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# Generalized superconnectedness

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**Abstract.** Á. Császár introduced and extensively studied the notion of generalized open sets. Following Csázar, we introduce a new notion superconnected. The main purpose of this paper is to study generalized superconnected spaces. Various characterizations of generalized superconnected spaces and preservation theorems are discussed.



**Keywords:** Generalized topology, connected, superconnected.

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## 1. Introduction

In recent years, Á. Császár [1] has studied various subjects of generalized topological spaces. The notion of generalized topological spaces in the sense of Á. Császár is a generalization of topological spaces. Various important collections of sets in a topological space from a generalized topology on it. Moreover, Á. Császár introduced and studied various basic operators related to generalized topological spaces. In this paper, the notion of generalized Superconnectedness spaces are investigated.

The purpose of this paper is to investigate superconnectedness on GTS's. We introduce the concept of superconnectedness on GTS's and give characterizations and properties of this notion.

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## 2. Preliminaires

Let $\exp X$ be the power set of $X$ for a nonempty set $X$. A subfamily $\mu \subset \exp X$ is called a generalized topology [1] (briefly, GT) on $X$ if $\emptyset \in \mu$ and unions of elements of $\mu$ belong to $\mu$. A set $X$ with a GT $\mu$ on $X$ is called a generalized topological space (briefly, GTS) and is denoted by $(X, \mu)$.

For a GTS $(X, \mu)$, the elements of $\mu$ are called $\mu$-open sets and the complements of $\mu$-closed sets containing $G$ is denoted by $c_\mu(G)$, i.e., the smallest $\mu$-closed sets containing $G$ and the union of all $\mu$-open sets contained in $G$ is denoted by $i_\mu(G)$, the largest $\mu$-open set contained in $G$ (see [2, 3]).

It is known that $i_\mu$ and $c_\mu$ are idempotent and monotonic where $\gamma : \exp X \longrightarrow \exp X$ is called idempotent if $G \subset X$ implies $\gamma(\gamma(G)) = \gamma(G)$ and monotonic if $G \subset H \subset X$ implies $\gamma(G) \subset \gamma(H)$ [3]. Moreover, it is known from [2, 3] that if $\mu$ is a GT on $X$ and $G \subset X$, $x \in X$, then $x \in c_\mu(G)$ if and only if $F \cap G \neq \emptyset$ for $x \in F \in \mu$ and $c_\mu(X \backslash G) = X \backslash i_\mu(G)$.

**Definition 2.1** [2, 3] A subset $G$ of a generalized topological space $(X, \mu)$ is called

1) $\mu$-preopen if $G \subset i_\mu(c_\mu(G))$.
2) $\mu$-semiopen if $G \subset c_\mu(i_\mu(G))$.
3) $\mu - \beta$-open if $G \subset c_\mu(i_\mu(c_\mu(G)))$.
4) $\mu r$-open if $G = i_\mu(c_\mu(G))$.

The complement of a $\mu$-preopen (resp. $\mu$-semiopen, $\mu-\beta$-open) set is called $\mu$-preclosed (resp. $\mu$-semiclosed, $\mu - \beta$-closed). The $\mu$-semi-interior of a subset $G$ of a GTS $(X, \mu)$ denoted by $i_\sigma(G)$, is defined by the union of all $\mu$-semiopen sets of $(X, \mu)$ contained in $G$ [3]. A GTS $(X, \mu)$ is said to be connected [6] if $X$ can not be written as the union of nonempty and disjoint $\mu$-open sets $G$ and $H$ in $(X, \mu)$.

## 3. Generalized superconnectedness

**Definition 3.1** [7] Let $(X, \mu)$ be a GTS and $G \subset X$.

1) $G$ is called $\mu$-dense if $c_\mu(G) = X$.
2) $(X, \mu)$ is called hyperconnected if $G$ is $\mu$-dense for every $\mu$-open subset $G \neq \emptyset$ of $(X, \mu)$.
3) $G$ is said to be $\mu$-nowhere dense if $i_\mu(c_\mu(G)) = \emptyset$.

