Target Tracking in MIMO Radar Systems Using Velocity Vector

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Abstract
The superiority of multiple-input multiple-output (MIMO) radars over conventional radars has been recently shown in many aspects. These radars consist of many transmitters and receivers located far from each other. In this scenario, the MIMO radar is able to observe the targets from different directions. One of the advantages of these radars is exploitation of Doppler frequencies from different transmitter-target-receiver paths. The extracted Doppler frequencies can be used for estimation of target velocity vector so that, the radar can be able to track the targets by use of its velocity vector with reasonable accuracy. In this paper, two different processing systems are considered for MIMO radars. First one is the pulse Doppler system, and the second one is continuous wave (CW) system without range processing. The measurement of the velocity vector of the target and its counterpart errors are taken into account. Also, the extended Kalman target tracking by using its velocity vector is considered. Besides, its performance is compared with those of MIMO target tracking without using the velocity vector and conventional radars. The simulation results show that the MIMO radars using velocity vector have superior performance over other above-mentioned radars in fast maneuvering target tracking. Since the range processing is ignored in CW MIMO radar systems, the complexity of this system is much lower than that of Pulse Doppler MIMO radar system, but has lower performance in tracking fast maneuvering target.

Keywords: MIMO Radar; Continuous Wave Radar; Pulse-Doppler Radar; Extended Kalman Filter; Target Tracking; Velocity Vector.

1. Introduction to MIMO Radar Systems

While the idea of Multistatic Radar is not new, Multiple Input Multiple Output (MIMO) radar is very different from it in some aspects. In design of detectors in MIMO radar, it is desired to estimate unknown parameters, such as Radar Cross-Section (RCS), as a part of detection algorithm [1, 2]. One of the specifications of MIMO radars is that the transmitted signals should be orthogonal or highly uncorrelated. We consider two different orthogonal signals for two different MIMO radar systems. The different carrier frequencies with arbitrary narrowband modulations are good candidate for CW radar systems. On the other hand, the Direct Sequence (DS) signaling can be a good candidate in pulse Doppler radars due to its security and easy implementation.

The various properties of DS codes have discussed in many references, for example [3]. The length of desired DS code depends on hardware capabilities. It is clear that more processing gain can be attained with longer DS code, but its implementation constraints the code length from being long. These codes are good candidate for Pulse-Doppler radars so that, each pulse is multiplied in special code sequence and then transmits. Figure 1 shows a block diagram of typical pulse Doppler MIMO radar. In this configuration it is shown that the signal of each transmitter can be separated in each receiver by passing the signal from match filter banks. After integrating K pulses, the intended signals are passed from FFT block and finally they arrive to detection and Doppler estimation unit. In this unit, the target is detected in the cell of under test so that, the location and its various Doppler frequencies are extracted, too. The output of this block which consists of the estimated target location passes through the extended Kalman filter in order to track the target.

The main contribution of this paper compared to previous works is to use the velocity vector in fast target tracking for two different MIMO radar systems. The velocity vector is obtained from various Doppler frequencies extracted from different transmitter-target-receiver paths. We will show that target tracking using velocity vector outperform that of conventional target tracking in MIMO radar systems especially in tracking fast maneuvering target.

In Figure 1, it is assumed that the MIMO radar consists of 2 Transmitters (TXs) and 2 Receivers (RXs) which are located at different positions. The transmitted waveform consists of two orthogonal signals as C1 and C2 (two DS codes) so that, these signals can be separated in each receiver easily by use of orthogonal property. Thus, every receiver can be able to extract two transmitted signals, appropriately. Hence, this configuration is equal to four virtual Bistatic Radars [2]. According to figure 1, the received signals from four virtual radars are processed in central processing unit. In this architecture, the task of

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control unit is to inform the exact frequency and timing of each TX to each RX, so it uses a pulse amplifier in TXs and accurate frequency synthesizer in both TXs implemented in each coherent pulse interval.

The operation of CW MIMO radar is somewhat different from Figure 1. In this system, it is assumed that the separation among carrier frequencies is sufficiently large so that the received signals from different transmitters are extracted easily. Then, the Doppler processing will be done on each extracted signal. In this paper, it is assumed that the CW MIMO radar does not have the range processing, thus the system complexity is much lower compared to pulse Doppler MIMO radar system. Furthermore, CW MIMO radar does not need synchronization among receivers and transmitters. The localization in this system is done based on transmit and receive angles.

In [4], the superiority of target tracking in MIMO radar compared to conventional and phased array radar is taken into account. Its authors use the maximum likelihood estimator for target location and velocity, but they have not considered the effect of Doppler frequencies estimation in target tracking. Target localization for MIMO radar using Doppler frequencies is considered in [5] and [6], but they use grid search method to find target location which has high computational cost. Our paper tries to show the superiority of MIMO radars over conventional and bistatic radars in fast manoeuvring target tracking by using velocity vector. The paper is organized as follows: the signal processing in MIMO radar is discussed in section 2. In section 3 the relation between target velocity and its various Dopplers are considered. Target tracking using extended Kalman filter is devoted in section 4. The simulation is run in section 5 to show that the performance of proposed processing algorithm in MIMO radar against conventional methods and finally, the paper ends with conclusions.

