use <math>w_i</math> for transmit beamforming.</li> </ul>
<p><b>Step 3-Iteration</b></p> <ul style="list-style-type: none"> <li>• Repeat steps 1 and 2 until convergence. The achievable rate will be the average of the results.</li> <li>• Using the optimum value of <math>P_x</math>, find the SER.</li> </ul>

The cooperative transmit power constraint in P2 makes such  $w_{ZFBF, \text{opt}}$  unique and the Theorem 1 is proved.

## 4. Performance Evaluation

In this section we explain the simulation results based on the proposed solutions for the problem P. However, before elaborating the results, we introduce a new parameter,  $\alpha$ , which controls the interference threshold at the PUs.  $\alpha$  is chosen so that allowable interference at the PUs is a fraction of PU SNR, i.e.  $P_i = \alpha \text{SNR}_{\text{PU}}$  at the PUs. To compare the different approaches, we use the measures of achievable rates and symbol error rates. These are the assumptions for the simulations:

- Achievable rates are determined by averaging over the results obtained from 1000 i.i.d. channels realizations.

- CVX package is used along with MATLAB for simulations [23].
- The SU TX and SU RX are equipped with 3 and 6 antennas, respectively. The codebook matrix is given by [10]

$$K = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} e^{\frac{2\pi j}{3}} & \frac{1}{\sqrt{2}} e^{\frac{4\pi j}{3}} & \frac{1}{\sqrt{2}} e^{\frac{4\pi j}{3}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} e^{\frac{4\pi j}{3}} & 0 & 0 & \frac{1}{\sqrt{2}} e^{\frac{2\pi j}{3}} & \frac{1}{\sqrt{2}} e^{\frac{4\pi j}{3}} \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} e^{\frac{4\pi j}{3}} & \frac{1}{\sqrt{2}} e^{\frac{2\pi j}{3}} & 0 & \frac{1}{\sqrt{2}} e^{\frac{2\pi j}{3}} \end{bmatrix}$$

Note that for highly correlated matrices,  $A_T$  and  $A_R$  have relatively high mean absolute values. The correlation matrices are the same as the correlation matrices proposed in [13], with mean absolute values of 0.96 and 0.37 for high and low correlation matrices, respectively. For the Rayleigh fading scenario,  $A_T = I_M$  and  $A_R = I_N$ . The correlation matrices remain the same throughout the transmission and adaptation phases; whereas,  $H_{SU,iid}$  changes for each new transmission.

In order to depict the BER performance, we assume that 4-QAM is used in the desired link as the modulation scheme. Fig. 2 shows the comparison between the BER performance of CCRN with high and low correlation channel. We assume that 10 cooperating SUs exist in the system and also 3 out of 6 antennas at SU RX have been selected. Note that using the proposed algorithm for a correlated channel scenario results in more than 2 dB performance improvement; while, for low correlation matrices, the BER performance remains the same for both of the TB with fixed codebook and the TB with adaptive codebook. This is because the adaptive codebook is designed to track the correlation matrix; for low correlation channels, the correlation matrices are close to the identity matrix, and thus, the TB with fixed codebook cannot be improved any further; whereas, for highly correlated channels, the correlation matrices have further deviated from the identity matrix and the proposed algorithm can improve the performance by approximating a reliable non-identity correlation matrix. The effect of cooperating SUs on the BER performance of joint TB (with adaptive codebook) and antenna selection in the CCRN with highly correlated channels is depicted in Fig. 3. 3 out of 6 antennas are selected at SU RX.

It can be implied from Fig. 4 that higher data rates can be attained by utilizing the cooperation of SUs. More specifically, by making use of only two cooperative users and for  $\alpha = 0.1$ , data rate is almost identical to the case with no cooperative users and  $\alpha = 0.1$ . Note that the case of highly correlated channel with adaptive

codebook for TB have been considered. An impossible case, i.e.  $\alpha = 1$ , has also been considered to confirm the necessity of cooperation of single-antenna SUs in the proposed system.

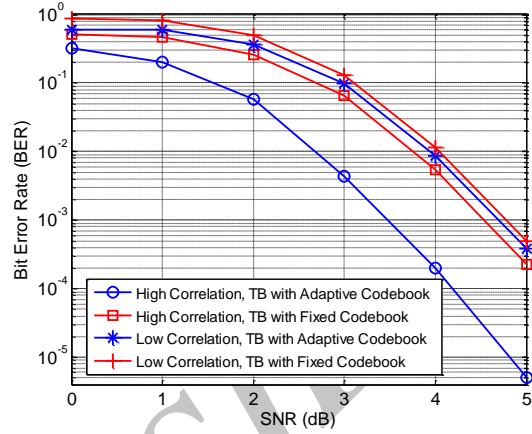


Fig. 2 BER versus SNR for CCRN with highly and lowly correlated channels ( $N_{SU} = 10$  and 3 out of 6 antennas selected at SU RX)

