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The Effects of Price Elasticity Dynamics on a Firm's Profit

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Abstract

This paper studies the dynamic behavior of price elasticity and its effects on the overall profit. Although price elasticity has a significant effect on sales, its dynamics have not been examined so far in pricing models. In this paper, a simple pricing model is suggested in which, price elasticity is considered dynamic. The suggested pricing model is concerned with a monopolist that its objective is to maximize profit by determining the optimal price. Dynamics of price elasticity is described by a quadratic model, with product lifetime as the single dependent variable. By solving the model using the theory of optimal control, a system of differential equations is obtained which can be used to find the optimal price trajectory. Finally, an example is provided to show how the dynamic behavior of price elasticity can influence the firm's overall profit.

Keywords:

Dynamic Behavior of Price Elasticity, Dynamic Pricing, Learning Effects, Optimal Control.

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Introduction

This study investigates the role of dynamic behavior of price elasticity in dynamic pricing over the product life cycle. A considerable number of researches have been done in the area of optimal pricing on new product decision models.

Chin Chun Wu, et al (2009) and Chin Chun Wu, et al (2006) developed two models to maximize the profit by making the optimal price, warranty length and production rate decisions in the first model and optimal price and warranty length decisions in the second one, using a predetermined product lifecycle. DeCroix (1999) derived the optimal price, warranty length and reliability for maximizing profit in an oligopoly. Teng and Thompson (1996) developed a general framework to determine the optimal price and quality policies of new products for a monopolistic manufacturer during a planning period. In the proposed model, they assumed that the demand is determined by price, quality level and cumulative demand. Pei-Chun Lin (2008) proposed a model to jointly determine the price, warranty length, and production rate to maximize the total expected profit of a new product for a monopolist. Glickman and Berger (1976) proposed a model in which they assumed that the customers' demand is determined by a function of price and warranty length. The optimal price and warranty length were obtained by maximizing the manufacturer's profit function.

What is common in all above researches is the invariability of price elasticity during the product lifecycle. This assumption, however, can be faulty because of marketing environment dynamics such as external influences including economic conditions and seasonal variation of customer demand.

For better understanding, assume that there is a monopolist which produces durable products. It is obvious that its customers' sensitivity about the price of the product in the primary stage of lifecycle is different from middle stages and consequently from final stage of product life cycle.

Price elasticity variability, however, may be low in few cases, and constant assumption of this factor does not affect optimal path of decision variables considerably, but in majority of cases, this assumption leads to remarkable deviation from optimal profit.

A considerable body of work has evolved over the last years on the dynamic behavior of price elasticity over the product life cycle in the marketing literature.

Simon (1979) analyzed the price elasticity of consumer products over the "brand" life cycle. In his paper, brand life cycle is distinguished from product life cycle significantly. The only few authors who noticed the distinction between brand and product life cycle, has referred to Mickwitz (1959) who presented some theoretical considerations without any empirical evidences.

According to Mickwitz (1959), Kotler (1971) and Lambin (1970) price elasticity increases in the first three stages of the product life cycle and decreases during the stage of decline, whereas Parsons (1975) does not seem to be fully consistent when he first states, "the absolute magnitudes of ... elasticity exhibit a nonlinear decline over time", and then, "At maturity ... both price elasticity and price cross-elasticity are high."

Simon (1979) derived his empirical study based on prices and quantities sold of 43 brands in seven different markets. Reasonable results however, were obtained only for 35 of these 43 brands. The following relationships are found finally:

1. In 18 of 19 cases (95%) the relation $\varepsilon_{Introduction} > \varepsilon_{Growth}$ is confirmed.
2. In 10 of 14 cases (71%) the relation $\varepsilon_{Growth} > \varepsilon_{Maturity}$ is confirmed.
3. In 8 of 8 cases (100%) the relation $\varepsilon_{Maturity} < \varepsilon_{Decline}$ is confirmed

Shoemaker (1986), though, is not in agreement with Simon. He attributed Simon's findings to particular used function in his research. Shoemaker has pointed out that Simon's findings might be correct if a different estimation process is applied.