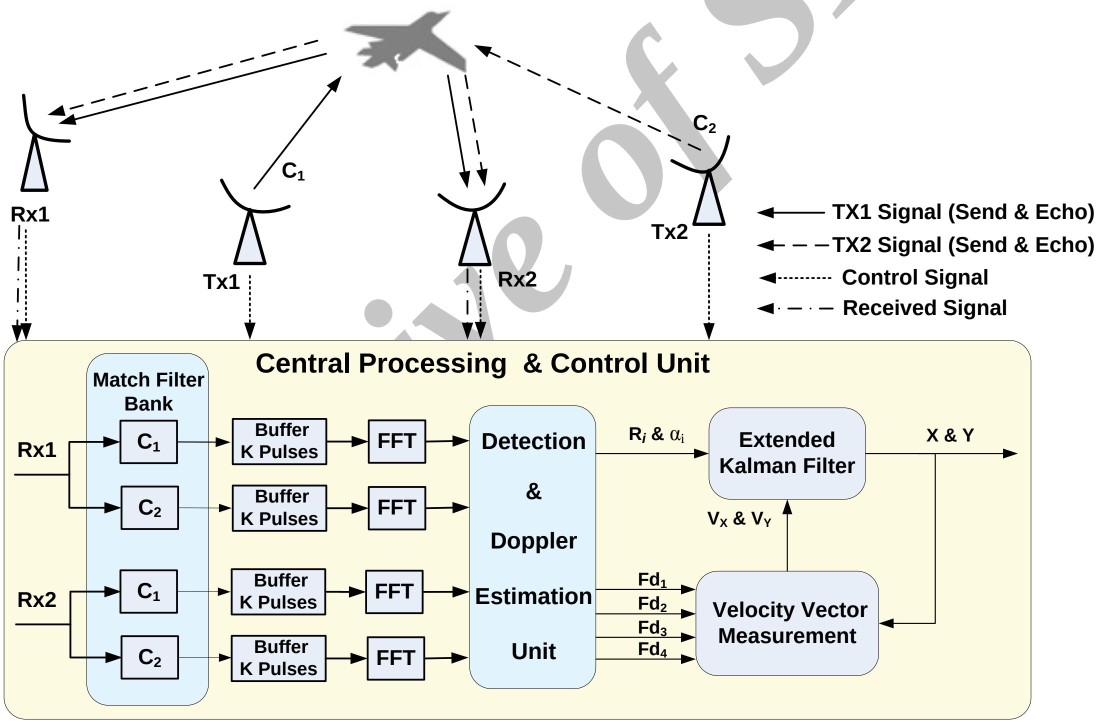

Fig. 1. Configuration of pulse Doppler MIMO Radar

2. Signal Processing in MIMO Radar Systems

The extraction of various Doppler frequencies of target from different paths between TXs-target-RXs with proper accuracy is one of the important aims of signal processing in MIMO radar. These Doppler frequencies are used in velocity vector calculation in section 3. This paper considers two different types of systems; the pulse-Doppler MIMO radar, and CW MIMO radar. We explain these two systems in following subsections.

2.1 Signal model for pulse Doppler MIMO radar

One of the good specifications of the pulse-Doppler system is its capability to obtain various Doppler frequencies of target. It means that pulse Doppler radars are good candidate to attain Doppler frequencies without any ambiguity. In these radars, the Doppler spectrum is periodic with a period equal to the Pulse Repetition Frequency (PRF) where, only the main period $[-\frac{PRF}{2}\ \frac{PRF}{2}]$ is needed [7]. The receiver's noise is considered to be uniformly distributed over the whole spectrum. Clutter occupies a portion of spectrum which contains only low frequencies around the

DC. Therefore, Doppler processing can be used to separate the target and clutter signals in the frequency domain. In order to obtain the sign of Doppler (plus or minus), two channel I & Q are used. The transmitted signals from TX1 and TX2 are considered as:

$$S_{t1} = \sqrt{\frac{E}{T_P}} C_1 Cos(2\pi f_c t)$$
$$S_{t2} = \sqrt{\frac{E}{T_P}} C_2 Cos(2\pi f_c t) \quad (1)$$

where C1 and C2 are two DS codes, $\|C_1\|^2 = 1$, $\|C_2\|^2 = 1$, f_c , is carrier frequency and *E/2* is the energy of singnal, and T_P is pulse duration. The received signals in RX1 and RX2 can be written as:

$$S_{r1} = \alpha_{11}\sqrt{\frac{E}{T_P}} C_1\, Cos\left(2\pi f_c\big(t-\tau_{11}(t)\big)\right) + \alpha_{12}\sqrt{\frac{E}{T_P}} C_2\, Cos\big(2\pi f_c(t-\tau_{12}(t))\big) + n_1(t)$$

$$S_{r2} = \alpha_{21}\sqrt{\frac{E}{T_P}} C_1\, Cos\left(2\pi f_c\big(t-\tau_{21}(t)\big)\right) + \alpha_{22}\sqrt{\frac{E}{T_P}} C_2\, Cos\big(2\pi f_c(t-\tau_{22}(t))\big) + n_2(t) \quad (2)$$

where, $\alpha_{ij}, i,j = 1,2$ models target fluctuations and assumed to be random variable with uniform distribution, $n_i(t), i = 1,2$ is the additive white Gaussian noise, i.e, $n \sim N(0, \sigma_n^2)$, $\tau_{ij}(t)$, $i,j = 1,2$, is equal to delay of echo signals of different paths.

