



Supplier Selection Using a DEA-TOPSIS Method

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Abstract

Supplier selection is one of the critical activities for firms to gain competitive advantage and achieve the objectives of the whole supply chain. In this paper based on a DEA-TOPSIS method for MADM problems a flexible strategy for supplier selection is introduced.

Keywords: Multiple criteria decision analysis, multiple criteria ranking, DEA, TOPSIS, supplier selection.

1 Introduction

Supply chain management (SCM) is one of the most important competitive strategies used by modern enterprises. Meanwhile, supplier selection plays an effective role in supply chain, [7]. Supplier selection problem is considered as a multiple attributes decision making (MADM) problem affected by several conflicting factors such as price, quality and delivery. Supplier selection requires the information about potential suppliers' credit history, performance history and other personal information, which are often not available to the public, so that, strengthening partnerships with suppliers is most important for enhancing competitiveness, [12]. In the other word, supplier selection is evaluated as a critical factor for the companies desiring to be successful in nowadays competition conditions and order allocation are the most significant issues in the purchasing division of enterprises, [9], [13]. TOPSIS method that was developed by Hwang and Yoon (1981) is a famous useful method for MADM problems. This method is based on the concept that the chosen alternative should have the shortest Euclidean distance from the ideal solution, and the farthest from the negative ideal solution. The ideal solution is a hypothetical solution for which all attribute values correspond to the maximum attribute values in the database comprising the satisfying solutions; the negative ideal solution is the hypothetical solution for which all attribute values correspond to the minimum attribute values in the database. TOPSIS thus gives a solution that is not only closest to the hypothetically best, that is also the farthest from the hypothetically worst, [11]. Data envelopment analysis (DEA) is an increasingly popular managerial decision tool that was initially proposed by Charnes et al. in 1978. As a nonparametric method for estimating production frontiers, DEA measures relative performance of a set of producers or decision making units where the presence of multiple inputs and outputs makes comparisons difficult. During the last thirty years, significant research has been conducted on DEA for both theoretical extensions and practical applications, including various DEA-based MCDA approaches. A comprehensive survey of DEA among the early attempts of combining DEA with MCDA, [3], explore the utilization of cross-

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efficiency analysis in DEA for evaluating alternatives in MCDA, [4], and suggests that cross efficiency-based DEA analysis could be a “Multi-attribute Choice (tool) for the Lazy Decision Maker: Let the Alternatives Decide!”. Stewart (1996) summarizes DEA and MCDA as “DEA arises from situations where the goal is to determine the productive efficiency of a system by comparing how well the system converts inputs into outputs, while MCDA models have arisen from the need to analyze a set of alternatives according to conflicting criteria.” A methodological connection between MCDA and DEA is that if “all criteria in an MCDA problem can be classified as either benefit criteria (benefits or output) or cost criteria (costs or inputs), then DEA is equivalent to MCDA using additive linear value functions” , [10]. In this paper using DEA-TOPSIS method, [1] a strategy for supplier selection is proposed. Since the TOPSIS method did not provide any relevant process to handle the uncertainty in ordinal criteria, this method adapts the method proposed by Cook & Kress, [1], [2] to address this problem. The DEA-TOPSIS method [1] provides a theoretically sound approach to quantifying qualitative criteria based on the aforesaid philosophy of individual performance optimization and so this method will be more reliable for supplier selection in real world. The remainder of the paper is organized as follows: Section 2 is allocated to the description of the concept of supplier selection, in section 3 the procedure of TOPSIS is introduced, section 4 is allocated to the DEA-TOPSIS [1] and finally as an application of DEA-TOPSIS method [1] a strategy for supplier selection is introduced.

2 Supplier Selection

One of the most important processes performed in enterprises today is the evaluation, selection and continuous measurement of suppliers. Also enterprise's ability to produce a quality product at a reasonable cost and in a timely manner is heavily influenced by its suppliers' capabilities.

On the other hand, Supplier selection is one of the key issues of Supply Chain Management because the cost of raw materials and component parts constitutes the main cost of a product Management.

