



## Transmission expansion planning based on merchandizing surplus of transmission lines

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### Abstract

This paper proposes an optimal transmission expansion planning (TEP) which is based on determining share of each line in merchandizing surplus (MS) of system, determining by Independent System Operator (ISO). The more share of a line in MS of system denotes the priority of a line for expansion. The procedure of determining MS of each line in a power system is based on determining the MS share of each energy exchange between certain generator and certain demand bus in the power system. By analyzing all energy exchange the optimal planning of transmission line is obtained by ISO. The variable revenue of a Transco is related to performing the optimal planning of transmission lines which is obtained through ISO. The proposed method determines the TEP economical resources and the procedure of receiving these resources (MS of system) from generators and customers. By performing proposed method Transco capacity withholding and misusing is prevented spontaneously caused by relating the revenue of Transco to its optimal performance.

*Keywords:* Transmission expansion planning, Merchandizing surplus, Line congestion.

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### 1. Introduction

TEP affects almost all aspect of power market e.g. price, profits of market participant, market power exercise and power market efficiency. Hence determining how TEP will be and how the revenue of Transco will be related to TEP, is a challenging issue in deregulated power market.

In some deregulated power markets like Brazil and Argentina, TEP is provided by an Independent System Operator (ISO) [1]. According to [2], power market policy makers implicitly consider the low ability for Transco to expand the present network. In other words they consider the "regulated revenue," for Transco which result in inefficient and uncompetitive power market. Under this model there is no motivation for Transco to reduce the congestion and also the revenue of a Transco is not related to its performance. So there is a great inertia in power markets to motivate Transco toward performing optimal TEP.

A relevant model for Transco revenue includes the maintenance cost of lines in addition to a part of initial investment of lines, which is provided by

power market participants as a transmission rent [3]. [4, 5] revised the modern method of transmission fixed cost allocation up to 2000. [6] Proposes a method to cover the transmission fixed cost which decreases the transmission capacity withholding. Some researches consider the MS of power market to cover the fixed cost of transmission lines which make a motivation for a Transco to increase the congestion of system so that the MS and its revenue will increase [7].

In all above researches a barrier to perform optimal TEP is that the Transco revenue is irrelevant to its performance also there is no certain plan for expansion of transmission lines and how the resources of this expansion plans should be supplied.

In this research work the Transco revenue is divided into two parts: the fixed part and the variable part. The fixed part provided by transmission network users through MW/mile method [8]. The Transco variable revenue resource is derived from MS of system which is considered as extra revenue for Transco for TEP. The more share of a line in MS

of system denotes the priority of a line for expansion. However how the MS share a line is determined, is a long story which will be expressed step by step in details in the following parts. So the Transco revenue will be related to the performance of optimal TEP, providing by ISO.

In the proposed method TEP resources is determined exactly and the procedure of receiving this resources (MS of system) from generators and customers is analyzed greatly in detail. The priority of lines for expansion is also determined in this method. The Transco capacity withholding and misusing are prevented spontaneously caused by relating the revenue of Transco to its optimal performance. Ultimately the proposed method cause that positive economic signal increase the competition and efficiency of power market.

The rest of this paper is organized as follows; section 2 describes Locational Marginal Pricing (LMP) decomposition. The share of each generator in load supply of each bus in the system is determined in section 3. Section 4 includes the calculation of the generator revenue from the load supply of each bus in the system, as well as the payments of customers for that supply load. Section 5 includes the calculation of each energy exchange MS. In section 6 the proposed method is tested on five bus test system and finally, section 7 concludes the paper.

**1. LMP decomposition**

*A) DCOPF Problem:*

DCOPF problem determines the optimal generation dispatch and LMPs subject to a set of constraints which represents the operational and physical limits of power system. Generators make offers to sell electricity as linear supply function and for the purpose of simplicity, no demand side bidding is considered and hence, loads are known constants for the dispatch.

It is assumed that the Generator' offers expressed by Eq.1 that is a straight lines with intercept  $a_i$  and slope  $b_i$  [7]:

$$\rho_i(P_i) = a_i + b_i P_i \tag{1}$$

Generator can change their pricing strategies by adjusting the slope and intercept of the line in (1). In [9, 10] it is assumed that generating units only manipulate the intercept  $a_i$  of the bid functions and their slope  $b_i$  is constant. Several reasons have been discussed for justification of this assumption in [8]. For instance, it has been stated that the slope of bid functions for individual generator is usually very

slight and therefore very steep slopes, resulting from manipulation of  $b_i$  are not plausible. According to presented discussion, in this paper it is assumed that the units may change their strategies by only adjusting the intercept values  $a_i$  and therefore  $b_i$  remains constant.

Therefore, the DCOPF can be stated as a problem of minimizing the total generation cost of generators subject to physical limits in the network:

$$\min \sum_{i=1}^N a_i * P_i + (b_i / 2) * P_i^2$$

Subject to:

$$\sum_{i=1}^N P_i = P_d \tag{2}$$

$$\alpha_l \leq \sum_{i=1}^N \gamma_{li} * P_i \leq \bar{\alpha}_l (\Gamma_l^{\min}, \Gamma_l^{\max})$$

$$(\Gamma_l^{\min}, \Gamma_l^{\max}) \quad l = 1, \dots, L$$

$$P_i^{\min} \leq P_i \leq P_i^{\max} (\mu_i^{\min}, \mu_i^{\max})$$

$$(\mu_i^{\min}, \mu_i^{\max}) \quad i = 1, \dots, N$$

Constraints (2) represent generation capacity constraint, transmission line constraint and load balance constraint respectively. By solving this optimization problem, ISO determines generation of every generator and LMPs which is the Lagrangian multiplier of constraint (2).