Delay of τ can be modelled in various types, but linear model is one of the practical models. Linear model of delay can be described as [8]:

$$\tau_{ij}(t) = \tau_0 + \frac{R_{ti}(t)+R_{rj}(t)}{C} = \tau_0 + \frac{V_{ti}(t)+V_{rj}(t)}{C} t \quad i,j = 1,2 \quad (3)$$

where, τ_0 is initial delay, $V_{ti}(t)$ is the radial velocity between TXs and target, $V_{rj}(t)$ is the radial velocity between RXs and target and C is the velocity of wave.

As shown in figure 1, the K echo pulses from target integrated in the buffer and are then fed to the FFT block. Therefore, the spectrum of the signal including the Doppler frequencies of the target can be exploited.

2.2 Signal model for CW MIMO radar

In this system, it is assumed that the transmitted signals are the different carrier frequencies with arbitrary narrowband modulations. Therefore, the transmitted signal from two TX1 and TX2 are considered as:

$$S_{t1} = \sqrt{\frac{E}{T_P}} A_1(t) Cos(2\pi f_1 t)$$
$$S_{t2} = \sqrt{\frac{E}{T_P}} A_2(t) Cos(2\pi f_2 t) \quad (4)$$

where *A₁(t)* and *A₂(t)* are arbitrary narrowband modulations with $\frac{1}{T_P}\int_{T_P}^{1} |A_1(t)|^2 dt = 1$ and $\frac{1}{T_P}\int_{T_P}^{1} |A_2(t)|^2 dt = 1$, f_1 and f_2 are carrier frequencies and $E/2$ is the energy of signal, and T_P is time duration for signal processing. The received signals in RX1 and RX2 can be written as:

$$S_{r1} = \alpha_{11}\sqrt{\frac{E}{T_P}} A_1(t)\, Cos\left(2\pi f_1\big(t-\tau_{11}(t)\big)\right) + \alpha_{12}\sqrt{\frac{E}{T_P}} A_2(t)\, Cos\big(2\pi f_2(t-\tau_{12}(t))\big) + n_1(t)$$

$$S_{r2} = \alpha_{21}\sqrt{\frac{E}{T_P}} A_1(t)\, Cos\left(2\pi f_1\big(t-\tau_{21}(t)\big)\right) + \alpha_{22}\sqrt{\frac{E}{T_P}} A_2(t)\, Cos\big(2\pi f_2(t-\tau_{22}(t))\big) + n_2(t) \quad (5)$$

These two signals are passed through the band pass filters which their center frequencies are f_1 and f_2 . Therefore, the received signals from different transmitters are extracted easily. The Doppler frequency of each extracted signal can be obtained by FFT processing.

3. Relation between Target Velocity and Various Doppler Frequencies

The locations of TX, RX and target for a typical scenario for bistatic radar in which 1 transmitter and 1 receiver located far from each other are shown in figure 2.

The imposed Doppler frequency at receiver can be calculated as follows [9]:

$$f_d = \frac{V_x}{\lambda}(Cos\alpha + Cos\beta) + \frac{V_y}{\lambda}(Sin\alpha + Sin\beta) \quad (6)$$

where, λ is the wavelength of transmitted signal, V_y and V_x are the elements of velocity vector in direction of *Y* axis and *X* axis, respectively.

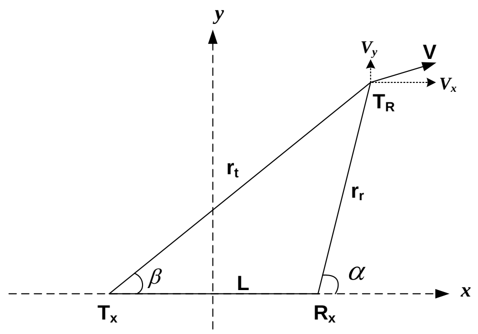

Fig. 2. A typical scenario for bistatic radar.

Now, we extend (6) for MIMO radar in which more than one transmitter and receiver exist. The MIMO radar that consists of *M* TXs and *N* RXs has *MN* various Doppler frequencies that can be obtained as follows:

$$\begin{bmatrix} f_{d1} \\ f_{d2} \\ \vdots \\ f_{dMN} \end{bmatrix} =$$
$$\frac{1}{\lambda}\begin{bmatrix} Cos\alpha_1 + Cos\beta_1 & Sin\alpha_1 + Sin\beta_1 \\ Cos\alpha_2 + Cos\beta_2 & Sin\alpha_2 + Sin\beta_2 \\ \vdots & \vdots \\ Cos\alpha_{MN} + Cos\beta_{MN} & Sin\alpha_{MN} + Sin\beta_{MN} \end{bmatrix}\begin{bmatrix} V_x \\ V_y \end{bmatrix} \quad (7)$$

Thus, the least square solution of velocity vector of target is abstained as:

$$\boldsymbol{V} = (\boldsymbol{G}^T\boldsymbol{G})^{-1}\boldsymbol{G}^T\boldsymbol{f}_d \quad (8)$$

where, $\boldsymbol{V} = [V_x \quad V_y]^{\boldsymbol{T}}$ is velocity vector, and $\boldsymbol{f}_d = [f_{d1} \quad \cdots \quad f_{dMN}]$ is Doppler vector. The matrix $\boldsymbol{G}$ is defined as follows:

$$\boldsymbol{G} = \frac{1}{\lambda}\begin{bmatrix} Cos\alpha_1 + Cos\beta_1 & Sin\alpha_1 + Sin\beta_1 \\ Cos\alpha_2 + Cos\beta_2 & Sin\alpha_2 + Sin\beta_2 \\ \vdots & \vdots \\ Cos\alpha_{MN} + Cos\beta_{MN} & Sin\alpha_{MN} + Sin\beta_{MN} \end{bmatrix} \quad (9)$$

The obtaining target velocity vector can be used in target tracking, where is discussed in next section.