In another research by Lilien and Yoon (1988), the trend of sales volume is used as the basic criterion for identifying the product life cycle phases as follows:

The introduction stage is defined as that time period when the annual sales growth rates are less than 5 percent.

The growth stage is defined as that time period when the annual sales growth rates are maintained at higher than 5 percent.

The maturity stage is defined as that time period when the sales growth slows down to less than 5 percent and stays between plus 5 and minus 5 percent.

The decline stage, finally, is defined as that time period when the sale decreases to less than 5 percent annually.

After examination and using time series data, they suggested that price elasticity generally follows a pattern of " stable → decrease → stable/decrease → stable/decrease" over the product life cycle. They summarized their findings as follows:

The level of price elasticity tends to be lower during the later stages of the product life cycle (maturity and decline) than during the earlier stages (introduction and growth).

There is no clear tendency of shift in the level of the price elasticity between the introduction and the growth stages.

Over the later two stages of the product life cycle (maturity and decline), price elasticity shows a tendency to be stable.

In another study by Parker (1992), the theory that "as the product innovation is adopted by non-innovators, elasticity increases" or in other words, the following hypothesis is tested.

H0: Price elasticity of adoption begins low and then increase as the adoption life cycle matures.

The modeling used in his study is based on the model proposed by Bass (1969). This model implies that a durable product category, the amount of first purchases at time t, is determined by the amount of cumulative first purchases up to but not including t, and the total number of potential adopters.

In their study, suggested time-varying relation for price elasticity of product category i is:

$$\eta(t) = \eta_{0i} + \eta_{1i}t + \eta_{2i}t^2$$

Where η_{0i}, η_{1i} and η_{2i} are constants associated with category i. An important advantage of this form over alternatives is its flexibility in fitting various alternative hypotheses (Shoemaker 1986). Equation 6 allows elasticity to be constant (η_{0i}), linear (η_{1i}), or quadratic (η_{2i}) functions of the age of the category (Parker 1992). Parker, in his study screens 70 categories of consumer durable goods which only 17 met the criterion of including multiple stages.

Parker, at last, states that the 17 categories can be considered in two groups:

For "necessities" or categories having reached and maintained penetration levels exceeding 90% and categories with increasing penetration levels, elasticity are either constant, not statistically different from zero, or decline toward the later stages of the adoption life cycle.

For non-necessities facing penetration decline or de-adoption or ones that have reached a stable penetration plateau (not necessities, not increasing, and not declining), elasticity increase during one or all stages of the adoption life cycle.

And finally, in his study, the hypothesis that "elasticity increase as the adoption life cycle matures" is rejected.

As it can be seen, a couple of researchers have tried to find a general framework to model dynamic behavior of price elasticity as a function of product lifetime or cumulative demand, which some of them are supported by empirical evidences and some of them are not. The obtained results in these researches are not totally in conformity with each other and can be summarized as follows:

Our objective is not to determine the dynamic behavior of price elasticity, but to find the optimal path for the single control variable of the suggested model in this paper, i.e. price.

The rest of this paper is arranged as follows. Section 2 details the formulation of the profit model and outlines the related assumptions. Section 3 solves the profit maximization model and analyzes obtained results. Section 4 uses an example to illustrate the effect of dynamic behavior of price elasticity on optimal profit. Section 5, finally concludes and suggests possible extensions to this research.

Table 1: Research on Dynamic Behavior of Price Elasticity (Lilien and Yoon, 1988)

Reference	*Product*	*Stage of Product Life Cycle*						
		Introduction	→	*Growth*	→	*Maturity*	→	*Decline*
Mickwitz[1] *(1959)*		*Increase*		*Increase*		*Increase*		*Decrease*
Parsons[1] *(1975)*		*Decrease*		*Decrease*		*Decrease*		
Wildt (1976)	Consumer products	*(Promotion elasticity decreases over time)*						
Simon[2] *(1979)*	Pharmaceutical, detergents		*Decrease* *Decrease*		*Decrease* *Decrease*		*Stable* *Increase*	
Liu and Hanssens (1981)	Inexpensive gift items	*Increase Over time*						
Lilien and Yoon (1988)	Industrialche micals		*Stable*		*Decrease / Stable*		*Stable/ Decrease*	

1. No empirical support was provided for these propositions
2. Simon's (1979) study was on the brand life cycle rather than product life cycle.

