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Performance evaluation of Iran universities with Stochastic Data Envelopment Analysis (SDEA)

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Abstract

Performance evaluation of universities is an important issue between researchers. Classic data envelopment analysis (DEA) models with deterministic data have been used by many authors to measure efficiency of universities in different countries. However, DEA with stochastic data are, rarely used to measure efficiency of universities. In this paper, input oriented model in stochastic data envelopment analysis is used to evaluate universities in Iran. In addition, super efficiency model is developed in stochastic DEA to rank stochastic efficient units. To obtain numerical results with stochastic data, a deterministic equivalent of each stochastic model is used which can be converted to a quadratic program. Thirty-three Iran universities have been considered in this study for which computational results of both classic and stochastic approaches are obtained.

Key words: Stochastic Data Envelopment Analysis (SDEA); Ranking; Efficiency, universities

1 Introduction

DEA as mathematical programming approach for determining the relative efficiency of decision making units (DMUs) was originated by Charnes, Cooper and Rhodes in 1978, and the first model in DEA was called CCR [3]. Since 1978 further models have been introduced in the literature, for instance, Banker, Charnes and Cooper in 1984 developed a variable returns to scale version of the CCR model that was called BCC model [2]. Khodabakhshi et al. [6] developed additive model for estimating returns to scale in imprecise DEA (see, e.g., [4-8]). In this paper, first, the classic input orientation model introduced in Banker et al.(1984) is used to evaluate Iran universities. Super efficiency model introduced in Anderson, Petersen (1993) is used to rank efficient universities [1]. In this paper, the stochastic super efficiency model is used to rank stochastic efficient universities. To obtain numerical results with Stochastic data, a deterministic equivalent of each stochastic model is used which can be converted to a quadratic program. Computational results of the classic and stochastic approaches are compared, too.

2 Classic DEA

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One of the basic models used to evaluate DMUs efficiency is BCC model introduced by Banker, Charnes, Cooper in 1984. This model is as follows.

$$
\begin{aligned}
&\text{Minimize} \quad \theta_o - \varepsilon\left(\sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+\right) \\
&s.t \quad \sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- = \theta x_{io}, \quad i=1,\dots,m \\
&\qquad \sum_{j=1}^{n} \lambda_j y_{rj} + s_r^+ = y_{ro}, \quad r=1,\dots,s \qquad (1) \\
&\qquad \sum_{j=1}^{n} \lambda_j = 1, \\
&\qquad \lambda_j \geq 0, s_i^- \geq 0, s_r^+ \geq 0 \;\; ; \;\; j=1,\dots,n, i=1,\dots,m, r=1,\dots,s
\end{aligned}
$$

Definition 1. (Efficiency according to model(1)): DMU0 is efficient when in optimal solution(s)

i) $\theta_o^* = 1$
ii) $s_i^{-*} = s_r^{+*} = \circ, i=1,\dots,m, r=1,\dots,s.,$

3 Super efficiency model

Excluding the column vector correspond to DMUo from the LP coefficients matrix of model (1) input oriented super-efficiency model introduced by Andersen and Petersen (1993) is defined as follows:

$$
\begin{aligned}
&\text{Minimize} \quad \theta_o^s \\
&s.t \sum_{j=1, j\neq o}^{n} \lambda_j x_{ij} \leq \theta_o^s x_{io}, \quad i=1,\dots,m \\
&\quad \sum_{j=1, j\neq o}^{n} \lambda_j y_{rj} - y_{ro} \geq \circ, \; r=1,\dots,s \qquad (2) \\
&\quad \sum_{j=1, j\neq o}^{n} \lambda_j = 1, \\
&\quad \lambda_j \geq 0, j=1,\dots,n, j\neq o
\end{aligned}
$$

Definition 2: If the optimal objective value of the super-efficiency model is greater than 1, DMUo that is efficient in the BCC model is super efficient. Otherwise, DMU_o is not super efficient. Thus, one can
solve the super-efficiency model for ranking efficient units without solving the BCC model (1).