The error due to velocity vector computation is a function of three elements:

1. Error of location estimation.
2. Error of Doppler estimation.
3. Target position with respect to RXs and TXs positions.

Error of location estimation: Because α_i and β_i are a function of target position, computation of sine and cosine of α_i and β_i are influence of this error.

Error of Doppler estimation: This error is due to FFT resolution. The interpolation or other such techniques can be used in order to decrease this undesired effect.

Target position with respect to RXs and TXs positions: The final error of velocity vector influenced with this error severely. In section 5, this effect will be discussed in more details.

4. Target Tracking by Extended Kalman Filter

Kalman filter is a method that recursively minimizes the mean squared error. The important advantage of Kalman filter against other prediction methods is that, it considers the observation noise in its model [10]. This subject is important in target tracking because of observation noise that exists in manoeuvring target [11]. Since Kalman filter considers process noise, it has better performance than other methods practically. We use extended Kalman filter in both pulse Doppler and CW MIMO radar systems due to the nonlinearity of observation equations.

4.1 Target tracking in pulse Doppler MIMO radar

The observation equations in pulse Doppler MIMO radar are range estimates $R_i[n]$ and bearing estimates $\alpha_i[n]$ of *i*th receiver which are derived as following:

$$R_i[n] = \sqrt{(x[n] - x_i^r)^2 + (y[n] - y_i^r)^2}$$
$$\alpha_i[n] = \tan^{-1}\left(\frac{y[n]-y_i^r}{x[n]-x_i^r}\right) \quad (10)$$

Where $(x[n], y[n])$ is the position of target at time *n*, and (x_i^r, y_i^r) is the position of ith receiver. Now, we choose the signal vector to be the target position and velocity components:

$$\mathbf{s}[n] = [x[n] \quad y[n] \quad V_x[n] \quad V_y[n]]^T \quad (11)$$

In general terms the observation equation is:

$$\mathbf{p}[n] = \mathbf{h}(\mathbf{s}[n]) + \mathbf{w}[n] \quad (12)$$

where $\mathbf{w}[n]$ is estimation error (or measurement noise) assumed to be Normal distribution with zero mean and covariance matrix:

$$\mathbf{C} = diag([\sigma_{R_1}^2 \quad \sigma_{\alpha_1}^2 \quad \cdots \quad \sigma_{R_N}^2 \quad \sigma_{\alpha_N}^2 \quad \sigma_{R_{Vx}}^2 \quad \sigma_{R_{Vy}}^2]) \quad (13)$$

$\mathbf{p}[n]$ and $\mathbf{h}(\mathbf{s}[n])$ for pulse Doppler MIMO Radar are obtained as:

$$\mathbf{p}[n] = [R_1[n] \quad \alpha_1[n] \quad \cdots \quad R_N[n] \quad \alpha_N[n] \quad V_x[n] \quad V_y[n]]^T \quad (14)$$

$$\mathbf{h}(\mathbf{s}[n]) = \begin{bmatrix} \sqrt{(x[n] - x_1^r)^2 + (y[n] - y_1^r)^2} \\ \tan^{-1}\left(\frac{y[n]-y_1^r}{x[n]-x_1^r}\right) \\ \vdots \\ \sqrt{(x[n] - x_N^r)^2 + (y[n] - y_N^r)^2} \\ \tan^{-1}\left(\frac{y[n]-y_N^r}{x[n]-x_N^r}\right) \\ V_x[n] \\ V_y[n] \end{bmatrix}_{(2N+2)\times 1} \quad (15)$$

The extended Kalman filter process is given in Table I [12].

The measurement matrix $\mathbf{H}$ is calculated as shown in (16). The process noise is assumed to be Normal distribution with zero mean and modified covariance matrix [13] as follows:

$$\mathbf{Q} = q\begin{bmatrix} \frac{T^3}{3} & 0 & \frac{T^2}{2} & 0 \\ 0 & \frac{T^3}{3} & 0 & \frac{T^2}{2} \\ \frac{T^2}{2} & 0 & T & 0 \\ 0 & \frac{T^2}{2} & 0 & T \end{bmatrix} \quad (17)$$

where *T* is sampling time, and *q* is a constant coefficient.

For considered model, the state transient matrix A can be given as:

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & T & 0 \\ 0 & 1 & 0 & T \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (18)$$

The estimation of vectors velocity, V_y and V_x, by estimation of different Doppler frequencies is one of the main specifications of MIMO radar. Thus, the extended

Kalman filter that is used in this type of radar consists of two inputs: positions (R_i, α_i) and velocity $\left(V_x, V_y\right)$. In order to demonstrate the fair comparison between MIMO radar and conventional radar in target tracking, we simulate them in the same condition. We show that the MIMO radar has better performance in target tracking against the conventional radars when it uses velocity vector in tracking filter. It should be noted that in conventional radars there is not target velocity in directions *X* and *Y* in order to estimate target position.