By running the DCOPF, generators are classified in three categories. The first category includes the generators with high generation cost which are restricted to their minimum limit. The second one includes the generator with marginal power generation and the third one comprises the generator with low generation cost which are restricted to their uppercase power limit. The below figure illustrate the stated classification.

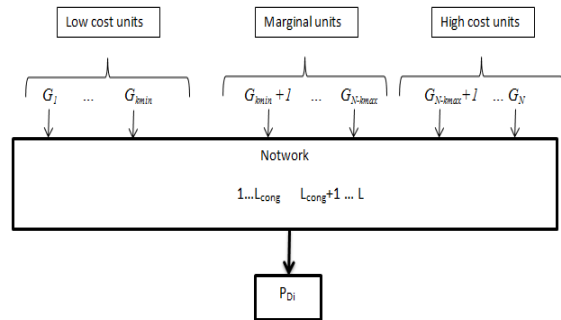


Fig. 1. The simple diagram of power system

The corresponding Lagrangian formulation for the minimization the problem (2) can be stated as Eq. (3).

$$\begin{aligned}
 & l(P_1, \dots, P_N, \lambda, \mu_1^{\min}, \mu_1^{\max}, \dots, \mu_N^{\min}, \\
 & \mu_N^{\max}, \Gamma_1^{\min}, \Gamma_1^{\max}, \dots, \Gamma_L^{\min}, \Gamma_L^{\max}) = \\
 & \sum_{i=1}^N (a_i * P_i + \frac{b_i}{2} * P_i^2) + \lambda (P_d - \sum_{i=1}^N P_i) + \sum_{i=1}^N \mu_i^{\min} (P_i^{\min} - P_i) \\
 & + \sum_{i=1}^N \mu_i^{\max} (P_i - P_i^{\max}) + \sum_{l=1}^L (\Gamma_L^{\min} (\alpha_l - \sum_{i=1}^N \gamma_{l,i} P_i)) + \\
 & \sum_{l=1}^L (\Gamma_L^{\max} (\sum_{i=1}^N \gamma_{l,i} P_i - \alpha_l))
 \end{aligned} \tag{3}$$

By solving the Kuhn-Tucker conditions for the above Lagrange equation, it has been proved in [7] that nodal prices are as equation (4).

$$\begin{aligned}
 LMP_n = & \frac{P_d}{C_1} - \sum_{i=1}^{K_{\min}} \frac{P_i^{\min}}{C_1} - \sum_{i=N-K_{\max}+1}^N \frac{P_i^{\max}}{C_1} + \\
 & \sum_{i=K_{\min}+1}^{N-K_{\max}} \left( \frac{a_i}{C_1 + b_i} \right) + \sum_{l=1}^{l_{cong}} \left( \frac{\sum_{i=K_{\min}+1}^{N-K_{\max}} \gamma_{l,i} / b_i}{C_1} - \gamma_{l,n} \right) \Gamma_l^{\max}
 \end{aligned} \tag{4}$$

$$\begin{aligned}
 \Gamma_k^{\max} = & \frac{1}{\sum_{k=1}^{l_{cong}} \left( \sum_{i=K_{\min}+1}^{N-K_{\max}} \frac{\sum_{j=K_{\min}}^{N-k} \gamma_{l,i} \gamma_{k,j} / b_j}{C_1 * b_i} - \frac{\gamma_{l,i} \gamma_{k,i}}{b_i} \right)} * \\
 [\bar{\alpha}_l - \sum_{j=1}^{k_{\min}} \gamma_{l,j} P_j^{\min} - \sum_{j=K_{\min}+1}^{N-k} \gamma_{l,j} \frac{P_d - \sum_{i=1}^{k_{\min}} P_i^{\min}}{C_1 * b_j} - \\
 \sum_{j=N-K_{\max}+1}^N (\gamma_{l,j} - \frac{\sum_{i=K_{\min}+1}^{N-k} \gamma_{l,i} / b_i}{C_1}) - \sum_{j=K_{\min}+1}^{N-k} (\frac{\sum_{i=K_{\min}+1}^{N-k} \gamma_{l,i} / b_i}{C_1 * b_j} - \frac{\gamma_{l,j}}{b_j}) a_j]
 \end{aligned} \tag{5}$$

By ignoring loss (Eq.6) the LMP formulation can be stated as Eq.7:

$$\begin{aligned}
 \sum_{i=1}^N P_i = P_d \longrightarrow \\
 P_d - \sum_{i=1}^{K_{\min}} P_i^{\min} - \sum_{i=N-K_{\max}+1}^N P_i^{\max} = \sum_{i=K_{\min}+1}^{N-K_{\max}} P_i
 \end{aligned} \tag{6}$$

Substituting Eq.6 into Eq. (4) yields:

$$\begin{aligned}
 LMP_n = & \sum_{i=K_{\min}+1}^{N-K_{\max}} P_i / C_1 + \sum_{i=K_{\min}+1}^{N-K_{\max}} \left( \frac{a_i}{C_1 * b_i} \right) + \\
 & \sum_{l=1}^{l_{cong}} \left( \frac{\sum_{i=K_{\min}+1}^{N-K_{\max}} \gamma_{l,i} / b_i}{C_1} - \gamma_{l,n} \right) \Gamma_l^{\max}
 \end{aligned} \tag{7}$$