4 Stochastic input oriented super-efficiency model

Following Cooper et al. [4], Khodabakhshi et al. [7], Khodabakhshi and Kheirollahi [8] let $\tilde{x}_j = (\tilde{x}_{1j}, \tilde{x}_{2j},\dots,\tilde{x}_{mj})^T$ and $\tilde{y}_j = (\tilde{y}_{1j}, \tilde{y}_{2j},\dots,\tilde{y}_{sj})^T$ represent $m\times 1$ and $s\times 1$ random input and output vectors, respectively, and $x_j = (x_{1j}, x_{2j},\dots,x_{mj})^T$ and $y_j = (y_{1j}, y_{2j},\dots,y_{sj})^T$ stand for the corresponding vectors of expected values of input and output for each DMUj; j = 1,. . . ,n. That is, we utilize these expected values in place of the observed values in (2). Let us consider all input and output components to be jointly normally distributed in the following chance-constrained stochastic DEA model which is stochastic version of model (2).

$$\begin{aligned}
&\text{Minimize} \quad \theta_o^s \\
s.t \quad & P\left(\sum_{j=1, j\neq o}^{n} \lambda_j \tilde{x}_{ij} + s_i^- \leq \theta_o^s \tilde{x}_{io} \right) \geq 1-\alpha, i=1,\dots,m \\
& P\left(\sum_{j=1, j\neq o}^{n} \lambda_j \tilde{y}_{rj} - \tilde{y}_{ro} \geq s_r^+ \right) \geq 1-\alpha, r=1,\dots,s \qquad (3) \\
& \sum_{j=1, j\neq o}^{n} \lambda_j = 1, \\
& \lambda_j \geq 0 \qquad j = 1,\dots,n
\end{aligned}$$

Where α , a predetermined value between 0 and 1, specifies the significance level, and P represents the probability measure.

Definition 3: DMUo is stochastically super-efficient at significance level α if the optimal value of the objective function is greater than 1. Therefore, if $\theta_o^{s*}>1$ it means that even if DMUo consumes θ_o^{s*} percent of its current input it can remain efficient, hence the greater the θ_o^{s*} , the better the DMU.

5 Deterministic equivalents

In this subsection, we exploit the Normality assumption to introduce a deterministic equivalent to model (3). It is assumed that x_{ij} and y_{rj} are the means of the input and output variables, which are, in application, observed values of the inputs and outputs.

$$\begin{aligned}
&\text{Minimize } \theta_{\circ}^{s} \\
&s.t \quad \sum_{j=1, j\neq o}^{n} x_{ij}\lambda_j + s_i^- - \phi^{-1}(\alpha)\omega_i^I = \theta_{\circ}^{s} x_{io}, i = 1,\dots,m \\
&y_{ro} - \sum_{j=1, j\neq o}^{n} y_{rj}\lambda_j + s_r^+ - \phi^{-1}(\alpha)\omega_r^0 = 0, r = 1,\dots,s \\
&\sum_{j=1, j\neq o}^{n} \lambda_j = 1, \qquad (4) \\
&(\omega_i^I)^2 = \sum_{j\neq 0}\sum_{k\neq 0} \lambda_j\lambda_k \operatorname{cov}(\tilde{x}_{ij}, \tilde{x}_{ik}) - 2\theta_{\circ}^{s} \sum_{j\neq o} \lambda_j \operatorname{cov}(\tilde{x}_{ij}, \tilde{x}_{i0}) + (\theta_{\circ}^{s})^2 \operatorname{var}(\tilde{x}_{io}) \\
&(\omega_r^o)^2 = \sum_{j\neq 0}\sum_{k\neq 0} \lambda_k\lambda_j \operatorname{cov}(\tilde{y}_{rk}, \tilde{y}_{rj}) - 2\sum_{k\neq o} \lambda_k \operatorname{cov}(\tilde{y}_{rk}, \tilde{y}_{ro}) + \operatorname{var}(\tilde{y}_{ro}) \\
&s_i^-, s_r^+, \lambda_j, \omega_i^I, \omega_r^o \geq 0 \qquad i=1,\dots,m \text{ , } r= 1,\dots,s \text{ , } j=1,\dots,n
\end{aligned}$$

Where ϕ is the cumulative distribution function (cdf) of a standard Normal random variable and ϕ^{-1} is its inverse.

6 Data and results

This study considers data of 33 Iran universities in year 2004. Table 1 shows the data and summary statistics for the case study. To compute results for stochastic data, $\alpha \cong 0.45$ has been chosen for which $\phi^{-1}(\alpha) \cong$ -0.12. This rather large value of α is deliberately chosen to illustrate differences between the results based on the classic and the stochastic one. It is assumed that all DMUs have the same variance, but they can have different means.