Table 1. Extended Kalman Filter Process

Initialization

1- $\mathbf{s}[0] = [x[0] \quad y[0] \quad V_x[0] \quad V_y[0]]^T$
2- $\mathbf{M}[0] = 10\mathbf{I}$

For $n = 1, \ldots, N_{Sample}$

a) $\hat{\mathbf{s}}[n|n-1] = \mathbf{A}\hat{\mathbf{s}}[n-1|n-1]$
b) $\mathbf{M}[n|n-1] = \mathbf{A}\mathbf{M}[n-1|n-1]\mathbf{A}^T + \mathbf{Q}$
c) $\mathbf{K}[n] = \mathbf{M}[n|n-1]\mathbf{H}^T[n](\mathbf{C} + \mathbf{H}[n]\mathbf{M}[n|n-1]\mathbf{H}^T[n])^{-1}$
d) $\hat{\mathbf{s}}[n|n] = \hat{\mathbf{s}}[n|n-1] + \mathbf{K}[n](\mathbf{p}[n] - \mathbf{h}(\hat{\mathbf{s}}[n|n-1]))$
e) $\mathbf{M}[n|n] = (\mathbf{I} - \mathbf{K}[n]\mathbf{H}[n])\mathbf{M}[n|n-1]$

4.2 Target tracking in CW MIMO radar

The observation equations in CW MIMO radar are transmit angle estimates $\beta_j[n]$ of *j*th transmitter and bearing estimates $\alpha_i[n]$ of *i*th receiver which are derived as following:

$$\begin{aligned} \beta_j[n] &= \tan^{-1}\left(\frac{y[n]-y_j^t}{x[n]-x_j^t}\right) \\ \alpha_i[n] &= \tan^{-1}\left(\frac{y[n]-y_i^r}{x[n]-x_i^r}\right) \end{aligned} \tag{19}$$

Where $(x[n], y[n])$ is the position of target at time *n*, and $\left(x_j^t, y_j^t\right)$ and (x_i^r, y_i^r) are the positions of jth transmitter and ith receiver, respectively. The signal vector and the observation equation are the same as (11) and (12), respectively. The covariance matrix of measurement noise is considered as:

$$\mathbf{H}[n] = \frac{\partial \mathbf{h}}{\partial \mathbf{s}[n]}\bigg|_{\mathbf{s}[n]=\hat{\mathbf{s}}[n|n-1]} = \begin{bmatrix} \frac{x[n]-x_1^r}{\sqrt{\left(x[n]-x_1^r\right)^2+\left(y[n]-y_1^r\right)^2}} & \frac{y[n]-y_1^r}{\sqrt{\left(x[n]-x_1^r\right)^2+\left(y[n]-y_1^r\right)^2}} & 0 & 0 \\ \frac{-\left(y[n]-y_1^r\right)}{\left(x[n]-x_1^r\right)^2+\left(y[n]-y_1^r\right)^2} & \frac{x[n]-x_1^r}{\left(x[n]-x_1^r\right)^2+\left(y[n]-y_1^r\right)^2} & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ \frac{x[n]-x_N^r}{\sqrt{\left(x[n]-x_N^r\right)^2+\left(y[n]-y_N^r\right)^2}} & \frac{y[n]-y_N^r}{\sqrt{\left(x[n]-x_N^r\right)^2+\left(y[n]-y_N^r\right)^2}} & 0 & 0 \\ \frac{-\left(y[n]-y_N^r\right)}{\left(x[n]-x_N^r\right)^2+\left(y[n]-y_N^r\right)^2} & \frac{x[n]-x_N^r}{\left(x[n]-x_N^r\right)^2+\left(y[n]-y_N^r\right)^2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{16}$$

$$\mathbf{H}[n] = \frac{\partial \mathbf{h}}{\partial \mathbf{s}[n]}\bigg|_{\mathbf{s}[n]=\hat{\mathbf{s}}[n|n-1]} = \begin{bmatrix} \frac{-\left(y[n]-y_1^t\right)}{\left(x[n]-x_1^t\right)^2+\left(y[n]-y_1^t\right)^2} & \frac{x[n]-x_1^t}{\left(x[n]-x_1^t\right)^2+\left(y[n]-y_1^t\right)^2} & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ \frac{-\left(y[n]-y_M^t\right)}{\left(x[n]-x_M^t\right)^2+\left(y[n]-y_M^t\right)^2} & \frac{x[n]-x_M^t}{\left(x[n]-x_M^t\right)^2+\left(y[n]-y_M^t\right)^2} & 0 & 0 \\ \frac{-\left(y[n]-y_1^r\right)}{\left(x[n]-x_1^r\right)^2+\left(y[n]-y_1^r\right)^2} & \frac{x[n]-x_1^r}{\left(x[n]-x_1^r\right)^2+\left(y[n]-y_1^r\right)^2} & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ \frac{-\left(y[n]-y_N^r\right)}{\left(x[n]-x_N^r\right)^2+\left(y[n]-y_N^r\right)^2} & \frac{x[n]-x_N^r}{\left(x[n]-x_N^r\right)^2+\left(y[n]-y_N^r\right)^2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{23}$$

$$\mathbf{C} = diag([\sigma^2_{\beta_1} \; \cdots \; \sigma^2_{\beta_M} \; \sigma^2_{\alpha_1} \; \cdots \; \sigma^2_{\alpha_N} \; \sigma^2_{R_{Vx}} \; \sigma^2_{R_{Vy}}]) \quad (20)$$

p[n] and h(s[n]) for CW MIMO Radar are obtained as:

$$\mathbf{p}[n] = diag([\hat{\beta}_1[n] \cdots \; \hat{\beta}_M[n] \; \hat{\alpha}_1[n] \; \cdots \; \hat{\alpha}_N[n] \; V_x[n] \; V_y[n]]) \quad (21)$$

$$\mathbf{h}(\mathbf{s}[n]) = \begin{bmatrix} \tan^{-1}\left(\frac{y[n]-y_1^t}{x[n]-x_1^t}\right) \\ \vdots \\ \tan^{-1}\left(\frac{y[n]-y_M^t}{x[n]-x_M^t}\right) \\ \tan^{-1}\left(\frac{y[n]-y_1^r}{x[n]-x_1^r}\right) \\ \vdots \\ \tan^{-1}\left(\frac{y[n]-y_N^r}{x[n]-x_N^r}\right) \\ V_x[n] \\ V_y[n] \end{bmatrix}_{(M+N+2)\times 1} \quad (22)$$

The measurement matrix **H** for CW MIMO radar is calculated as shown in (23).

5. Simulation and Results

Different configurations are proposed for RXs and TXs of MIMO radar in [14]. In this paper, we consider two different configurations for two different MIMO radar systems as in following subsections.

5.1 Pulse Doppler MIMO radar

For pulse Doppler MIMO radar, it is assumed that TXs are omni-directional and RXs are directional with stacked beam. By using of stacked beams that is processed digitally, it is possible to search the whole surveillance area simultaneously. In this case, the search time is substantially reduced, but the computational burden is so high.

The range cell of the proposed MIMO radar with two TXs, two RXs and the target trajectory are shown in Figure 3.

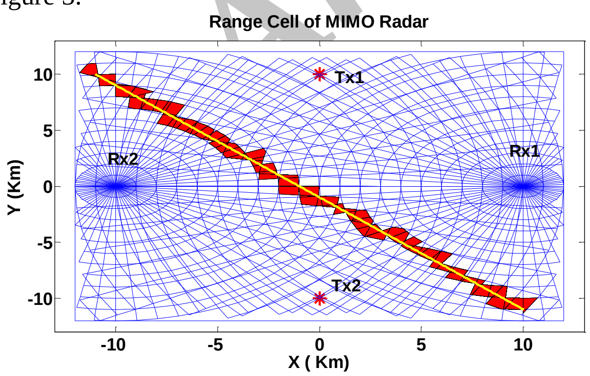

Fig.3. Configuration of Proposed pulse Doppler MIMO Radar

In pulse Doppler MIMO radar configuration, each range cell does not have any symmetric shape. Because it is obtained by subscription of several range cells that lead to smaller range cell. The parameters of system under simulation are considered as follows:

$f_c = 800MHz$	$T_c = 24.4nsec$	$N_C = 1024$
$PRF = 8kHz$	$N_P = 128$	$D.C = \%20$

Where f_c, T_c and N_C are carrier frequency, chip duty and code length, respectively. Also PRF, N_P and $D.C$ are pulse repetition frequency, number of integrated pulses and duty cycle, respectively. The carrier frequency and PRF are determined such a manner that there is not any ambiguity in Doppler and range calculation. We assume the maximum radial velocity is 800m/s, so that the maximum Doppler frequency will be 4kHz. In order to eliminate ambiguity in Doppler calculation, the PRF can be chosen 8 kHz. The number of integrated pulses is considered to be 128. Thus, in Doppler frequency spectrum computation, the FFT with 128 point can be used.

In this paper it is assumed that the target has fast maneuvering movement equations as follows:

$$X = 5000 + 100t + 300Cos(3\pi t)/(3\pi t)$$
$$Y = 3000 - 100t - 300Sin(3\pi t)/(3\pi t) \quad (24)$$

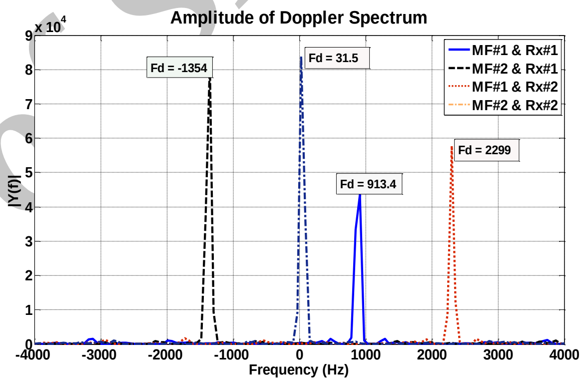

Fig. 4. Signals after FFT Computation

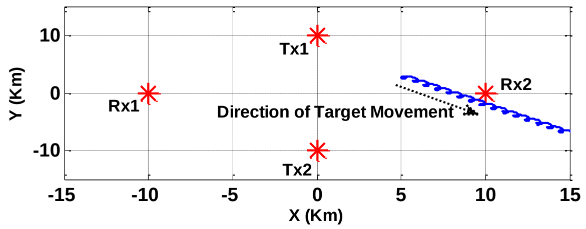

Fig. 5. The Path of moving target from the surveillance area of MIMO radar

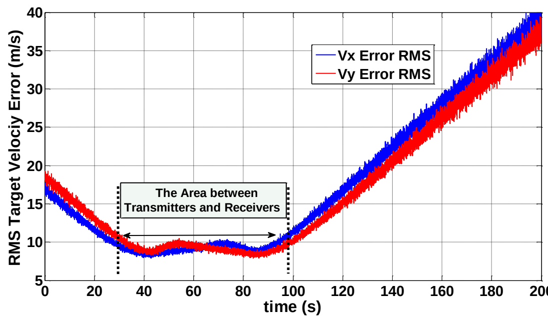

Fig. 6. RMS error of Velocity

The received signals after passing through matched filter and FFT block in RX1 and RX2 are shown in Figure

4. In this case, the signals are in frequency domain so that the Doppler frequencies of different paths can be obtained.

From Figure 4 it is observed that each TX-target-RX paths have different Doppler frequencies. It should be noted that the different amplitude of spectrum between channels is due to RCS fluctuations of target in which is considered in simulation.