By some calculation Eq. (8) is obtained from Eq. (7) as below:

$$\begin{aligned}
 LMP_n = & \sum_{i=K_{\min}+1}^{N-K_{\max}} \left( \frac{P_i}{C_1} + \frac{a_i}{C_1 * b_i} \right) + \\
 & \sum_{l=1}^{l_{cong}} \left( \frac{\sum_{i=K_{\min}+1}^{N-K_{\max}} \gamma_{l,i} / b_i}{C_1} - \gamma_{l,n} \right) \Gamma_l^{\max} =
 \end{aligned} \tag{8}$$

$$\sum_{i=K_{\min}+1}^{N-K_{\max}} \left( \frac{P_i b_i + a_i}{C_1 * b_i} \right) + \sum_{l=1}^{l_{cong}} \left( \frac{\sum_{i=K_{\min}+1}^{N-K_{\max}} \gamma_{l,i} / b_i}{C_1} - \gamma_{l,n} \right) \Gamma_l^{\max}$$

Substituting Eq. (1) into Eq. (8) yields:

$$\begin{aligned}
 LMP_n = & \sum_{i=K_{\min}+1}^{N-K_{\max}} \left( \frac{P_i}{C_1 * b_i} \right) + \\
 & \sum_{l=1}^{l_{cong}} \left( \frac{\sum_{i=K_{\min}+1}^{N-K_{\max}} \gamma_{l,i} / b_i}{C_1} - \gamma_{l,n} \right) \Gamma_l^{\max}
 \end{aligned} \tag{9}$$

The obtained formula for LMP includes two terms. The first term (left part in Eq. (9)) is a common term in all buses price formula and depends on marginal units bids. If there is no congestion in the network, all buses of the system have same price that is equal to the first term of Eq. (9) (common part of all buses price) and second term is omitted. If congestion exists in the system, the second part of formula causes the advent of different price in different bus. The second term depends on each generator share in lines flow of the system.

The first part of Eq. (9) is called  $lmp_n^{energy}$  that is related to marginal unities bids and the second part of formula is called  $lmp_n^{cong}$  that causes different price in different buses.

In this research work,  $lmp_n^{energy}$  is considered as the base price .

So Eq.9 can be stated as below:

$$lmp_n = lmp_n^{energy} + lmp_n^{cong} \tag{10}$$

$$\begin{aligned}
 lmp_n^{cong} &= \frac{\sum_{i=1}^{i^{cong}} \left( \frac{\sum_{j=k_{min}+1}^{N-k_{max}} \gamma_{l,i} / b_j}{C_1} - \gamma_{l,n} \right)}{\sum_{k=1}^{i^{cong}} \left( \sum_{j=k_{min}+1}^{N-k_{max}} \frac{\gamma_{l,i} \gamma_{k,j} / b_j}{C_1 * b_i} - \frac{\gamma_{l,i} \gamma_{k,i}}{b_i} \right)} * \\
 [\bar{\alpha}_i - \sum_{j=1}^{k_{min}} \gamma_{l,j} P_j^{min} - \sum_{j=k_{min}+1}^{N-k_{max}} \gamma_{l,j} \frac{P_d - \sum_{i=1}^{k_{min}} P_i^{min}}{C_1 * b_j} - \\
 \sum_{j=N-k_{max}+1}^N (\gamma_{l,j} - \frac{\sum_{i=k_{min}+1}^{N-k_{max}} \gamma_{l,i} / b_i}{C_1}) - \sum_{j=k_{min}+1}^{N-k_{max}} (\frac{\sum_{i=k_{min}+1}^{N-k_{max}} \gamma_{l,i} / b_i}{C_1 * b_j} - \frac{\gamma_{l,j}}{b_i}) a_j] \\
 LMP_n^{energy} &= \sum_{i=K_{min}+1}^{N-K_{max}} \left( \frac{\rho_i}{C_1 * b_i} \right)
 \end{aligned} \tag{11}$$

**2. Nodal supplying of a generator**

By running DCOPE, the generation vector of generator is obtained ( $p_g$ ). To calculate the delivered power of a generator to each bus as Ref [10] the below equations are applied to the generation vector  $p_g$ .

According to the below picture which represents the inflow power, outflow power, load ( $P_{d_n}$ ) and generation ( $pg_n$ ) at bus n of system, the below equation can be obtained:

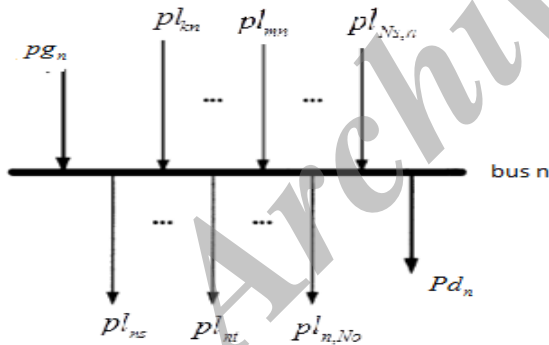


Fig. 2. A general node n in the system

According to above figure the below equation can be obtained:

$$PI_n = \sum_{k \in N_s} pl_{kn} + pg_n \tag{12}$$

Where  $PI_n$  is total power inflow into the bus n. The  $pl_{kn}$  denotes the inflow power from bus k to bus n and the number of inflows to bus n is  $N_s$ . The above equation can be written as below for N node system.

