

# Soft Switching MBC Controller for MIMO Linear Hybrid Systems

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## ABSTRACT

*Switching supervisory is the most important section of a feedback control process in MIMO hybrid systems. By choosing a non-compatible controller, system may go to unstable mode or high overshoot response. In this paper, a new method of switching for selecting MBC controllers is discussed. Results of the simulation show the MIMO (2-in 2-out) linear hybrid system can be switched stable and low overshooting in switching time. This method can be expanded to nonlinear MIMO systems.*

## 1. INTRODUCTION

One of the most common tasks in nonlinear control is to design a feedback algorithm that robustly, asymptotically steers a dynamical system to a target set. This fact motivates the extensive literature on asymptotic stabilization for nonlinear differential equations and difference equations. In a quest to provide more flexible tools for achieving the stabilization task and wider relevance for its solution, recent research efforts have focused on developing control algorithms that produce closed-loop systems where continuous variables interact with variables that make instantaneous jumps. A special case is when the control algorithm contains logic variables which take on discrete values. Systems with continuous variables and variables that jump are called hybrid systems.

Stability is a very important property of feedback control systems. For MIMO linear feedback control systems [1,2],

methods for proving or disproving system stability are well known. In control practice it is quite common to use several different controllers and switch between them with some type of logical device. One example is systems with selectors which have been used for constraint control for a long time. Systems with gain scheduling are another example. Both selectors and gain scheduling are commonly used for control of chemical processes, power stations and in flight control. Other examples of systems with mode switching are used in robotics.

It is well known that hybrid systems [3,7] are difficult to analyze. Nonetheless they are used more and more. The reason for this is that they give better performance than ordinary systems and that they can solve problems that cannot be dealt with by conventional control. In process control it is common practice to use of several MBC controls [4,5] for hybrid systems

with a controller selection supervisor. Such a controller can be designed very elegantly using the results of this paper.

A continuous-time, the autonomous hybrid system [6] is a system of the form:

$$\begin{aligned} \dot{X}(t) &= f(X(t), m(t)) \\ m(t^+) &= \varphi(X(t), m(t)) \end{aligned} \quad (1)$$

Where  $x(t)$  is the continuous state vector of the Hybrid System at time instant  $t$  and  $m(t) \in M := \{1, j\}$  its discrete state.  $m(t^+)$  denotes the updated discrete state right after time instant  $t$ .  $H = \mathfrak{R}^n \times M$  is called hybrid state space. The function  $f: H \rightarrow \mathfrak{R}^n$  describes the behavior of the continuous state and  $\varphi: H \rightarrow M$  describes the behavior of the discrete state of the Hybrid System.  $f$  is assumed to be continuously differentiable.  $S_{m_1, m_2} = \{x: \varphi(x, m_1) = m_2\}, m_1 \neq m_2$ , denotes the switch set from discrete state  $m_1$  to discrete state  $m_2$ . A solution  $x(t)$  of the Hybrid System for a particular tuple  $(x(0), m(0))$  of starting points is called a trajectory. A Hybrid System is called piecewise affine if for each  $m \in M$ ,  $f(x, m)$  is affine in  $x$ , i.e. there exist a

matrix  $A_m \in \mathfrak{R}^{n \times n}$  and a vector  $b_m \in \mathfrak{R}^n$ , such that  $f(x, m) = A_m x + b_m$  for all  $x$ .

## 2. NEW FORM MBC CONTROLLER

MBC controllers are one of the most stable controllers, because they have both state detector and state feedback. This type of controllers needs for two signals  $U$  (in) and  $Y$  (in) as inputs and  $U$  (out) as output. Whereas MBC controller has an output signal ( $U$ ) which connects to the main system (Plant). Figure 1 shows an MBC controller and a plant.