- Supplier Selection: “The stage in the buying process when the intending buyer chooses the preferred supplier or suppliers from those qualified as suitable.” (West burn Dictionary)

3 The Procedure of TOPSIS

The TOPSIS method is a distance-based approach, and its general procedure consists of the following steps [8]:

Step 1: Construct a performance matrix: An $n \times q$ matrix contains the raw consequence data for all alternatives against all criteria as following:

$$Y = \begin{matrix} & A_1 & A_2 & \dots & A_n \\ \begin{matrix} C_1 \\ C_2 \\ \vdots \\ C_q \end{matrix} & \begin{bmatrix} y_{11} & y_{12} & \dots & y_{1n} \\ y_{21} & y_{22} & \dots & y_{2n} \\ \vdots & \vdots & & \vdots \\ y_{q1} & y_{q2} & \dots & y_{qn} \end{bmatrix} \end{matrix}$$

Step 2: Normalize performance matrix as following:

$$A_1 \quad A_2 \quad \dots \quad A_n$$

$$Y = \begin{matrix} C_1 \\ C_2 \\ \vdots \\ C_q \end{matrix} \begin{bmatrix} v_{11} & v_{12} & \dots & v_{1n} \\ v_{21} & v_{22} & \dots & v_{2n} \\ \vdots & \vdots & & \vdots \\ v_{q1} & v_{q2} & \dots & v_{qn} \end{bmatrix} ; \quad v_{ij} = \frac{y_{ij}}{\sqrt{\sum_{i=1}^q y_{ij}^2}}$$

Step 3: Define the ideal and anti-ideal point: Set the ideal point, A^+ and anti-ideal point, A^- , based on the normalized performance matrix. For a benefit criterion, $C_i, v_i(A^+) = \max_{j=1}^n v_{ij}$ and $v_i(A^-) = \min_{j=1}^n v_{ij}$ but for a cost criterion, $C_k, v_i(A^+) = \min_{j=1}^n v_{ij}$ and $v_i(A^-) = \max_{j=1}^n v_{ij}$ respectively.

Step 4: Assign weights to criteria: Set $w_i, \left(w_i \in R^+ \text{ and } \sum_{i=1}^q w_i = 1 \right)$ to represent the relative importance of criterion C_i . Note that R is the set of all real numbers.

Step 5: Calculate the distances of A^j to the two ideal points, A^+ and A^- : A commonly used distance definition is the Euclidean distance. Compute the distances of A_j to A^+ and A^- using Euclidean distance function,

$$D(A^j)^+ = \sqrt{\sum_{i=1}^q w_i (v_i(A^+) - v_i(A^j))^2}, \quad D(A^j)^- = \sqrt{\sum_{i=1}^q w_i (v_i(A^-) - v_i(A^j))^2}$$

Note that $v_i(A^j), ((1 \leq i \leq q), (1 \leq j \leq n))$ represents the i^{th} element of j^{th} alternative vector.

Step 6: Obtain an integrated distance A^j to these two extreme points: The distances of A^j to the ideal and anti-ideal points have to be integrated to reach a final result. One way to integrate these two distances into an overall distance of $A_j, D(A^j)$, can be expressed as:

$$D(A^j) = \frac{D(A^j)^-}{D(A^j)^- + D(A^j)^+}$$

where a larger value of $D(A^j)$ represents a better overall performance.

4 The DEA-TOPSIS method

In this section the method introduced in [1] is proposed.

4.1 Flexible Settings of A^+ and A^-

Setting of ideal and anti-ideal points in the original TOPSIS is based upon value data that are normalized consequences reflecting the Decision-Makers' (DM's) preference directions over different criteria. A^+ and A^- are set as the combinations of either maximum or minimum values of $v_i(A^j)$, $(\forall C_i \in C, (C \text{ is the set of all criterion}) \text{ and } \forall A^j \in A (A \text{ is the set of all alternatives}))$, depending on whether a criterion is benefit or cost. In practice, a DM may often have ideal or anti-ideal alternatives (points) directly on consequences, rather than on normalized values. For example, in business analysis, various benchmarks have been identified for company performance evaluations. To improve the

flexibility in setting A^+ and A^- , the approach reported in this method allows a DM to define A^+ and A^- in the consequence space directly with the following conditions:

- $\forall A^j \in A, D(A^j)^+ \leq D(A^-)^+$: The normalized distance from A^+ and A^- should be larger than that between any alternative A_j in A and A^+ .
- $\forall A^j \in A, D(A^j)^- \leq D(A^+)^-$: The normalized distance from A^+ and A^- should be larger than that between any alternative A_j in A and A^- .