$$PI_n - \sum_{k \in N_s} \left( \frac{pl_{kn}}{PI_k} * PI_k \right) = pg_n, n = 1, 2, \dots, N \tag{13}$$

The matrix form of above equation can be written as:

$$M * PI = pg \tag{14}$$

Where  $PI$  denotes the vector of nodal supplying power,  $P_g$  is the vector of nodal generations, and  $M$  is the distribution matrix with element  $m_{kn}$ .

$$m_{kn} = \begin{cases} 1 \rightarrow \text{if } : k = n \\ -\frac{pl_{kn}}{PI_n} \rightarrow \text{if } : k \in N_s \\ 0 \rightarrow \text{otherwise} \end{cases} \tag{15}$$

$$PI = M^{-1} * pg \tag{16}$$

Each generator contribution in load of bus n can be written as below:

$$P_{i,n} = \frac{P_{d,n}}{PI_n} [M^{-1}]_{i,n} * P_i \tag{17}$$

Where  $P_{d,n}$  denotes bus n load and  $P_i$  is total generations of generator i. By applying Eq.17 the share of each generator in each bus demand of system is obtained. The generator share in each demand bus of system is the basis of calculating the congestion revenue of generator as well as the congestion payment of customers as stated in the following.

**3. The revenue of each generator from demand supplying of a certain bus and payment of each customer for that demand**

A) Revenue of each generator from each energy exchange:

As it is stated in section 2 LMP is decomposed into LMP of energy and LMP of congestion. Also generator share in bus load of system was determined in section 3. Now the nodal revenue of generator i from bus k can be stated as below:

$$d_{i,n} = P_{i,n} * LMP_k \tag{18}$$

$LMP_k$  denotes the LMP of bus k which generator i is connected it. As it was stated previously in Eq.12, LMP consists of two parts (energy and congestion), so replacing Eq.12 into Eq.18 yields:

$$d_{i,n} = lmp_k^{energy} * P_{i,n} + lmp_k^{cong} * P_{i,n} \tag{19}$$

Substituting Eq.11a and Eq.11b into Eq.22 yields:

$$d_{i,n} = \sum_{i=K_{\min}+1}^{N-K_{\max}} \left( \frac{\rho_i}{C_1 * b_i} \right) * P_{i,n} + \sum_{l=1}^{l_{cong}} \left( \frac{\sum_{i=K_{\min}+1}^{N-K_{\max}} \gamma_{l,i} / b_i}{C_1} - \gamma_{l,k} \right) \Gamma_l^{\max} P_{i,n} \quad (20)$$

Finally by replacing Eq.17 into Eq.20, revenue of generator *i* (which is connected to bus *k*) from selling energy to bus *n* can be stated as:

$$d_{i,n} = \sum_{i=K_{\min}+1}^{N-K_{\max}} \left( \frac{\rho_i}{C_1 * b_i} \right) * \frac{P_{d,n}}{PI_n} [M^{-1}]_{n,i} P_i + \sum_{l=1}^{l_{cong}} \left( \frac{\sum_{i=K_{\min}+1}^{N-K_{\max}} \gamma_{l,i} / b_i}{C_1} - \gamma_{l,k} \right) \Gamma_l^{\max} \frac{P_{d,n}}{PI_n} [M^{-1}]_{n,i} P_i \quad (21)$$

According to Eq.21, revenue of generator *i* from selling electricity to bus *n*, can be divided into two parts. The first part (left side of Eq.21) is common among all generators which is related to marginal unites bid. But the second part denotes the increase or decrease of generator *i* revenue from energy sale to bus *n* which is related to congestion and more precisely to structure of system. So the revenue of generator *i* can be stated as below:

$$d_{i,n} = d_{i,n}^{energy} + d_{i,n}^{cong}$$

$$d_{i,n}^{energy} = \sum_{i=K_{\min}+1}^{N-K_{\max}} \left( \frac{\rho_i}{C_1 * b_i} \right) * \frac{P_{d,n}}{PI_n} [M^{-1}]_{n,i} P_i$$

$$d_{i,n}^{cong} = \frac{\sum_{l=1}^{l_{cong}} \left( \frac{\sum_{i=K_{\min}+1}^{N-K_{\max}} \gamma_{l,i} / b_i}{C_1} - \gamma_{l,k} \right)}{\sum_{k=1}^{l_{cong}} \left( \sum_{j=K_{\min}+1}^{N-K_{\max}} \frac{\gamma_{l,i} \gamma_{k,j} / b_j}{C_1 * b_i} - \frac{\gamma_{l,i} \gamma_{k,i}}{b_i} \right)} \left[ \bar{\alpha}_i - \sum_{j=1}^{k_{\min}} \gamma_{l,j} P_j^{\min} - \sum_{j=k_{\min}+1}^{N-k_{\max}} \gamma_{l,j} \frac{P_d - \sum_{i=1}^{k_{\min}} P_i^{\min}}{C_1 * b_j} - \sum_{j=N-k_{\max}+1}^N \left( \gamma_{l,j} - \frac{i=K_{\min}+1}{C_1} \right) - \sum_{j=k_{\min}+1}^{N-k_{\max}} \left( \frac{i=K_{\min}+1}{C_1 * b_j} - \frac{\gamma_{l,j}}{b_i} \right) a_j \right] * \frac{P_{d,n}}{PI_n} [M^{-1}]_{n,i} P_i \quad (22)$$

Eq. 22 denotes the revenue of generator *i* from energy sale to bus *n* which is decomposed in two separable parts. Eq.22a denotes the common revenue

of generator from bus *n*, while Eq.22b denotes congestion revenue of generator from bus *n*. So the increase or decrease in generator *i* revenue depends on congested lines. The amounts of  $d_{i,n}^{cong}$  denotes the effect of congestion of line on the revenue of generators. If this term is positive the revenue of generator increase where as if it is negative it causes that revenue of generator decreases. Most of the time this term is negative, caused that revenue of generator decreases.

$$d_i^{cong} = \Delta d_i = \sum_{n=1}^N (d_{i,n} - d_{i,n}^{energy}) = \sum_{n=1}^N P_{i,n} (lmp_k - lmp_k^{energy}) \quad (23)$$

The above equation denotes the variation of generator *i* revenue caused by line congestion.