Table 1
The data of Inputs and outputs of Iran universities

DATA / DMU	INPUTS							OUTPUTS				
	I1	**I2**	**I3**	**I4**	**I5**	**I6**	**I7**	**O1**	**O2**	**O3**	**O4**	**O5**
D1	0.16	0.27	0.46	0.67	0.60	0.23	0.10	0.28	0.09	0.02	0.18	0.88
D2	0.29	0.38	0.53	0.48	0.86	0.35	0.31	0.55	0.20	0.23	0.39	0.89
D3	0.17	0.30	0.34	0.66	0.35	0.18	0.13	0.24	0.09	0.04	0.14	0.99
D4	0.16	0.30	0.45	0.66	0.39	0.19	0.09	0.29	0.09	0.09	0.20	0.90
D5	0.27	0.37	0.49	0.63	0.81	0.42	0.33	0.47	0.15	0.05	0.47	0.90
D6	1	1	0.99	0.90	0.59	1	1	1	1	1	1	0.91
D7	0.13	0.27	0.47	0.57	0.46	0.19	0.09	0.31	0.10	0.07	0.20	0.88
D8	0.12	0.32	0.72	0.42	0.84	0.34	0.15	0.34	0.05	0.07	0.17	0.91
D9	0.22	0.53	0.68	0.56	0.53	0.30	0.23	0.47	0.13	0.05	0.29	0.90

D10	0.33	0.34	0.39	0.36	0.32	0.39	0.40	0.43	0.19	0.30	0.36	0.92
D11	0.25	0.38	0.62	0.24	0.60	0.43	0.29	0.40	0.16	0.14	0.30	0.92
D12	0.34	0.41	0.50	0.77	0.68	0.58	0.51	0.56	0.54	0.11	0.49	0.91
D13	0.36	0.53	0.75	0.71	0.52	0.49	0.40	0.64	0.21	0.26	0.51	0.93
D14	0.17	0.24	0.36	0.46	0.53	0.24	0.19	0.39	0.11	0.07	0.24	0.90
D15	0.17	0.36	0.56	0.40	0.73	0.22	0.17	0.42	0.08	0.05	0.26	0.90
D16	0.15	0.23	0.37	0.90	0.48	0.17	0.14	0.36	0.05	0.06	0.21	0.92
D17	0.06	0.15	0.23	0.33	0.38	0.07	0.16	0.13	0.03	0.02	0.09	0.86
D18	0.06	0.11	0.20	0.32	0.51	0.09	0.04	0.09	0.01	0.02	0.04	0.91
D19	0.09	0.19	0.24	0.57	0.59	0.12	0.04	0.24	0.02	0.02	0.09	0.92
D20	0.05	0.17	0.23	0.45	0.50	0.11	0.06	0.12	0.03	0.08	0.11	0.90
D21	0.05	0.09	0.20	0.75	0.66	0.09	0.04	0.11	0.02	0.01	0.06	0.92
D22	0.10	0.34	0.69	0.16	0.19	0.14	0.09	0.20	0.01	0.02	0.04	0.93
D23	0.09	0.20	0.27	0.57	0.37	0.12	0.07	0.12	0.05	0.02	0.10	0.90
D24	0.06	0.14	0.21	0.50	0.51	0.10	0.04	0.11	0.02	0.04	0.07	0.90
D25	0.07	0.13	0.17	0.89	0.89	0.10	0.04	0.09	0.05	0.01	0.05	0.92
D26	0.06	0.14	0.16	0.44	0.54	0.07	0.03	0.09	0.02	0.04	0.05	0.96
D27	0.07	0.12	0.22	0.36	0.35	0.11	0.06	0.16	0.05	0.03	0.12	0.92
D28	0.07	0.14	0.23	0.43	0.62	0.10	0.04	0.17	0.03	0.03	0.08	0.88
D29	0.8	0.20	0.42	0.38	0.29	0.10	0.07	0.12	0.04	0.03	0.08	0.92
D30	0.6	0.15	0.20	0.70	0.64	0.08	0.04	0.18	0.03	0.07	0.07	0.90
D31	0.4	0.07	0.18	0.24	0.36	0.06	0.03	0.06	0.01	0.02	0.04	0.92
D32	0.03	0.07	0.13	0.59	0.63	0.08	0.05	0.08	0.01	0.03	0.03	0.93
D33	0.04	0.11	0.17	0.32	0.39	0.06	0.04	0.09	0.02	0.01	0.04	0.90
s.d	0.04	0.11	0.17	0.32	0.39	0.06	0.04	0.21	0.19	0.18	0.20	0.02

Table 2
Ranking Results of classic model (2) and stochastic model (4)