To describe the distance definitions of different types of criteria more easily, let $C = C^c \cup C^o$, where C , C^c and C^o represent the whole criteria set, quantitative (cardinal) criteria set and qualitative (ordinal) criteria set, respectively. Furthermore, let $C^c = \{C_1^c, C_2^c, \dots, C_{q_c}^c\}$ and $C^o = \{C_1^o, C_2^o, \dots, C_{q_o}^o\}$.

4.2 Definitions Over C^c

Let $m_i^c(A^j)$ be the consequence measurement of A_j on a quantitative criterion, C_i^c . When $A^j = A^+$ or A^- , $m_i^c(A^j) = m_i^c(A^+)$ or $m_i^c(A^-)$. For each $C_i^c \in C^c$, the distances from A_j to the predefined extreme points, A^+ and A^- , are denoted as $|m_i^c(A^+) - m_i^c(A^j)|$ and $|m_i^c(A^-) - m_i^c(A^j)|$, respectively. Then, an appropriate normalization function can be chosen to obtain the normalized distances of A^j to A^+ and A^- , denoted by $d_i^c(A^j)^+$ and $d_i^c(A^j)^-$, respectively. In this paper vector-based normalization is used as detailed below. Note that in order to validate the two conditions in Section 4.1, the distance between A^+ and A^- , $|m_i^c(A^+) - m_i^c(A^-)|$, is included in the following normalization process. As a unique property of the new distance definitions, the following normalization function can be used over all kind of criterion (benefit, cost, or non-monotonic). There isn't any require to explicitly differentiate these three types of criteria in this normalization.

- *Vector-based normalization* [1]:

$$\varepsilon_i^+ = \sqrt{\sum_{j=1}^n (m_i^c(A^+) - m_i^c(A^j))^2 + (m_i^c(A^+) - m_i^c(A^-))^2}$$

as the ideal normalization factor, and

$$\varepsilon_i^- = \sqrt{\sum_{j=1}^n (m_i^c(A^-) - m_i^c(A^j))^2 + (m_i^c(A^-) - m_i^c(A^+))^2}$$

as the anti-ideal normalization factor. Then, the normalized distance between $A^j \in A$ and A^+ over criterion C_i is defined as:

$$d_i^c(A^j)^+ = \frac{|m_i^c(A^+) - m_i^c(A^j)|}{\varepsilon_i^+}$$

and the normalized distance between $A_j \in A$ and A^- over criterion C_i is

$$d_i^c(A^j)^- = \frac{|m_i^c(A^-) - m_i^c(A^j)|}{\varepsilon_i^-}$$

By plugging A^- in $d_i^c(.)^+$ and A^+ in $d_i^c(.)^-$ we have:

$$d_i^c(A^-)^+ = \frac{|m_i^c(A^+) - m_i^c(A^-)|}{\varepsilon_i^+}, \quad d_i^c(A^+)^- = \frac{|m_i^c(A^-) - m_i^c(A^+)|}{\varepsilon_i^-}$$

4.3 Definitions Over C^o

Nowadays linguistic terms are commonly used for measuring consequences over qualitative criteria, C^o . Let $L = \{l_1, l_2, \dots, l_m\}$ as the linguistic terms set, where l_1 represents the best level, l_2 the next best, ..., and l_m the worst grade. Then, $m_i^o(A^j) = l_r$ means that A^j has the grade l_r over criterion. Since the linguistic grade set represents a preference order, obviously, $m_i^o(A^+) = l_1$ and $m_i^o(A^-) = l_m$, because the linguistic grade for A^+ on criterion C_i^o should be the best one, l_1 , and in the same way the grade of A^- should be the worst, l_m . Now suppose that $d_i^o(A^j)^+$ and $d_i^o(A^j)^-$ represent the distance between A^j and A^+ , and between A^j and A^- over the criterion C_i^o , respectively. Similar to qualitative criterion case distances should be normalized to between 0 and 1, in this paper we supposed that the distance between A^j and A^- over C_i^o is 1, $(d_i^o(A^-)^+ = d_i^o(A^+)^- = 1)$. Using piecewise linear interpolation, if $m_i^o(A^j) = l_r$, then we have the following conditions:

$$\frac{r-1}{m} \leq d_i^o(A^j)^+ \leq \frac{r}{m}, \text{ and } \frac{m-r+1}{m} \leq d_i^o(A^j)^- \leq \frac{m-r}{m}.$$

After obtaining the normalized distances from each alternative A^j to A^+ and A^- , an aggregated distance related to the so-called p -norm, where $p \geq 1$, will be used to obtain the integrated normalized distances, $d_i^+(A^j)$ and $d_i^-(A^j)$, over each criterion. The norms $p=1$ and $p=2$ are most used. If

$w^c = (w_1^c, w_2^c, \dots, w_{q_c}^c)$ and $w^o = (w_1^o, w_2^o, \dots, w_{q_o}^o)$ represent the weight information for C^c and

C^o respectively Then, the weighted p -power distance of A^j to A^+ over C^c and C^o will be

$$D(A^j)^+ = \left\{ \sum_{i=1}^{q_c} w_j^c \cdot (d_i^c(A^j)^+)^p + \sum_{i=1}^{q_o} w_j^o \cdot (d_i^o(A^j)^+)^p \right\}^{1/p} \quad (4-1)$$

and

$$D(A^j)^- = \left\{ \sum_{i=1}^{q_c} w_j^c \cdot (d_i^c(A^j)^-)^p + \sum_{i=1}^{q_o} w_j^o \cdot (d_i^o(A^j)^-)^p \right\}^{1/p} \quad (4-2)$$

Obviously, when $A^j = A^-$, $D(A^j)^+ = D(A^-)^+$ and $D(A^j)^- = D(A^+)^-$ for $A^j = A^+$.

4.4 Imprecise Intrinsic Preference Expressions

Furthermore, the DM could provide rough information about weights to ensure that the results reflect his or her intrinsic preferences, insofar as they are known. The imprecise preference expressions designed in [5], [6] can be used for this purpose. In this paper the weight preference expression given below is used: $0 \prec w_i \leq (\prec) w_j$

4.5 A DEA-based Model

Before adopting TOPSIS method the parameters, w^c , w^o , $d_j^o(A^j)^+$ and $d_j^o(A^j)^-$, $\forall C_i^o \in C^o$ and $A^j \in A$, should be obtained. In this paper the following optimization is used for this purpose:

$$\begin{aligned} \max \theta &= \frac{D(A^j)^-}{D(A^j)^- + D(A^j)^+} \\ \text{s.t.} \\ \forall A^j \in A, \quad D(A^j)^+ &\leq D(A^-)^+ \leq 1; \\ \forall A^j \in A, \quad D(A^j)^- &\leq D(A^+)^- \leq 1; \\ \forall A^j \in A, \text{ if } m_j^o(A^j) &= l_r, \text{ then } \frac{r-1}{m} \leq d_j^o(A^j)^+ \leq \frac{r}{m} \text{ and } \frac{m-r-1}{m} \leq d_j^o(A^j)^- \leq \frac{m-r}{m}; \\ \forall C_i^o \in C^o, \quad d_i^o(A^-)^+ &= 1; \\ \forall C_i^o \in C^o, \quad d_i^o(A^+)^- &= 1; \\ \sum_{i=1}^{q_c} w_i^c + \sum_{i=1}^{q_o} w_i^o &= 1; \\ \forall C_i^c \in C^c, \quad w_i^c &\geq \rho \text{ and } \forall C_i^o \in C^o, \quad w_i^o \geq \rho \end{aligned} \quad (4-3)$$

Note that the two conditions of setting A^+ and A^- in Section 4.1 have to be verified.