**B) Customer payment for each energy exchange:**

Customer payment at bus *n* to buy from generator *i* (that is connect to bus *k*) can be state

$$s_{i,n} = P_{i,n} * LMP_n \quad (24)$$

The customer's payment can be divided into two parts like generator revenue (Eq.24 & Eq.25). First part is corresponding to consumed energy and second part denotes the increase or decrease in payments of customers which caused by lines congestion.

$$s_{in} = \sum_{i=K_{\min}+1}^{N-K_{\max}} \left( \frac{\rho_i}{C_1 * b_i} \right) * P_{i,n} + \sum_{l=1}^{l_{cong}} \left( \frac{\sum_{i=K_{\min}+1}^{N-K_{\max}} \gamma_{l,i} / b_i}{C_1} - \gamma_{l,n} \right) \Gamma_l^{\max} P_{i,n} \quad (25)$$

$$s_{i,n} = s_{i,n}^{energy} + s_{i,n}^{cong} \quad (26)$$

The above equations denote the payment of customers of bus *n* to buy energy from generator *i*. This term is consisting of two parts. The first part is the money that customers pay to buy energy from generators and the second part denotes the extra money that customers of bus *n* pay as a transmission rent to ISO as well as the share of customers of bus *n* in MS of this energy exchange (energy exchange between bus *n* customers and generator *i*). By considering all generator of system (Ng) the above equation can be written as below:

$$s_n^{cong} = \Delta S_n = \sum_{i=1}^{N_g} (s_{i,n} - s_{i,n}^{energy}) = \sum_{i=1}^{N_g} P_{i,n} (lmp_n - lmp_n^{energy}) \tag{27}$$

The above equation denotes the share of bus  $n$  customers in MS of system as well as the share of bus  $n$  customers in transmission lines rent.

Since  $lmp^{energy}$  is equal in all buses, we can result that energy parts of customer payment and generator revenue are equal as below:

$$s_{i,n}^{energy} = d_{i,n}^{energy} \tag{28}$$

#### 4. Congestion surplus determination for each energy exchange and its allocation

##### A) Congestion surplus determination for each energy exchange:

MS of an energy exchange between generator  $i$  (at bus  $k$ ) and demand at bus  $n$  is the difference between payment of customer at bus  $n$  to buy energy from generator  $i$  and revenue of generator  $i$  from selling energy to bus  $n$ . So MS of each energy exchange can be formulated as below:

$$MS_{i,n} = s_{i,n} - d_{i,n} \xrightarrow{s_{i,n}^{energy} = d_{i,n}^{energy}} MS_{i,n} = s_{i,n}^{cong} - d_{i,n}^{cong} = P_{i,n} (LMP_n^{cong} - LMP_k^{cong}) \tag{29}$$

Substituting Eq.11 a into Eq.27 yields:

$$MS_{i,n} = \sum_{l=1}^{l_{cong}} \left( \frac{\sum_{i=K_{min}+1}^{N-K_{max}} \gamma_{l,i} / b_i}{C_1} - \gamma_{l,n} \right) \Gamma_l^{max} P_{i,n} - \sum_{l=1}^{l_{cong}} \left( \frac{\sum_{i=K_{min}+1}^{N-K_{max}} \gamma_{l,i} / b_i}{C_1} - \gamma_{l,k} \right) \Gamma_l^{max} P_{i,n} \tag{30}$$

$$MS_{i,n} = \sum_{l=1}^{l_{cong}} \left( \frac{\sum_{i=K_{min}+1}^{N-K_{max}} \gamma_{l,i} / b_i}{C_1} - \gamma_{l,n} - \frac{\sum_{i=K_{min}+1}^{N-K_{max}} \gamma_{l,i} / b_i}{C_1} + \gamma_{l,k} \right) \Gamma_l^{max} P_{i,n} \tag{31}$$

$$\Gamma_l^{max} P_{i,n}$$

$$MS_{i,n} = \sum_{l=1}^{l_{cong}} (\gamma_{l,k} - \gamma_{l,n}) \Gamma_l^{max} P_{i,n} \tag{32}$$

that generator  $i$  receive for generation of that energy.