**Definition 3.2** Let $(X, \mu)$ be a GTS and $G \subset X$.

1) $G$ is called $\mu$-scaled-dense if there exists $x_1, \ldots, x_p \in X$ such that $c_\mu(G \cup \sqcup_{i=1}^{p}\{x_i\}) = X$.
2) $(X, \mu)$ is called superconnected if $G$ is $\mu$-scaled-dense for every $\mu$-open subset $G \neq \emptyset$ of $(X, \mu)$.

***Example* 3.3** Let $X = \{a, b, c, d\}$ and $\mu = \{\emptyset, \{a\}, \{b, c\}, \{a, c\}\}$. Then the GTS $(X, \mu)$ is superconnected.

***Remark 1*** *For a GTS $(X, \mu)$, the following holds:*
*$(X, \mu)$ is hyperconnected $\Longrightarrow$ $(X, \mu)$ is superconnected.*

This implication is not reversible as shown in the following example:

***Example* 3.4** Let $X = \{a, b, c, d\}$ and $\mu = \{\emptyset, \{a\}, \{c, d\}, \{a, b, c\}\}$. Then the GTS

$(X, \mu)$ is superconnected but it is not hyperconnected.

**Definition 3.5** A subset $G$ of a generalized topological space $(X, \mu)$ is said to be $\mu$-nowhere scaled-dense if there exists $x_1, \ldots, x_p \in X$ such that, $i_\mu(c_\mu(G \cup \sqcup_{i=1}^{p}\{x_i\})) = \emptyset$

**Theorem 3.6** Let $(X, \mu)$ be a GTS where $c_\mu(\emptyset) = \emptyset$. The following properties are equivalent:

1) $(X, \mu)$ is superconnected,
2) $G$ is $\mu$-scaled-dense or $\mu$-nowhere scaled-dense for every subset $G$ of $(X, \mu)$,
3) for every nonempty $\mu$-open subsets $G$ and $H$ of $(X, \mu)$ there exists $x_1, \ldots, x_p \in X$ such that, $(G \cup \sqcup_{i=1}^{p}\{x_i\}) \cap H \neq \emptyset$.

**Proof.** 1) $\Rightarrow$ 2). Let $(X, \mu)$ be a superconnected GTS and $G \subset X$. Suppose that $G$ is not $\mu$-nowhere scaled-dense. We have

$$c_\mu(X \backslash (c_\mu(G \cup \sqcup_{i=1}^{p}\{x_i\}))) = X \backslash i_\mu(c_\mu(G \cup \sqcup_{i=1}^{p}\{x_i\})) \neq X.$$

This implies from (1) that for $i_\mu(c_\mu(G \cup \sqcup_{i=1}^{p}\{x_i\})) \neq \emptyset$, $c_\mu(i_\mu(c_\mu(G \cup \sqcup_{i=1}^{p}\{x_i\}))) = X$. Since

$$X = c_\mu(i_\mu(c_\mu(G \cup \sqcup_{i=1}^{p}\{x_i\}))) \subset c_\mu(G \cup \sqcup_{i=1}^{p}\{x_i\}).$$

Then $c_\mu(G \cup \sqcup_{i=1}^{p}\{x_i\}) = X$. Hence, $G$ is a $\mu$-scaled-dense set.

2) $\Rightarrow$ 3). Suppose that for some nonempty $\mu$-open subsets $G$, $H$ of $(X, \mu)$ and for every $x_1, \ldots, x_p \in X$, $(G \cup \sqcup_{i=1}^{p}\{x_i\}) \cap H = \emptyset$. We have,

$$c_\mu(G \cup \sqcup_{i=1}^{p}\{x_i\} \cap H) \subset c_\mu(G \cup \sqcup_{i=1}^{p}\{x_i\}) \cap c_\mu(H) \subset c_\mu((G \cup \sqcup_{i=1}^{p}\{x_i\}) \cap H) = c_\mu(\emptyset) = \emptyset.$$

Hence, $c_\mu(G \cup \sqcup_{i=1}^{p}\{x_i\}) \cap H = \emptyset$. Thus $G$ is not a $\mu$-scaled-dense set. Since $G$ is $\mu$-open, then

$$\emptyset \neq G \subset G \cup \sqcup_{i=1}^{p}\{x_i\} \subset i_\mu(c_\mu(G \cup \sqcup_{i=1}^{p}\{x_i\})).$$

Then $G$ is not a $\mu$-nowhere scaled-dense set. This is a contradiction. Thus,

$$(G \cup \sqcup_{i=1}^{p}\{x_i\}) \cap H \neq \emptyset$$

for every nonempty $\mu$-open subsets $G$ and $H$ of $(X, \mu)$.