In this form of MBC controller  $U$  (in) and  $U$  (out) joint practically together. Figure 2 shows how to connect of the feedback controller to the hybrid system. That means, at this time the other controllers have non-conventional signal to control. In this method, a new section which called supervisor is used which works based on signal error  $(y - \hat{y})$ . So, it is necessary to redesign the classic form of MBC controller as figure 3. In the new MBC controller,  $U$  (in) is disconnected from  $U$  (out) point. But it must be used as  $U$  (control signal). Figure 4 shows the final form of a feedback control of hybrid system based on new form MBC controllers.

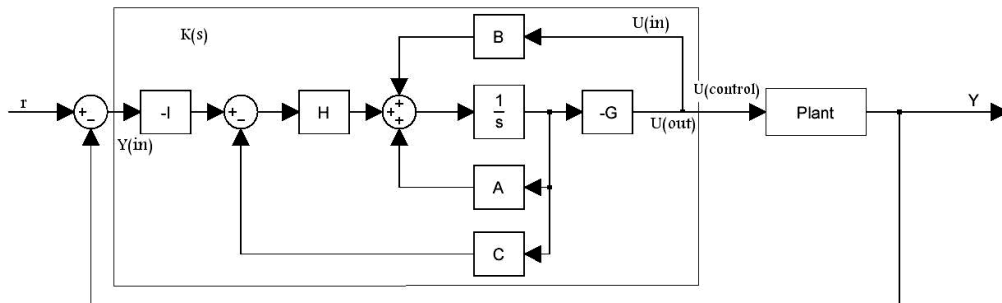


Figure.1. Classic MBC controller

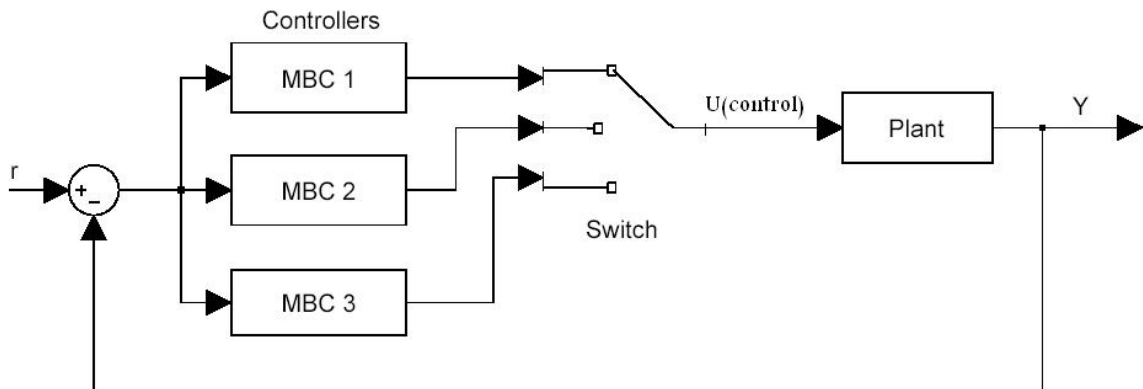


Figure .2. Classic switch control of hybrid systems

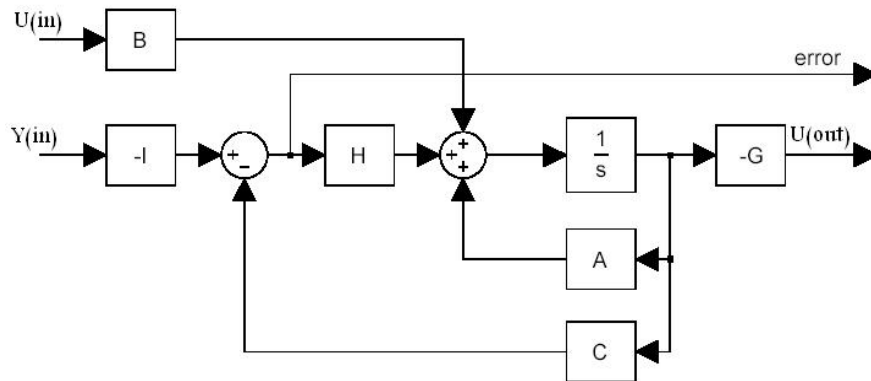


Figure. 3. New form for MBC controller

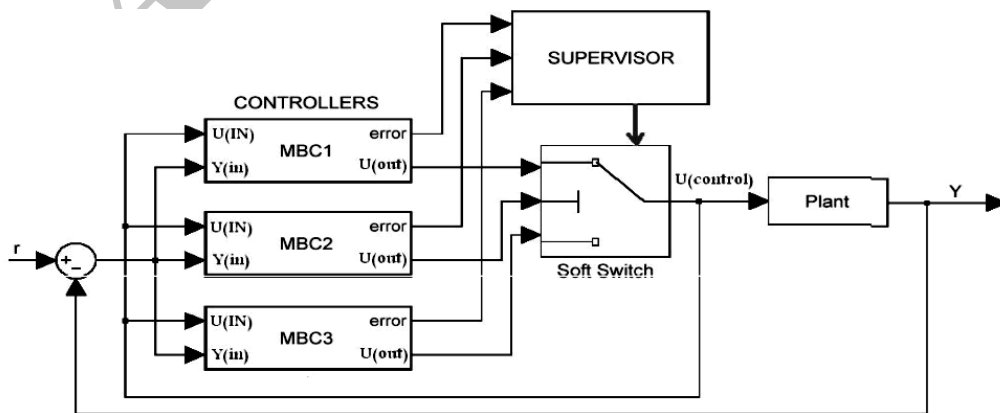


Figure .4. New design switch control of hybrid systems

## 2. SUPERVISOR FOR SOFT SWITCHING

Supervisor [3] is one of the important sections in hybrid system feedback control. At first, how to calculate outputs, which had done based on error vectors and state detectors, is discussed. Finally, the proposed supervisory block diagram is discussed. This block diagram can