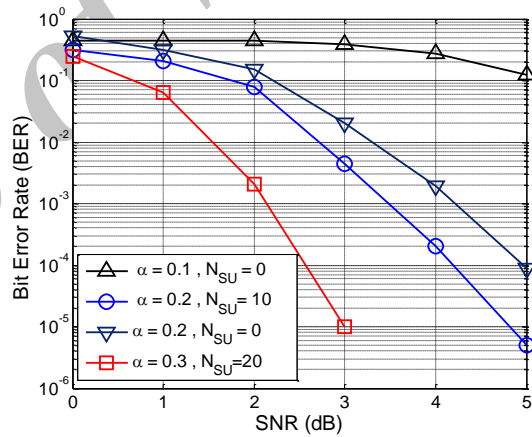


Fig. 3 Symbol Error Rates versus SNR for different values of  $\alpha$  and  $N_{SU}$  (highly correlated channel)

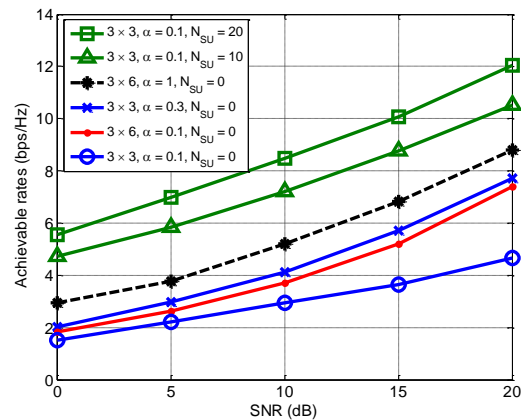


Fig. 4 Achievable data rates versus SNR for different number of  $N_{SU}$ , different value of  $\alpha$  and different number of selected antennas at SU RX

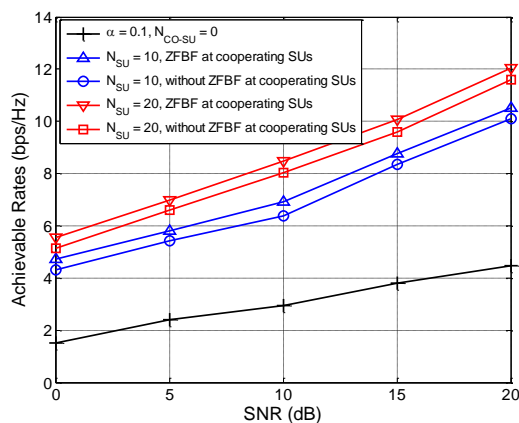


Fig. 5 The effect of ZF beamforming on the data rates of the system

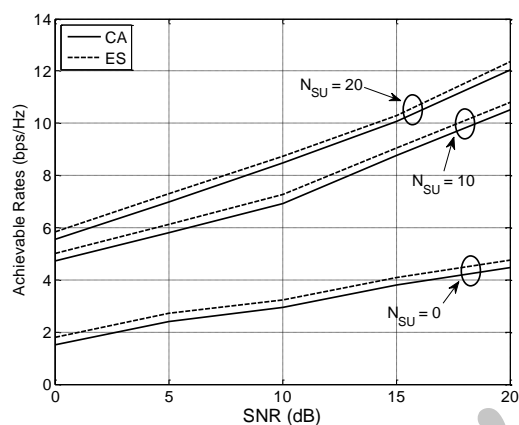


Fig. 6 Comparison between the performance of Convex Approximation and Exhaustive Search method

In Fig. 5, we demonstrate that CB not only removes the interference on PUs, due to cooperation of single-antenna SU with desired SU link, but also leads to an increase in the achievable rates of the desired link. Note that for

all graphs in Fig. 5,  $\alpha = 0.1$ . Note the highly correlated channel case was considered. A comparison between the achievable rates of the multi-antenna SU link, using the CA and ES methods has been performed in Fig. 6. Evidently, the proposed CA method performs very close to ES method, which is very promising.

## 5. Conclusions

Jointly determining the optimum TB and CB weight vectors and antenna selection in the MIMO-CCRN was discussed, considered the correlation in the wireless channels. The scenario consists of 2 multi-antenna SUs and a number of single-antenna SUs and PUs. The problem was formulated and to achieve a computationally efficient solution with much less complexity, we utilized convex approximation method. It was shown through simulations that using the proposed method, a rather complex problem can be solved with reduced complexity. It was further proved, using simulations, that taking advantage of cooperation of single-antenna SUs and ZF beamforming, along with adapting the codebook of the TB to the correlated channel is an inevitable task in the proposed scenario.

## Acknowledgments

This work was supported in part by CSRI (CyberSpace Research Institute) under contract number 17169/500.

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