3) $\Rightarrow$ 1). Let $(G \cup \sqcup_{i=1}^{p}\{x_i\}) \cap H \neq \emptyset$ for every nonempty $\mu$-open subsets $G$ and $H$ of $(X, \mu)$. Suppose that the GTS $(X, \mu)$ is not superconnected. This implies that there exists a nonempty $\mu$-open subset $A$ of $(X, \mu)$ such that $A$ is not $\mu$-scaled-dense in $(X, \mu)$, i.e. for every $x_1, \ldots, x_p \in X$, $c_\mu(A \cup \sqcup_{i=1}^{p}\{x_i\}) \neq X$. We have $X \backslash c_\mu(A \cup \sqcup_{i=1}^{p}\{x_i\}) \neq \emptyset$. This implies that $X \backslash c_\mu(A \cup \sqcup_{i=1}^{p}\{x_i\})$ and $A$ are nonempty $\mu$-open subsets of $(X, \mu)$ such that $X \backslash c_\mu(A \cup \sqcup_{i=1}^{p}\{x_i\}) \cap A = \emptyset$. This is a contradiction. Consequently, the GTS $(X, \mu)$ is superconnected. ■

**Definition 3.7** [6] A GTS $(X, \mu)$ is called irreducible if for each nonempty $U, V \in \mu$, i.e., $U, V \in \mu \backslash \{\emptyset\}$, we have $U \cap V \neq \emptyset$.

**Definition 3.8** A GTS $(X, \mu)$ is called scaled-irreducible if for each nonempty $U, V \in \mu$, there exists $x_1, \ldots x_n \in X$, we have $(U \cup \sqcup_{i=1}^{n}\{x_i\}) \cap V \neq \emptyset$.

***Remark 2*** *It follows from Theorem 3.6 that a GTS $(X,\mu)$ is superconnected if and only if for every nonempty , $\mu$-open subsets G and H of $(X,\mu)$ we have $(G\cup\sqcup_{i=1}^{n}\{x_i\})\cap H\neq\emptyset$. Thus, Theorem 3.6 implies that a GTS $(X,\mu)$ is superconnected if and only if $(X,\mu)$ is scaled-irreducible.*

**Definition 3.9** [3] The $\mu$-semiclosure (resp. $\mu$-preclosure) of a subset G of a GTS $(X,\mu)$, denoted by $c_\sigma(G)$ (resp. $c_\pi(G)$), is defined by the intersection of all $\mu$-semiclosed (resp. $\mu$-preclosed) sets of $(X,\mu)$ containing G.

**Theorem 3.10** Let $(X,\mu)$ be a GTS where $c_\mu(\emptyset)=\emptyset$. Then the following properties are equivalent:

1) $(X,\mu)$ is superconnected,
2) G is $\mu$-scaled-dense for every $\mu$-preopen subset $\emptyset\neq G\subset X$,
3) for every $\mu$-preopen subsets $\emptyset\neq G\subset X$ there exists $x_1,\dots,x_p\in X$ such that,

$$c_\sigma(G\cup\sqcup_{i=1}^{p}\{x_i\})=X,$$

4) for every $\mu$-semiopen subsets $\emptyset\neq G\subset X$ there exists $x_1,\dots,x_p\in X$ such that,

$$c_\pi(G\cup\sqcup_{i=1}^{p}\{x_i\})=X.$$

**Proof.** 1) $\Rightarrow$ 2). Let $(X,\mu)$ be a superconnected GTS. Suppose that G is a nonempty $\mu$-preopen subset of $(X,\mu)$. This implies that

$$\emptyset\neq G\subset i_\mu(c_\mu(G))\subset i_\mu(c_\mu(G\cup\sqcup_{i=1}^{p}\{x_i\}))$$

where $x_1,\dots,x_p\in X$. Thus, $G$ is not $\mu$-nowhere scaled-dense. Then by Theorem 3.6, $G$ is $\mu$-scaled-dense.