be used practically in most hybrid system control. In proposed MBC controller, all of the state detectors are always in active mode. But, only one of them has the slightest error output ( $E_i$ ), which it is used to control of hybrid system.

### 2.1 EQUATIONS

In the output of controllers,  $E_i$  shows the error of  $i_{th}$  MBC controller state vector.  $D_i$  (weighted error for  $i_{th}$  detector) and  $N_i$  (error for  $i_{th}$  detector) is calculated use of  $E_i$  and  $W_i$  (weight function for  $i_{th}$  detector).  $W_i$  must be selected until  $D_i < N_i$ .

$$\begin{aligned} D_i &= E_i^T \cdot W_i \cdot E_i \quad i=1,2,\dots,j \\ N_i &= E_i^T \cdot E_i \end{aligned} \quad (2)$$

Where  $j$  is the number of states for hybrid system and also we have  $E_i = [e_{i1} \ e_{i2} \ \dots \ e_{iL}]^T$  and  $W_i = \text{diag} [W_{i1} \ W_{i2} \ \dots \ W_{iL}]$ .

$L$  is the number of outputs in hybrid system.  $N$  as total number of error signals is calculated with the equation (3).

$$N = \left( \sum_{i=1}^j N_i \right) \quad (3)$$

Where  $KE_i$  (first effective multiplier for  $i_{th}$  MBC controller) is calculated by equation (4).

$$KE_i = \frac{\beta \cdot D_i}{N} \quad 0 < KE_i < \beta \quad (4)$$

$\beta$  is defined as soft switching multiplier which usually is the positive and larger than 1 constant.