In order to show the effect of target position relative to TX and RX antennas, we consider the target manoeuvre as Figure 5. In this case, the best target position relative to TX and RX antennas is obtained by minimum RMS in estimation of target velocity.

The RMS error of velocity is depicted in Figure 6. It is shown that when the target is located between RXs and TXs, the RMS error is less than other positions. This is due to the MIMO radar capability in different Doppler observation of target lead to target velocity be estimated accurately. As a result, The MIMO radar has its best performance when the target located between RXs and TXs antennas. After estimation the target velocity, it is possible to track the target by using of extended Kalman filter, accurately.

It should be noted that, we considered fast manoeuvring targets in target tracking to show that the MIMO radar outperforms in this scenarios. The initial conditions for extended Kalman filter are considered as follows:

$$\mathbf{s}[0] = [4950 \quad 2950 \quad 200 \quad -200]^T \tag{25}$$

$$T = 16msec \qquad q = 10^6 \tag{26}$$

$$\sigma_{f_{d1}} = \sigma_{f_{d2}} = \sigma_{f_{d3}} = \sigma_{f_{d4}} = 50Hz \tag{27}$$

$$\sigma_{R_1} = \sigma_{R_2} = 30m \qquad \sigma_{\alpha_1} = \sigma_{\alpha_2} = 0.1rad \tag{28}$$

Target position estimations for extended Kalman filter in MIMO radar with and without using velocity vector and in conventional radar are demonstrated in Figures 7, 8, and 9, respectively. Clearly, it is observed that the designed extended Kalman filter by using velocity vector in MIMO radar has good performance in target tracking, but in the other cases, the result is not acceptable.

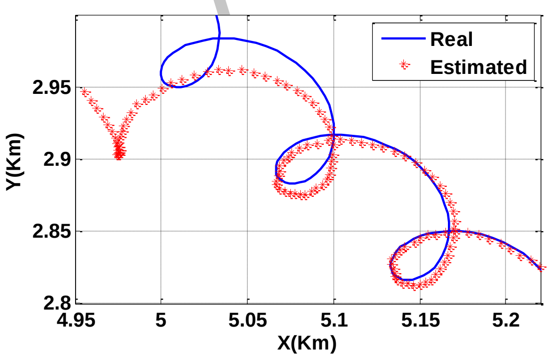

Fig. 7. Location Estimation in pulse Doppler MIMO Radar using velocity vector

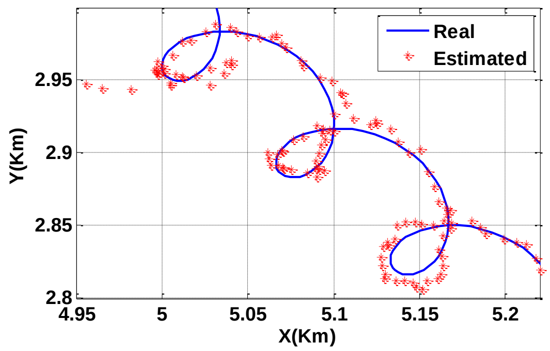

Fig. 8. Location Estimation in pulse Doppler MIMO Radar without using velocity vector

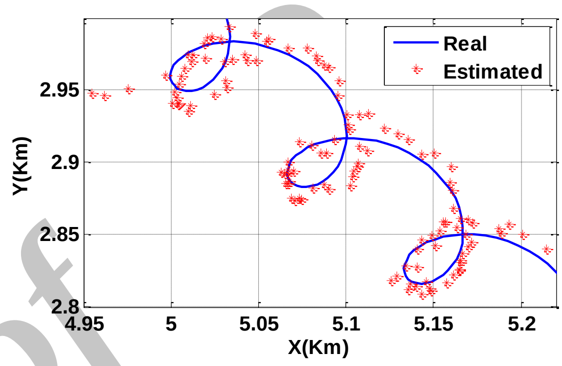

Fig. 9. Location Estimation in Conventional pulse Doppler Radar

The RMS error of target position for standard deviation of measurement noise $\sigma_{R_1} = \sigma_{R_2} = 30m$ and $\sigma_{\alpha_1} = \sigma_{\alpha_2} = 0.1rad$, is shown in Figure 10. As observed from Figure 10, in conventional pulse Doppler radar, the RMS error decreases to 26m, and in pulse Doppler MIMO radar without using velocity vector it decreases to 19m but in pulse Doppler MIMO radar using velocity vector it decreases to 6m (Figure 10).