2) $\Rightarrow$ 3). Suppose that there exists a nonempty $\mu$-preopen set $G$ such that for every $x_1,\dots,x_p\in X$, we have $c_\sigma(G\cup\sqcup_{i=1}^{p}\{x_i\})\neq X$. Then there exists a nonempty $\mu$-semiopen set $A$ such that $A\cap(G\cup\sqcup_{i=1}^{p}\{x_i\})=\emptyset$. Thus, $i_\mu(A)\cap(G\cup\sqcup_{i=1}^{p}\{x_i\})=\emptyset$. Hence,

$$\emptyset=i_\mu(A)\cap c_\mu(G\cup\sqcup_{i=1}^{p}\{x_i\})=i_\mu(A)$$

by 2). Since $A$ is a nonempty $\mu$-semiopen set, we have $A\subset c_\mu(i_\mu(A))=c_\mu(\emptyset)=\emptyset$. This is a contradiction.

3) $\Rightarrow$ 4). Suppose that there exists a nonempty $\mu$-semiopen set $G$ such that

$$c_\pi(G\cup\sqcup_{i=1}^{p}\{x_i\})\neq X.$$

Then there exists a nonempty $\mu$-preopen set $A$ such that

$$(A\cup\sqcup_{i=1}^{p}\{x_i\})\cap G=\emptyset.$$

Thus, $(A\cup\sqcup_{i=1}^{p}\{x_i\})\cap i_\mu(G)=\emptyset$. Hence,

$$\emptyset=c_\mu(A\cup\sqcup_{i=1}^{p}\{x_i\})\cap i_\mu(G)\supset c_\sigma(A\cup\sqcup_{i=1}^{p}\{x_i\})\cap i_\mu(G)=X\cap i_\mu(G)=i_\mu(G).$$

Since $G$ is $\mu$-semiopen set, then $G\subset c_\mu(i_\mu(G))=c_\mu(\emptyset)=\emptyset$, this is a contraduction.

4) $\Rightarrow$ 1). Let $G$ be a nonempty $\mu$-open set of $(X, \mu)$. Since $G$ is $\mu$-semiopen, by 4), there exists $x_1, \ldots, x_p \in X$ such that $c_\mu(G \cup \sqcup_{i=1}^p \{x_i\}) \supset c_\pi(G \cup \sqcup_{i=1}^p \{x_i\}) = X$. Consequently, $(X, \mu)$ is a superconnected GTS. ■

**Lemma 3.11** [3] The following holds for a subset $G$ of a GTS $(X, \mu)$:

$$c_\sigma(G) = G \cup i_\mu(c_\mu(G)).$$

**Theorem 3.12** Let $(X, \mu)$ be a GTS where $c_\mu(\emptyset) = \emptyset$. Then the following properties are equivalent:

1) $(X, \mu)$ is superconnected,
2) $G$ is $\mu$-scaled-dense for every $\mu - \beta$-open subset $\emptyset \neq G \subset X$,
3) for every $\mu - \beta$-open subset $\emptyset \neq G \subset X$ there exists $x_1, \ldots, x_p \in X$ such that,

$$c_\sigma(G \cup \sqcup_{i=1}^p \{x_i\}) = X.$$

**Proof.** 1) $\Rightarrow$ 2). Let $(X, \mu))$ be a superconnected GTS. Assume that $G$ is a nonempty $\mu$-$\beta$-open subset of $(X, \mu)$. It follows that $\emptyset \neq i_\mu(c_\mu(G)) \subset i_\mu(c_\mu(G \cup \sqcup_{i=1}^p \{x_i\}))$. Then,

$$X = c_\mu(i_\mu(c_\mu(G \cup \sqcup_{i=1}^p \{x_i\}))) = c_\mu(G \cup \sqcup_{i=1}^p \{x_i\}).$$

2) $\Rightarrow$ 3). Let $G$ be any nonempty $\mu$-$\beta$-open subset of $(X, \mu)$. By Lemma 3.11, we have

$$c_\sigma(G \cup \sqcup_{i=1}^p \{x_i\}) = (G \cup \sqcup_{i=1}^p \{x_i\}) \cup i_\mu(c_\mu(G \cup \sqcup_{i=1}^p \{x_i\})) = G \cup \sqcup_{i=1}^p \{x_i\} \cup i_\mu(X) = X.$$