Formulation

Consider a monopolistic firm which wants to maximize the profit in the market with discount rate r. As it is mentioned in the previous section, the price elasticity has been supposed to be constant over the product lifecycle in nearly all of the dynamic pricing studies.

According to table 1, there is no absolute agreement between researchers about the dynamic behavior of price elasticity. Some of them suggest decreasing trend, and some of them achieved increasing trending some researches, however, the increasing then decreasing or decreasing then increasing patterns have been suggested. In this paper, similar to the Parker (1992) study, a time-varying quadratic relation is suggested for the dynamic behavior of price elasticity which is:

$$\varepsilon(t)=\varepsilon_0+\varepsilon_1 t+\varepsilon_2 t^2 \quad (1)$$

Where ε (t) is the price elasticity at t, the past time from product introduction, and ε_0, ε_1 and ε_2 are constant.

Demand function

The model considered in this paper, is a separable function and includes two parts, one for presenting the dependency between price and demand, and the other for consideration of learning effects in demand side, learning effects is brought into the model by the famous Bass model. The Bass model is a very useful tool for forecasting the adoption (first purchase) of an innovation (more generally, a new product) for which no closely competing alternatives exist in the marketplace. This model expresses that demand will shift over time as a function of cumulative demand. The first version of this model, however, did not consider price as a determining variable. In this paper, the proposed function for demand is:

$$q(p(t),Q(t))=k_1\, p^{-\varepsilon(t)}[m(M-Q(t)+n(M-Q(t))Q(t)/M] \qquad (2)$$

Where:

$k_1 > 0$, $\varepsilon(t) > 1$, and

k_1: Amplitude factor,

$\varepsilon(t)$: Dynamic price elasticity

M: the Bass model potential market, the maximum number of cumulative adopters

m: The Bass model coefficient of innovation

n: The Bass model coefficient of imitation

Cost function

The cost function involves only production-related costs. These cost components are described in this section. To determine the reasonable price for the product, production-related costs should be accurately estimated by the producer.

The unit production cost, $C(Q(t))$ consists of two elements- the fixed C_0 and the variable $C_1/(Q(t))$. The fixed cost, in this paper, includes raw material costs and remains constant during the production period. The variable cost, however, is a function of cumulative demanded quantity and decreases with increased production rate and can be interpreted as the labor costs. The reduction of variable cost is due to learning effect which implies there is a progressive improvement in productivity as the number produced increases. The cost function, finally, could be written as follows:

$C(Q(t)) = C_0 + C_1/(Q(t))$ (3)

The decision model

With the above considerations, and the firm's objective which is to find the optimal trajectory of p(t) that maximize the total profit, the mathematical model with a discount rate r and a planning horizon T of the product may be formulated as

$$\underset{p(t)}{Max} \int_0^T e^{-rt}\{[p(t) - c(Q(t))] \cdot q(p(t), Q(t))\} dt \quad (4)$$

Subject to:

$$\dot{Q}(t) = q(p(t), Q(t)) \quad (5)$$

Where$p(t)$, is the unit price at time t, $c(Q(t))$ is the unit production cost, $q(p(t), Q(t))$ is the demand quantity and production rate at time t. The constraint in Eq. (5) indicates that the demand is a function of price and cumulative demand quantity.