Then  $KE_i$  is filtered by LPF ( $FE_i$ ) and is limited to  $[0 \ - \ 1]$  by a saturation function.

$$FE_i = KE_i \left( \frac{a}{s+a} \right) \quad (5)$$

$$SE_i = \text{SAT}(FE_i, 0, 1) \quad (6)$$

The least value of  $SE_i$ 's is calculated with  $MIN$  function by equation (7).

$$\min SE = \text{MIN}(SE_1, SE_2, \dots, SE_j) \quad (7)$$

Then,  $K_i$  (final effective multiplier of  $i_{th}$  MBC controller) is calculated by equation (8). Practically, If we assume that  $K_i$  is the active detector, it means that the value of  $K_i$  is close to '1' and the others have values close to '0'. It may be 2 or more of  $K_i$ 's greater than zero when state switching occurs.

$$K_i = \frac{SE_i}{\min SE} \quad (8)$$

$G_i$  (optimal state feedback multiplier) and  $H_i$  (optimal estimation multiplier) can be calculated by LQR and KALMAN (MATLAB commands) for

each state of hybrid system. Also  $Q_i$  and  $R_i$  are independently selected for all states of hybrid system. Control signal ( $U$ ) is provided by equation (9).

$$U = -G \hat{X} = \sum_{i=1}^j K_i \cdot (-G_i \cdot \hat{x}_i) \quad (9)$$

## 2.2 FINAL DIAGRAMS

Figure.5 shows a block diagram of supervisory for soft switching.  $E_1, E_2, \dots, E_j$  are inputs from MBC detectors and  $K_1, K_2, \dots, K_j$  are outputs to soft switching box.

Figure 6 shows the hybrid system which is controlled by proposed MBC controllers and soft switching strategy.