DMU. No.	Super efficiency score(classic)	Super efficiency score(Sto)	Rank
1	0.9094	0.6818	29
2	1.2538	1.0653	14
3	1.2145	1.9626	33
4	1.0728	0.9174	20
5	1.1838	1.0811	19
6	2.3336	1.0854	3
7	1.2316	0.8820	18
8	0.9871	0.8805	22
9	0.9031	0.8975	27
10	1.4529	1.1676	13
11	1.5993	1.1266	12
12	1.5009	1.4242	6
13	1.1767	1.3048	7
14	0.9983	0.9796	25
15	1.0755	0.9656	24
16	1.1155	0.9867	22
17	1.1017	0.8465	23
18	0.9054	0.7417	30
19	1.1794	0.9490	5
20	2.0070	0.9711	9
21	1.0341	0.6898	28
22	1.8042	1.5444	4
23	0.8782	0.8023	32
24	0.9253	0.6945	31
25	0.9495	0.6430	16
26	1.1435	1.4594	2
27	1.2235	0.9432	15
28	1.0876	0.7830	26
29	1.2541	0.8973	17
30	1.2137	0.9718	8
31	1.5404	1.1966	10
32	1.3631	1.0722	11
33	1.0744	0.8857	1

The results of super efficiency model and rank of DMUs in cases classic and stochastic data are presented in columns 2 and 3 of table 2, respectively. Based on the results of classic super efficiency model, university 6, with $\theta_o^{s*} = 2.3336$ is ranked the first. It means that, even if DMU6 consumes 2.3336 times of its current inputs, in comparison to other companies, it remain efficient. Also, Based on the results of stochastic super efficiency model, university 3, with $\theta_o^{s*} = 1.9626$ is ranked the first. It means that, even if DMU3 consumes 1.9626 times of its current inputs, in comparison to other companies, it remain efficient. Note that the higher the super efficiency score, the better the DMUs. Based on the results of classic super efficiency model, the worst DMU is DMU23 with $\theta_o^{s*} = 0.8782$ and based on the results of stochastic super efficiency model, DMU23 with $\theta_o^{s*} = 0.643$ is the worst DMU. In fact, DMU23 with 87.82 percent of its current inputs can be efficient, while 12.12 percent of its current inputs is wasted.

7 Conclusion

In this paper, Iran universities are evaluated by classic and stochastic DEA. Input orientation BCC model introduced in Banker et al. (1984) is used to measure technical efficiency of Iran universities in classic DEA. Furthermore, super efficiency model of Andersen and Peterson (1993) is used to rank efficient universities. Stochastic version of the input oriented model is also used to measure stochastic efficiency of universities. Moreover, super Efficiency model is developed in stochastic DEA to rank stochastic efficient universities. Numerical results obtained by the two approaches are compared, too.

References

[1] P. Andersen and N. C. Petersen, 1993. A procedure for ranking efficient units in DEA. Management Science, Vol 39, No. 10, pp. 1261-1264.

[2] R.D. Banker, A. Charnes, W.W. Cooper, 1984. Some models for estimating technical and scale inefficiencies in DEA, Management Science 30, 1078–-1092.

[3] Charnes A., W.W. Cooper and E. Rhodes, 1978. Measuring the Efficiencies of DMUs, European Journal of Operational Research, 2(6)429-444.

[4] W.W. Cooper, H. Deng, Z. Huang, Susan X. Li, Chance constrained programming approaches to technical efficiencies and inefficiencies in stochastic data envelopment analysis, J. Oper. Res. Soc. 53 (2002) 1347–1356.

[5] Khodabakhshi M. 2007. A super-efficiency model based on improved outputs in DEA. Applied Mathematics and Computation 184 (2)695-703.

[6] Khodabakhshi M., M. Asgharian, and G. N. Gregoriou, 2009a. An input-oriented super efficiency measure in stochastic DEA: Evaluating chief executive officers of US public banks and thrifts. Expert Systems with Applications: doi:10.1016/j.eswa.2009.06.091.

[7] Khodabakhshi M., Y. Gholami, and H. Kheirollahi 2009b. An additive model approach for estimating returns to scale in imprecise DEA. Applied Mathematical Modeling, in press. earch, Vol. 143, No. 1, pp. 32-41.

[8] Khodabakhshi M., H. Kheirollahi. An additive model approach for estimating returns to scale in stochastic Data Envelopment Analysis. 4th International Conference of Iranian Operations Research Society May 18-19, 2011, Guilan, Iran.

[9] Khodabakhshi M., H. Kheirollahi. An additive model approach for estimating returns to scale in fuzzy Data Envelopment Analysis (FDEA). 4th International Conference of Iranian Operations Research Society May 18-19, 2011, Guilan, Iran