Solution procedure

The optimization problem in Eq. (4) and (5)can be solved by applying the maximum principle. To apply the maximum principle, we first obtain the Hamiltonian, which is written as:

$$H(\mathrm{t}) = e^{-rt}\{[p(t) - c(Q(t))] \cdot q(p(t), Q(t)) + e^{rt}\lambda(t) \cdot q(p(t), Q(t))\} \quad (6)$$

Where λ(t)is the adjoint variable of the model and should satisfy the following condition (for convenience, a dot above a variable denotes the first derivative with respect to time):

$$\dot{\lambda} = -H_Q = -e^{-rt}\left((p - c)q_Q - c_Q q + e^{rt}\lambda q_Q\right) \quad (7)$$

The partial derivative of the Hamiltonian with respect to p along with the optimal trajectory would satisfy the following necessary condition held for an optimal solution as follows:

$$\frac{\partial H}{\partial p} = 0 \Rightarrow \frac{\partial H}{\partial p} = q + (p - c)q_p + e^{rt}\lambda q_p = 0 \qquad (8)$$

Solving Eq. (8) for $e^{rt}\lambda$ yields:

$$e^{rt}\lambda = \left(-(p - c) + \frac{p}{\varepsilon}\right) \qquad (9)$$

The first and second derivatives of the demand function with respect to related variables can be expressed by:

$$q\big(p(t), w(t), Q(t)\big) = k_1 p^{-\varepsilon(t)}\left[m(M - Q(t) + n(M - Q(t))\frac{Q(t)}{M}\right]$$

$$q_p = \frac{-\varepsilon(t)}{p}q \Rightarrow q_{pp} = \frac{\varepsilon(t)(\varepsilon(t) + 1)}{p^2}q$$

$$q_{p\varepsilon} = \left(-\frac{q}{p} - \frac{\varepsilon q_\varepsilon}{p}\right)$$

$$q_Q = k_1 p^{-\varepsilon(t)}\left[-m + n - \frac{2nQ}{M}\right]$$

$$q_{pQ} = -\frac{\varepsilon(t)}{p}q_Q$$

The time derivative of the Eq. (8) can be written as:

$$\frac{\partial}{\partial t}\big(q + (p - c)q_p + e^{rt}\lambda q_p\big) = 0 \ \Rightarrow$$

$$2\dot{p}q_p + qq_Q + \dot{\varepsilon}q_\varepsilon - qc_Qq_p + (p - c)\dot{p}q_{pp} + (p - c)qq_{pQ} + (p - c)\dot{\varepsilon}q_{p\varepsilon} + re^{rt}\lambda q_p + e^{rt}\dot{\lambda}q_p + e^{rt}\lambda q_{pp}\dot{p} + e^{rt}\lambda q_{pQ}q + e^{rt}\lambda q_{p\varepsilon}\dot{\varepsilon} = 0 \ \Rightarrow$$

$$\big(2q_p + (p - c)q_{pp} + e^{rt}\lambda q_{pp}\big)\dot{p} = -\big(qq_Q + \dot{\varepsilon}q_Q - qc_Qq_p + (p - c)qq_{pQ} + (p - c)\dot{\varepsilon}q_{p\varepsilon} + re^{rt}\lambda q_p + e^{rt}\dot{\lambda}q_p + e^{rt}\lambda q_{pQ}q + e^{rt}\lambda q_{p\varepsilon}\dot{\varepsilon}\big) \qquad (10)$$

Based on the above equation (Eq. (10)) and the state equation (Eq. (5)), the following system of differential equations will be obtained:

$$\begin{cases} \dot{p} = rp + \frac{r\varepsilon c - pq_Q}{(1-\varepsilon)} + p\frac{\dot{\varepsilon}}{\varepsilon(1-\varepsilon)} \\ \dot{Q} = k_1 p^{-\varepsilon(t)}\left[m(M - Q(t) + n(M - Q(t))\frac{Q(t)}{M}\right] \end{cases} \qquad (11)$$

Based on the above system of differential equations and the model parameters, the optimal trajectory for control variable, price, can be obtained over the time. As it can be seen, the dynamic behavior of price elasticity is appeared in the price differential equation by the ratio of $p\frac{\dot{\varepsilon}}{\varepsilon(1-\varepsilon)}$. Considering assumption a > 1 and the positivity of price, the sign of the mentioned ratio has the reverse sign to the$\dot{\varepsilon}$. It implies that, if the firm's objective is to maximize its profit, and the price elasticity slope is negative, the price should be increased and vice versa, if the price elasticity slope is positive, the price should be decreased. The effect of dynamic behavior of price elasticity equals to product of $\frac{p}{\varepsilon(1-\varepsilon)}$ and $\dot{\varepsilon}$. So consideration of dynamic behavior of price elasticity depends on the magnitudes of these two components: $\frac{p}{\varepsilon(1-\varepsilon)}$ and $\dot{\varepsilon}$. If the amount of $\frac{p}{\varepsilon(1-\varepsilon)}$ is considerable, the deviation from optimal profit will be remarkable. In these kinds of cases, one who has not considered the dynamic behavior of price elasticitycannot claim that his trajectory is optimal. In the continuation, an example will be presented to clarify the importance of consideration of the dynamic behavior of price elasticity in pricing models.