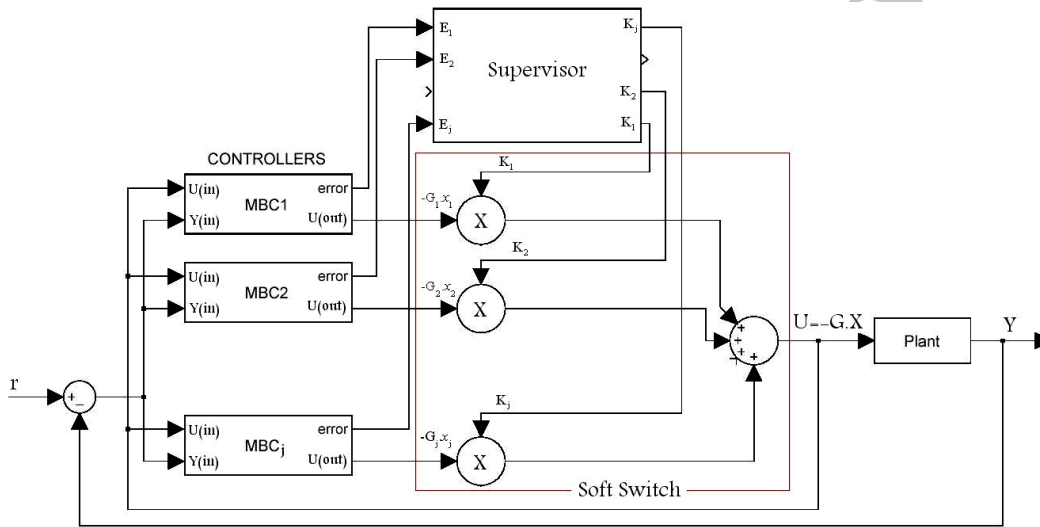


Figure. 5. Supervisory for soft switching

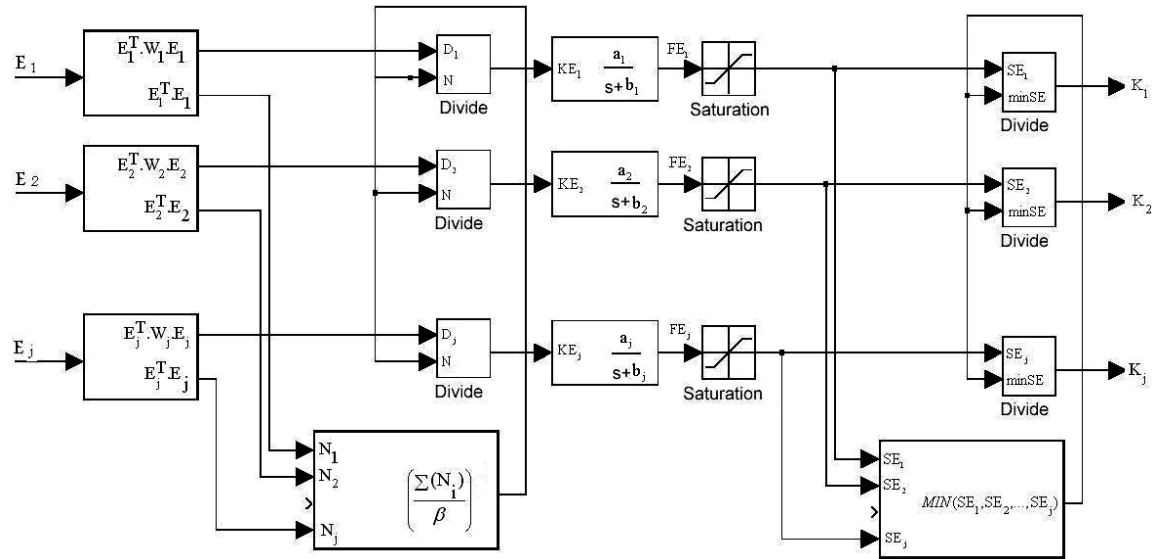


Figure.6. Hybrid System and Feedback Control Using Proposed MBC

### 3. APPLICATION EXAMPLE

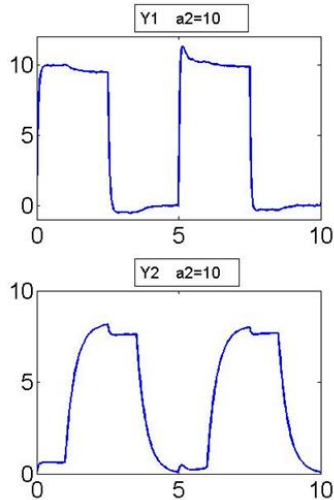
For an illustrative example, a two mode hybrid system was selected which have 2-input and 2-output. This system has two modes which mode1 is stable and mode2 is unstable. The hybrid system dynamics are as below:

Dynamics of Mode 1:			
$A_1 = \begin{bmatrix} -4.19 & 0 & 0 \\ 0 & -6.19 & -2.09 \\ 0 & 4 & 0 \end{bmatrix}$	$B_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0.5 \\ 0 & 0 \end{bmatrix}$		
$C_1 = \begin{bmatrix} 1.875 & 0 & -0.075 \\ 0.1194 & 0.4446 & 0.4718 \end{bmatrix}$	$D_1 = 0$		
$G_1 = \begin{bmatrix} 15 & 0.15 & -0.45 \\ 0.06 & 2 & 2.14 \end{bmatrix}$	$H_1 = \begin{bmatrix} 7.99 & 0.52 \\ -0.049 & 0.92 \\ -0.11 & 1.24 \end{bmatrix}$		

Dynamics of Mode 2:

$A_2 = \begin{bmatrix} -2.19 & 0 & 0 \\ 0 & -0.19 & -2.197 \\ 0 & 2 & 0 \end{bmatrix}$	$B_2 = \begin{bmatrix} 2 & 0 \\ 0 & 0.5 \\ 0 & 0 \end{bmatrix}$
$C_2 = \begin{bmatrix} 1.4375 & 0 & -0.15 \\ 0.0597 & 0.4446 & 0.4989 \end{bmatrix}$	$D_2 = 0$
$G_2 = \begin{bmatrix} 13.33 & 0.079 & -1.29 \\ 0.019 & 10 & 11.08 \end{bmatrix}$	$H_2 = \begin{bmatrix} 18.5 & 1.02 \\ -0.34 & 6.4 \\ -0.24 & 4.26 \end{bmatrix}$