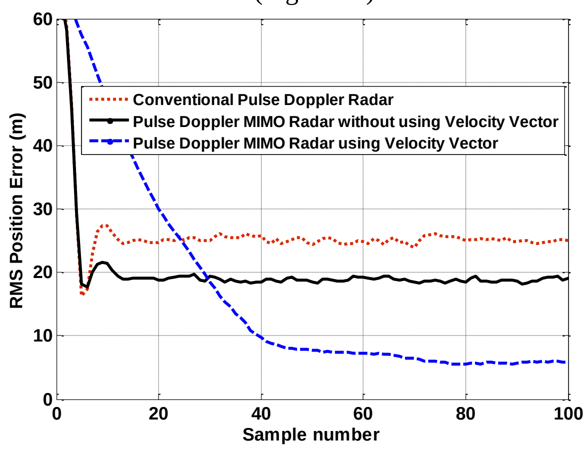

Fig. 10. RMS Error of Location Estimation

In figure 10, we see that the proposed method converges with more delay respect to the other methods. The reason is that in other methods we estimate the target velocity by obtaining difference between new and old estimated target positions and then divide by the sampling time. Due to the big difference between initial position and estimated position at beginning, the value of velocity is big and causes the curve converges quickly. But in our method, target velocity is estimated by Doppler method, so its value is close to target velocity and it converges with more delay. To solve this problem, we can start tracking without using target velocity at several initial steps, then use target velocity vector in tracking filter.

5.2 CW MIMO radar

For CW MIMO radar, it is assumed that both TXs and RXs are directional. In this case, the angle of transmitted signal to target and received signals from target will be estimated. Then, EKF will track the target by using these angles and velocity vector. As mentioned in introduction, there is no range processing in this system, therefore the complexity of this system is much lower than MIMO radar system.

We compare this system with CW bistatic radar in which the angle of transmitted and received signal take part in EKF. Similar to CW MIMO radar, it is assumed that there is no range processing in CW bistatic radar. It should be noted that in bistatic radars, we cannot estimate target velocity in directions X and Y because there is only one equation, and two unknowns in (5). Therefore, this equation is underdetermined, and does not have unique solution.

Since the range processing is ignored in CW MIMO radar systems, they cannot track fast manoeuvring target as pulse Doppler MIMO radars can do. Therefore, we reduce the acceleration of target in movement equations as following:

$$X = 2000 + 100t + 300Cos(0.5\pi t)/(0.5\pi t)$$
$$Y = 6000 - 100t - 300Sin(0.5\pi t)/(0.5\pi t) \quad (29)$$

The initial conditions for extended Kalman filter are considered as follows:

$$\mathbf{s}[0] = [2900 \quad 6900 \quad 200 \quad -200]^T \quad (30)$$

$$T = 16msec \qquad q = 10^7 \quad (31)$$

$$\sigma_{f_{d1}} = \sigma_{f_{d2}} = \sigma_{f_{d3}} = \sigma_{f_{d4}} = 50Hz \quad (32)$$

$$\sigma_{\beta_1} = \sigma_{\beta_2} = 0.1rad \qquad \sigma_{\alpha_1} = \sigma_{\alpha_2} = 0.1rad \quad (33)$$

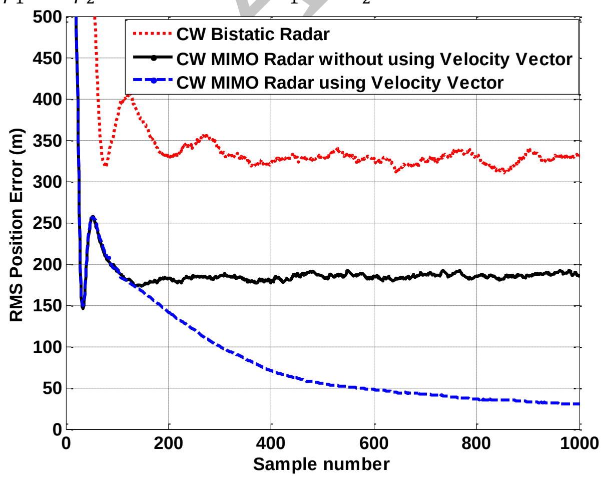

Fig. 11. RMS Error of Location Estimation

Figure 11 shows the RMS position error of different CW radars. For CW MIMO radar using velocity vector, we start tracking without using target velocity at 100 initial steps, then use target velocity vector in tracking filter. It makes this method converge with less delay. As seen in this figure, in CW bistatic radar, the RMS error decreases to 325m, and in CW MIMO radar without using velocity vector it decreases to 180m but in CW MIMO radar using velocity vector it decreases to 30m.

The above results show the superiority of MIMO radar in target tracking against to conventional radar when it uses velocity vector in tracking filter. The MIMO radar can reach to lower RMS error and track the target accurately when it uses velocity vector which is extracted from Doppler processing, and it is only possible in MIMO Radar with widely separated antenna. By using velocity vector in tracking, extended Kalman filter is able to track fast manoeuvring target.

6. Conclusions

In this paper, the problem of target tracking in MIMO radar by using velocity vector is considered. Also, two different types of processing system are taken into account; first on was pulse Doppler system, and the other was CW system. The result that is taken in simulations shows that target tracking by MIMO radar using target velocity is more accurate than that without using velocity vector in MIMO and conventional radar. Because of the MIMO radar capability to exploit the different Doppler frequencies of target, it can be able to estimate the velocity vector of target. By using of this vector and location of target, the radar can be able to track the target accurately. Also, the proposed MIMO radar with CW system without rang processing has lower performance in tracking fast maneuvering targets compared to pulse Doppler system which exploits range processing in target tracking.

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