Example

Suppose that there is a monopolist which its objective is to maximize the profit during a predetermined period of time (for example2 years). The firm wantsto sell its products to 95% of the potential market after determined period. Assume that the product and the market have the following parameters (all of these parameters are arbitrary and can be changed by the model examiner).

k1	50	C_0	8
m	0.3	C_1	5
n	3	r	0.1
M	1000		

According to the above parameters, firm wants to have the potential market purchased 950 of its products after 2 years. Assume that the product's price elasticity is in one of the following forms in table 2. As it was mentioned, price elasticity has a quadratic form with product lifetime as the dependent variable (in table 2 various forms of price elasticity relation are titled from S1 to S5).

Table 2: Various forms for price elasticity relation

State	ε_2	ε_1	ε_0
S1	0.4	-0.8	2.2665
S2	-0.4	0.8	1.7333
S3		0.267	1.7333
S4		-0.2665	2.2665
S5			2

If the price elasticity curve is drawn for each of the above forms, the following diagram is obtained (Figure 1).

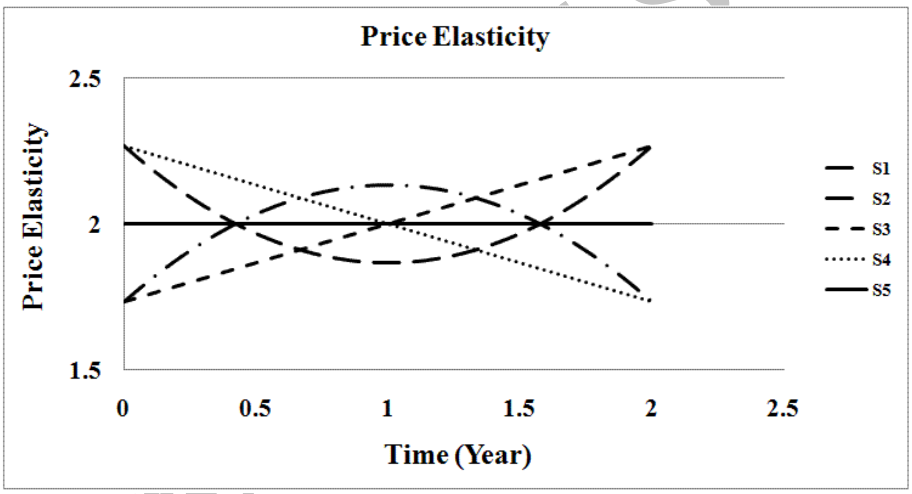

Figure 1: Different forms of price elasticity relation

Figure 2 shows the obtained curves for optimal price trajectories by using the achieved system of differential equations and the above product and market parameters, regardingto each of the price elasticity relations.