Congestion surplus for each energy exchange between demand of bus  $n$  and generation of bus  $k$  can be calculated through above equation. Eq.30 is multiply of two parts: the first part is share of

generator  $i$  (there is bus  $k$ ) in bus  $n$  power supply ( $P_{i,n}$ ) and the second part denotes the difference

between the share of each generator bus in flow of congested line and the share of each customer bus in flow of congested line ( $\gamma_{l,k} - \gamma_{l,n}$ )

Consider line  $l$  that delivers the generated power of generator  $i$  to bus  $n$ , the MS of this line can be stated as below:

$$MS_{i,n,l} = P_{i,n} (\gamma_{l_{cong},k} - \gamma_{l_{cong},n}) \Gamma_l^{max} \tag{33}$$

Total MS to supply the demand of bus  $n$  is equal to:

$$MS_n = s_n - \sum_{i=1}^{N_g} P_{i,n} * LMP_i = LMP_n * P_{d,n} - \sum_{i=1}^{N_g} P_{i,n} * LMP_i \tag{34}$$

$$MS_n = \sum_{i=1}^{N_g} MS_{i,n} = \sum_{i=1}^{N_g} P_{i,n} \sum_{l=1}^{l_{cong}} (\gamma_{l,i} - \gamma_{l,n}) \Gamma_l^{max} \tag{35}$$

So total MS of system can be stated as:

$$MS = \sum_{n=1}^N MS_n = \sum_{i=1}^{N_g} s_i - \sum_{i=1}^{N_g} d_i \tag{36}$$

##### B) Congestion surplus allocation among transmission line with notice to its payment origin between Transco's:

As it was stated previously in some power markets the MS of system pay as a transmission cost to Transco. In this section we pursuit that how MS should be allocated to transmission lines as the lines which has more effect in creation of MS have more share in MS of system to expand more quickly in comparison to the other lines. So ISO identify the congested line that have more share in MS of system and determines the candidate line for expansion. As much as the performance of Transco is correspond to the optimal TEP, the revenue of Transco increase.

Eq.33 denotes the share of line  $l$  in MS which is derived from energy exchange between bus  $n$  and generator  $i$ .  $MS_{i,n,l}$  is a congestion rent that line  $l$  is received for energy exchange between Generator  $i$  and customer of bus  $n$ , in fact  $MS_{i,n,l}$  denotes the share of line  $l$  in MS which is derived from energy exchange between bus  $n$  and generator  $i$ .

Now if the summation of  $MS_{i,n,l}$  is calculated for each energy exchange for line l, the share of line l is obtained in MS of system as well as the money which is received by the Transco that is the owner of line l. As much as this money increase the priority of line for expansion increases.

$$MS_{i,n,l} = P_{i,n}(\gamma_{l_{cong},n} - \gamma_{l_{cong},k})\Gamma_l^{\max} = \frac{P_{i,n}(\gamma_{l_{cong},n} - \gamma_{l_{cong},k})}{\sum_{k=1}^{l_{cong}} \left( \sum_{i=k^{\min}+1}^{N-k^{\max}} \frac{\gamma_{l,i}\gamma_{k,j}/b_j}{C_1 * b_i} - \frac{\gamma_{l,i}\gamma_{k,i}}{b_i} \right)} \left[ \bar{\alpha}_l - \sum_{j=1}^{k^{\min}} \gamma_{l,j} P_j^{\min} - \sum_{j=k^{\min}+1}^{N-k^{\max}} \gamma_{l,j} \frac{P_d - \sum_{i=1}^{k^{\min}} P_i^{\min}}{C_1 * b_j} - \sum_{j=N-k^{\max}+1}^N \left( \gamma_{l,j} - \frac{\sum_{i=k^{\min}+1}^{N-k^{\max}} \gamma_{l,i}/b_i}{C_1} \right) - \sum_{j=k^{\min}+1}^{N-k^{\max}} \left( \frac{\sum_{i=k^{\min}+1}^{N-k^{\max}} \gamma_{l,i}/b_i}{C_1 * b_j} - \frac{\gamma_{l,j}}{b_i} \right) a_j \right] \quad (37)$$

## 5. Case Study

The selected test case to study is PJM 5 bus test system. Fig. 3 shows the diagram of this test case and Tables 1 and 2 depict its lines and generation data. Here there are 4 Gencos and 3 loads (Genco A has 2 generator Alt and Park city). The system may be roughly divided into two areas, a generation center consisting of Buses A and E including three low-cost generation units and a load center consisting of Buses B, C, and D including two high-cost generation units.

Generation limits of Gencos and their bid coefficient are depicted in table 2. The result of decomposing LMP as described previously in section 2 depicted in Table 4.

Table 5 contains share of each generator in load supplying of each bus. It shows all exchanges between system's buses ( $P_{i,n}$ ).

Table 6 contains MS of energy exchanges in the system.

Table.7 denotes the congestion payments of customers due to the congestion of lines AB & ED. According to below table the most disadvantage of congestion of line ED&AB received by customers at bus B. in another word the customers at bus B have the maximum share in transmission rent. With respect to tables.7 & 8 the share of each line in transmission rent is calculable.

Table 8 denotes the congestion revenue of generators due to the congestion of lines AB & ED.

According to above table the most generators benefits from line congestion except generator E. congestion cause that generator E revenue decrease. This signifies that generator E like customers has share in transmission rent.

Since in this load level line ED&AB have congestion the MS of system belongs to these lines to expand in future. The share of these line in depicted in below tables. ( $MS_{i,n,ED}$ ) ( $MS_{i,n,AB}$ ).

According to below figure the MS of energy exchange between Genco A and customer at bus B is equal to 610.875\$, that is an extra money which customers pay due to congestion to buy energy. The share of line ED in the MS of this energy exchange is 163.95 \$ and the share of line AB is 446.92\$.

The MS of all energy exchange is calculated and depicted in below figure.