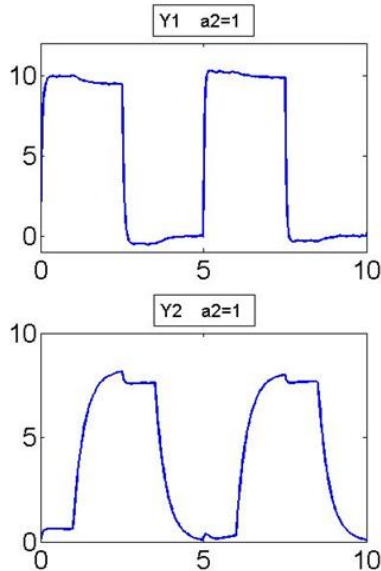
The parameter of modes is selected as :  $R = 0.05$  ,  $\beta = 2$  ,  $a_1 = b_1 = b_2 = 10$  ,  $a_2=1$  ,  $W_1 = W_2 = Q = I_{2 \times 2}$ . The System runs from mode1 (stable) and then is switched to mode2 (unstable) at time 2s. Figure 7 shows outputs  $Y_1$  and  $Y_2$  when the parameter  $a_2$  is 10 ( $a_2=10$ ) . In this simulation,  $U_1$  and  $U_2$  as inputs have square waveform with  $10^v$  amplitude and period 5s, but  $U_2$  has 1s delay compared to  $U_1$ .



**Figure.7.**

- a) Results for output  $Y_1$
- b) Results for output  $Y_2$

**Figure.8.** shows outputs  $Y_1$  and  $Y_2$  when the parameter  $a_2$  is 1 ( $a_2=1$ ).

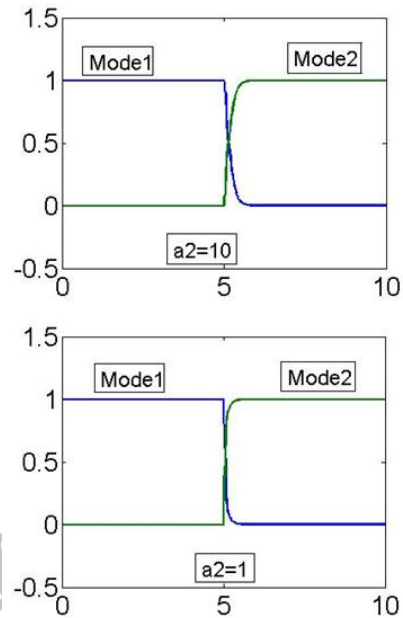


**Figure.8.**

- a) Results for output  $Y_1$
- b) Results for output  $Y_2$

Also Fig.9 shows the final effective multiplier of two modes for 10 seconds which the hybrid system mode is

changed at  $t=5s$  (1 as active and 0 as disable).



**Figure. 9.**

- a) Final effective multipliers with  $a_2=10$
- b) Final effective multipliers with  $a_2=1$

The Simulations were prepared by MATLAB with P4, 2.4G full, 512M RAM. The Runtime of proposed strategy simulations is less than one second, but for fast switching method was more than 5 seconds.

#### 4. CONCLUSIONS

The results of the simulation show that stable and optimal feedback controller using an MBC algorithm for a hybrid system is realized by fine adjusting  $\beta$  and proper filtering on supervisory parameters. Even the response of system for pulse form input commands will have less overshooting by proper supervisory parameters. As well as, the soft switching system can be changed controllers to active and best mode without any shock. Also The results of Simulations show the proposed

technique has less runtime comparing to fast switching method. This method can be used to control MIMO hybrid with nonlinear subsystems.

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