Then, $c_\sigma(G \cup \sqcup_{i=1}^p \{x_i\}) = X$

(3) $\Rightarrow$ (1). Let $G$ be a nonempty $\mu$-open set of $(X, \mu)$. It follows from 3) that

$$c_\sigma(G \cup \sqcup_{i=1}^p \{x_i\}) = (G \cup \sqcup_{i=1}^p \{x_i\}) \cup i_\mu(c_\mu(G \cup \sqcup_{i=1}^p \{x_i\})) = X.$$

Thus, $c_\mu(G \cup \sqcup_{i=1}^p \{x_i\}) = X$. Consequently, $(X, \mu)$ is a superconnected GTS. ■

**Corollary 3.13** Let $(X, \mu)$ be a GTS where $c_\mu(\emptyset) = \emptyset$. Then the following properties are equivalent:

1) $(X, \mu)$ is superconnected,
2) for every nonempty $\mu$-semiopen subset $A$ and every nonempty $\mu - \beta$-open subset $B$, there exists $x_1, \ldots, x_p \in X$ such that $A \cap (B \cup \sqcup_{i=1}^p \{x_i\}) \neq \emptyset$.
3) for every nonempty $\mu$-open subset $A$ and every nonempty $\mu - \beta$-open subset $B$, there exists $x_1, \ldots, x_p \in X$ such that $A \cap (B \cup \sqcup_{i=1}^p \{x_i\}) \neq \emptyset$.

**Proof.** The proof follows from Theorem 3.12. ■

**Theorem 3.14** Let $(X, \mu)$ be a GTS where $c_\mu(\emptyset) = \emptyset$. Then the following properties are equivalent:

1) $(X, \mu)$ is superconnected,
2) $G$ is $\mu$-scaled-dense for every $\mu$-semi-open subset $\emptyset \neq G \subset X$,
3) for every $\mu$-semi-open subset $\emptyset \neq G \subset X$ there exists $x_1, \ldots, x_p \in X$ such that,

$$c_\sigma(G \cup \sqcup_{i=1}^p \{x_i\}) = X.$$

**Proof.** It is similar to that of Theorem 3.12. ■

## 4. Preservations of generalized superconnectedness

**Definition 4.1** [7] Let $(X, \mu)$ and $(Y, \mu)$ be two generalized topological spaces. A function $f : (X, \mu) \to (Y, \mu)$ is called

1) almost feebly $(\mu, \lambda)$-continuous if for each nonempty $\lambda r$-open set G of $(Y, \lambda)$, $f^{-1}(G) \neq \emptyset$ implies $i_\sigma(f^{-1}(G)) \neq \emptyset$
2) $(\mu, \lambda)$-semicontinuous if for each $\lambda$-open set G of $(Y, \lambda)$, $f^{-1}(G)$ is $\mu$-semiopen in $(X, \mu)$.

**Theorem 4.2** [7] Every $(\mu, \lambda)$-semicontinuous function $f : (X, \mu) \to (Y, \lambda)$ is almost feebly $(\mu, \lambda)$-continuous.

**Theorem 4.3** Let $(X, \mu)$ be a superconnected GTS where $c_\mu(\emptyset) = \emptyset$. If $f : (X, \mu) \to (Y, \lambda)$ is an almost feebly $(\mu, \lambda)$-continuous surjection, then $(Y, \lambda)$ is superconnected.

**Proof.** Assume that $(Y, \lambda)$ is not a superconnected GTS. This implies that there exist nonempty $\lambda$-open sets $G \subset Y$ and $H \subset Y$ such that $(G \cup \sqcup_{i=1}^{p} \{x_i\}) \cap H = \emptyset$, where $x_1, \ldots, x_p \in X$. Put $O = i_\lambda(c_\lambda(G \cup \sqcup_{i=1}^{p} \{x_i\}))$ and $P = i_\lambda(c_\lambda(H))$. Then $O$ and $P$ are nonempty $\lambda r$-open sets and $O \cap P = \emptyset$. This implies

$$i_\sigma(f^{-1}(O)) \cap i_\sigma(f^{-1}(P)) \subset f^{-1}(O) \cap f^{-1}(P) = \emptyset.$$