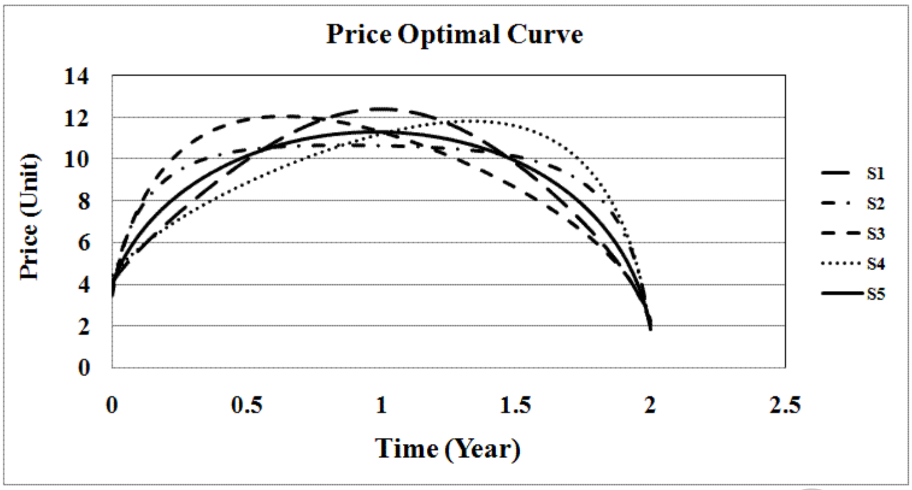

Figure 2: Optimal price curves for different forms of price elasticity relation

In addition, figure 3 shows the cumulative demand curves that correspond to each form of price elasticity relation.

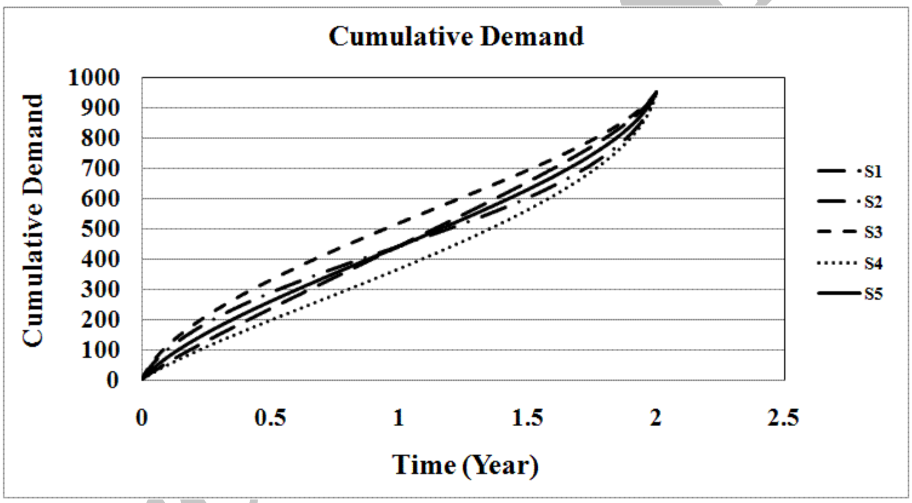

Figure 3: Cumulative demand curves for different forms of price elasticity relation

Now, regarding to the optimal prices and incurred costs to the firm, the following profits are obtained for every price elasticity relation (table 3).

Table 3: The overall profit for each of the price elasticity relations

State	Overall Profit (after 2 years, Unit of Price)
S1	661
S2	396
S3	731
S4	560
S5	388

In this example, the profit related to the fixed price elasticity equals to 388 and is the minimum value of all other profits corresponding to other forms of price elasticity.The maximum deviation from the fixed price elasticity can be seen in S3. The profit in S3 is 88 percent higher than the minimum. These deviations show that (at least in this example), the dynamic behavior of price elasticity can play a crucial role and much attention should be paid to this issue.

Conclusion and suggestions

The present study investigated the effects of the dynamic behavior of price elasticity in pricing models and optimal trajectories. As it was mentioned in introduction section, only a limited number of researches have focused on this issue and designed a practical model.Even those researchers, who have paid attention to the dynamic behavior of price elasticity, have not reached to a common conclusion. In this paper, a general quadratic model with lifetime as the dependent variable is considered to present the dynamic behavior of price elasticity.This quadratic model covers nearly all the obtained results bythe researchers. A simple pricing model was suggested with the price as the single control variable. Then, an example wasprovided to show the importance of the dynamic behavior of price elasticity.

For future works, the considered models for representing the dynamic behavior of price elasticity can be strengthened by appropriate mathematical relations and empirical evidences. Suggesting a model that has therequired accuracy can be very applicable for pricing models.

The pricing model suggested in this paperis a simple one. One potential topic for further research is to develop amodel which can successfully represent the dynamics of the business environment and includes the most important variables of the marketing field.

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