It should be noted that in some cases energy exchange between two buses is negative. It signify that energy exchange between these buses not only cause MS but also cause an under budget and ISO faces an under budget for this energy exchange that should be supplied. For instance MS of energy exchange between customer at bus C and Genco D is negative. This under budget is received from rent of line AB & ED cause the decrease in revenue of Transco.

According to table 10, the rent (MS) of line AB at 8:00 pm is equal to  $MS_{AB} = 4432.44$  and for line ED is  $MS_{ED} = 8057.22$ . According to table.6 the whole MS of system is equal to 12489.66\$ that exactly equal to the MS summation of line AB & ED.

The amount of MS for each line denotes the priority of expansion for this line. In this load level line ED has more priority for expansion since the more share of this line in MS of system. This expansion causes the more flat price profile of buses. More precisely customers which pay for congestion of line ED, should benefit more from TEP.

But in different load level different congestion occurs. For instance in load level 720MW line AB is not congested, so in this case whole MS of system allocate to line ED for expansion. Below figure denotes the assumed load model for 24 hours of 5 bus test system.

Below figure denotes the MS share of each line which is allocated to them for expansion in different hour of system.

$$MS_{AB} = 28849.65$$

$$MS_{ED} = 58877.34$$

According to above figures by variation of load from 500 to 950MW the MS share of each line varies too. Line ED has more MS share in comparison to line AB in different hour of a day. It signifies the priority of line ED for expansion. ISO should present a TEP plan which this line has more priority for expansion. By expansion of line ED, the more competitive market and more flat profile of price is obtained.

**6. Conclusion**

In this paper a method is presented in which the MS of power market considers as a variable rent of Transco. The MS of system is an extra money that customers pays and so should be utilized in a way that customers benefits from it. So in this paper the TEP is determined by ISO in the way that the lines which have more share in MS of system, have more priority for expansion. As much as the Transco performance is correspond to determined TEP, the variable revenue of Transco increases or decreases. The basis of determining the MS share of a line is determining the MS share of each energy exchange between certain generator and certain demand bus in power system. By applying the proposed method TEP economical resources is determined exactly. Transco revenue relates to optimality of its performance and the capacity withholding and misusing is prevented spontaneously. Ultimately the proposed TEP method increases the competition and efficiency of power market.

Table.1.  
Line impedance and flow limits of them

Line	ED	EA	AB	AD	DC	CB
Limit(MW)	240	700	400	800	900	900
X(%)	2.97	0.64	2.81	3.04	2.97	1.08

Table.1.  
Generation limits of Gencos and their bid coefficient

GEN	$a_i$ (\$/MWh)	$b_i$ (\$/MW <sup>2</sup> h)	$p_i^{max}$ (MW)
Brighton	10	0	600
Alta	14	0.00559	40
Park city	15	0.02148	170
Sundance	35	0.365	200
Solitude	30	0.37937	520

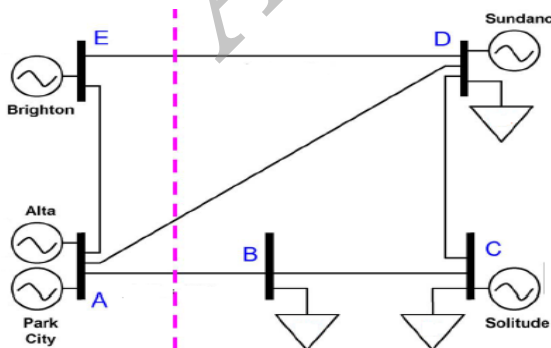


Fig. 3. Diagram of modified PJM five-bus

Table.2.  
Bus demand data at 8:00 pm is depicted in table 3.

BUS	$P_{d,n}$ (MW)
BUS A	0
BUS B	200
BUS C	300
BUS D	300
BUS E	0

Table.3.  
 $lmp$ ,  $lmp^{energy}$  and  $lmp^{cong}$  in each bus

\$	Lmp	$lmp^{energy}$	$lmp^{cong}$
BUS A	19.4368	18.4368	+1
BUS B	26.9368	18.4368	+8.5
BUS C	30.4368	18.4368	+12
BUS D	38.4368	18.4368	+20
BUS E	13.4368	18.4368	-5

Table.4.  
The share of each Genco in load supplying of each bus

Nodal supplying power of Gencos	Genco A (MW)	Genco C (MW)	Genco D (MW)	Genco E (MW)	SUM (MW)
BUS B	692.32	0	0	118.5	200
BUS C	67.19	0	25.24	207.55	300
BUS D	63.22	0	36.25	200.51	300

Table.5.  
Congestion surplus from each energy exchange ( $MS_{i,n}$ )

( $MS_{i,n}$ )	Genco A (\$)	Genco C (\$)	Genco D (\$)	Genco E (\$)
BUS B	610.875	0	0	1599.75
BUS C	739.09	0	-201.92	3528
BUS D	1201.18	0	0	5012.75

Table.6.  
variation of customer payment in comparison to case that system has no congestion

( $\Delta S_n$ )	Genco A (\$)	Genco C (\$)	Genco D (\$)	Genco E (\$)	SUM (\$)
BUS B	610.875	0	0	1007.25	1618.1
BUS C	806.28	0	302.88	2490.6	3599.7
BUS D	1264.4	0	0	4010.22	5274.6

Table.7.  
Variation of generator revenue in comparison to case that system has no congestion

( $\Delta d_i$ )	Genco A (\$)	Genco C (\$)	Genco D (\$)	Genco E (\$)
BUS B	692.325	0	0	-592.5
BUS C	67.19	0	504.8	-1037.75
BUS D	63.22	0	725	-1002.55
SUM	822.735	0	1229.8	-2632.8