Since $f : (X, \mu) \to (Y, \lambda)$ is an almost feebly $(\mu, \lambda)$-continuous surjection, then

$$i_\sigma(f^{-1}(O)) \neq \emptyset$$

and $i_\sigma(f^{-1}(P)) \neq \emptyset$. It follows from corollary 3.13 that $(X, \mu)$ is not a superconnected GTS. This is a contradiction. ■

**Corollary 4.4** Let $(X, \mu)$ be a superconnected GTS where $c_\mu(\emptyset) = \emptyset$. If $f : (X, \mu) \to (Y, \lambda)$ is a $(\mu, \lambda)$-semicontinuous surjection, then $(Y, \lambda)$ is superconnected.

**Proof.** It follows from Theorem 4.2 and 4.3. ■

**Definition 4.5** [1] Let $(X, \mu)$ and $(Y, \lambda)$ be GTSs. Then a function $f : (X, \mu) \to (Y, \lambda)$ is said to be $(\mu, \lambda)$-continuous if $f^{-1}(G) \in \mu$ for every $G \in \lambda$.

**Theorem 4.6** [7] Every $(\mu, \lambda)$-continuous function is $(\mu, \lambda)$-semicontinuous.

**Corollary 4.7** Let $(X, \mu)$ be a superconnected GTS where $c_\mu(\emptyset) = \emptyset$. If $f : (X, \mu) \to (Y, \lambda)$ is a $(\mu, \lambda)$-continuous surjection, then $(Y, \lambda)$ is a superconnected GTS.

**Proof.** Since every $(\mu, \lambda)$-continuous function is almost feebly $(\mu, \lambda)$-continuous, it follows from Theorem 4.3. ■

**Definition 4.8** [7] A function $f : (X, \mu) \to (Y, \lambda)$ is said to be almost feebly $(\mu, \lambda)$-open if $i_\sigma(f(G)) \neq \emptyset$ for each nonempty $\mu r$-open set $G \subset X$.

**Theorem 4.9** Let $(X, \mu)$ be a superconnected GTS where $c_\mu(\emptyset) = \emptyset$. If $f : (X, \mu) \to (Y, \lambda)$ is an almost feebly $(\mu, \lambda)$-open injection, then $(X, \mu)$ is a superconnected GTS.

**Proof.** Let $G$ and $H$ be any nonempty $\mu$-open sets of $(X, \mu)$. Put $O = i_\lambda(c_\lambda(G \cup \sqcup_{i=1}^{p} \{x_i\}))$ and $P = i_\lambda(c_\lambda(H))$. It follows that $O$ and $P$ are nonempty $\mu r$-open sets. Since $f : (X, \mu) \to (Y, \lambda)$ is an almost feebly $(\mu, \lambda)$-open, then $i_\sigma(f(O)) \neq \emptyset$ and $i_\sigma(f(P)) \neq \emptyset$.

Since $(Y, \lambda)$ is a superconnected GTS, then

$$\emptyset \neq i_\sigma(f(O)) \cap i_\sigma(f(P)) \subset f(O) \cap f(P).$$

Since $f : (X, \mu) \to (Y, \lambda)$ is an injection function, then $0 \cap P \neq \emptyset$. Hence, $G \cup \sqcup_{i=1}^{p} \{x_i\} \neq \emptyset$. Consequently, $(X, \mu)$ is a superconnected GTS. ■

**Definition 4.10** [5] Let $(X, \mu)$ and $(Y, \lambda)$ be GTSs. Then a function $f : (X, \mu) \to (Y, \lambda)$ is called $(\mu, \lambda)$-open if $f(G) \in \lambda$ for every $G \in \mu$.

**Corollary 4.11** Let $(Y, \lambda)$ be a superconnected GTS where $c_\mu(\emptyset) = \emptyset$. If $f : (X, \mu) \to (Y, \lambda)$ is a $(\mu, \lambda)$-open injection, then $(X, \mu)$ is a superconnected GTS.

**Proof.** Since every $(\mu, \lambda)$-open function is almost feebly $(\mu, \lambda)$-open, it follows from Theorem 4.9. ■

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