Table.8.  
Share of line ED in MS of each energy exchange

$MS_{i,n,ED}$	Genco A (\$)	Genco C (\$)	Genco D (\$)	Genco E (\$)
BUS B	163.95	0	0	527.13
BUS C	274.60	0	-92.30	1526.42
BUS D	787.02	0	0	4870.41

Table.9.  
Share of AB in congestion surplus from each energy exchange

$(MS_{i,n,AB})$	Genco A (\$)	Genco C (\$)	Genco D (\$)	Genco E (\$)
BUS B	446.92	0	0	1072.61
BUS C	464.48	0	-109.61	2001.57
BUS D	414.15	0	0	142.33

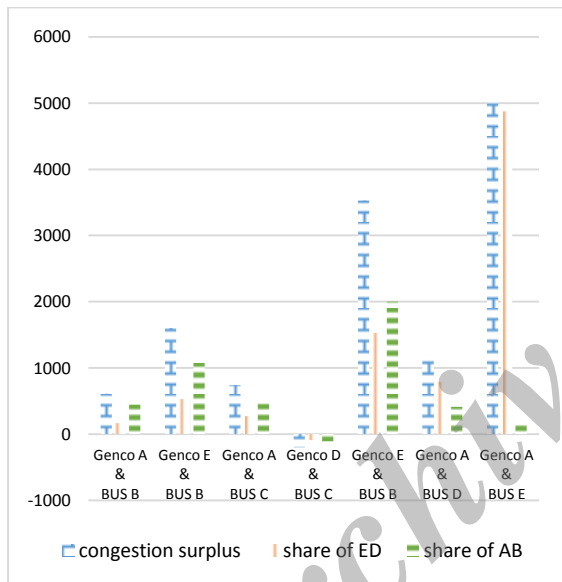


Fig. 4. MS of each energy exchange and the share of congested line in MS of each energy exchange

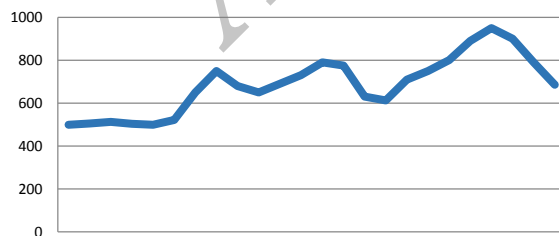


Fig. 5. Hourly load of 5 bus test system during a day

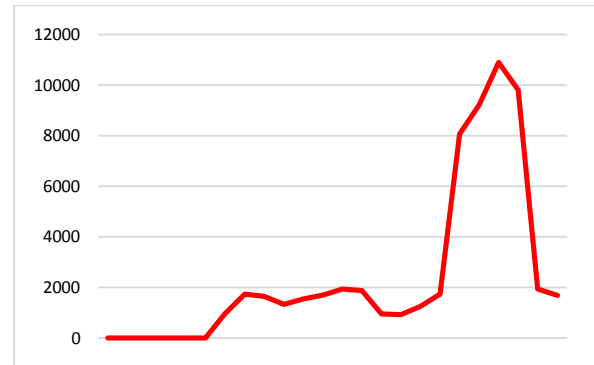


Fig. 6. MS share of each line in different hour of system

## References:

- [1] B Murray, *Electricity Markets: Investment, Performance and Analysis*. New York: Wiley, 1998.
- [2] L. S. Hyman, "Transmission, congestion, pricing and incentive," *IEEE 2. Power Eng. Rev.*, vol. 19, 1999.
- [3] Shirmohammadi, X. V. Filho, B. Gorenstin, M.V.P. Pereira, "Some Fundamental Technical Concepts About Cost Based Transmission Pricing", *IEEE Trans. Power Syst*, Vol. 11, No. 2, pp. 1002-1008, 1996.
- [4] J. Pan, Y. Teklu, S. Rahman, and K. Jun, "Review of Usage-Based Transmission Cost Allocation Methods under Open Access", *IEEE Trans. Power Syst*, Vol. 15, No. 4, pp. 1218-1224, 2000.
- [5] F.J. Rubio-Oderiz and I.J. Perez-Arriaga, "Marginal Pricing of Transmission Services: A Comparative Analysis of Network Cost Allocation Methods", *IEEE Trans. Power Syst*, Vol. 15, No. 1, pp. 448-454, 2000.
- [6] Hogan. W W, "Market-Based Transmission Investments and Competitive Electricity Markets", August 1999.
- [7] L. Y. C. Amarasinghe, Member, IEEE, and U. D. Annakkage "Determination of Network Rental Components in a Competitive Electricity Market" *IEEE TRANSACTIONS ON POWER SYSTEMS*, VOL. 23, NO. 3, 2008.
- [8] H.A. Gil, E.L. da Silva, and F.D. Galiana, "Modeling Competition in Transmission Expansion", *IEEE Trans. Power Syst*, Vol. 17, No. 4, pp. 1043-1049, 2002.
- [9] Hajiabadi, Mohammad Ebrahim, and Habib Rajabi Mashhadi. "LMP decomposition: A novel approach for structural market power monitoring." *Electric Power Systems Research* 99,: 30-3, 2013.
- [10] Wang, Peng, Yu Xiao, and Yi Ding. "Nodal market power assessment in electricity markets" *Power Systems, IEEE Transactions on* 19.3, 1373-1379, 2004.