5 Supplier Selection Using DEA-TOPSIS method

In this section our proposed strategy for supplier selection is introduced.

Let n supplier and q attributes there exist:

Step1. Construct the decision matrix Y.

Step2. Construct the weights order as mentioned in 4.4 and obtain equivalent constraints.

Step3. Obtain Positive Ideal Supplier (PIS) and Negative Ideal Supplier (NIS).

Step4. Using (4-2) and (4-3) and Feeding constraints obtained in *Step2* in to the optimization model (4-5) obtain the relative closeness between each supplier and ideal suppliers.

Step5. Order the suppliers with respect to their relative closeness obtained in previous step, the larger the higher efficiency and subsequently lower ranking order.

Considering the fact that the preference of each supplier and inefficiency of suppliers obtain in step 4, it should be noted that these two steps propose the best supplier and evaluate the suppliers.

6 Numerical example

In this section an illustrative numerical example [1] is introduced. Let eight suppliers, S^j , ($1 \leq j \leq 8$), and seven attributes (all of them are cost attributes) there exist. The basic structure of problem is introduced in table 6.1 and the weights orders are as follows:

$$w_1^c \succ w_2^c \succ w_3^c, w_1^c \succ w_2^c \succ w_4^c, w_1^c \succ w_5^c, w_6^c \succ w_1^o$$

To strengthen the expression of “preferred” or “more important”, it is assumed that the weight gap between the above inequalities is greater than or equal to 0.1, hence the above preference relationships can be translated into the following constraints:

$$w_1^c \geq w_2^c + 0.1, \quad w_2^c \geq w_3^c + 0.1, \quad w_1^c \geq w_4^c + 0.1, \quad w_2^c \geq w_4^c + 0.1, \quad w_1^c \geq w_5^c + 0.1, \\ w_6^c \geq w_1^o + 0.1$$

Table 6.1

Basic structure of the problem

Alternatives	Criteria						
	C_1^c	C_2^c	C_3^c	C_4^c	C_5^c	C_6^c	C_1^o
S^1	82	1,027	372,650	6	389.80	137.53	l_4
S^2	489	1,097	85,249	45,165	967.91	0.15	l_4
S^3	3,530	13,124	10,009,750	28,529	7,234.94	8.44	l_1
S^4	2,496	5,105	2,722,850	28,740	3096.30	2.26	l_2
S^5	386	2,139	668,024	681	333.00	1.56	l_3
S^6	1,108	1,407	123,229	13,244	289.20	7.08	l_3
S^7	1,969	7,743	6,864,977	21,271	1128.05	22.00	l_2
S^8	1,872	620	896,571	8,460	554.49	0.79	l_3
S^+	82	620	85,249	6	289.20	137.53	l_1
S^-	3,530	13,124	10,009,750	45,165	7,234.94	0.15	l_4

Table 6.2Normalized distance information to S^+

Alternatives	Criteria						
	C_1^c	C_2^c	C_3^c	C_4^c	C_5^c	C_6^c	C_1^o
S^1	0.46486	0.3915	0.3868	0.4623	0.3772	0.0000	[0.75,1]
S^2	0.4099	0.3892	0.3983	0.0000	0.3454	0.3668	[0.75,1]
S^3	0.0000	0.0000	0.0000	0.1703	0.0000	0.3447	[0,0.25]
S^4	0.1394	0.2595	0.2924	0.1682	0.2281	0.3612	[0.25,0.5]
S^5	0.4238	0.3555	0.3749	0.4554	0.3803	0.3631	[0.5,0.75]
S^6	0.3265	0.3792	0.3968	0.3268	0.3828	0.3483	[0.5,0.75]
S^7	0.2104	0.1741	0.1262	0.2446	0.3365	0.3085	[0.25,0.5]
S^8	0.2235	0.4046	0.3657	0.3758	0.3681	0.3651	[0.5,0.75]
S^-	0.4648	0.4046	0.3983	0.4623	0.3828	0.3668	1
Note that " $z \in [a, b]$ " represent " $a \leq z \leq b$ ".							

Table 6.3Normalized distance information to S^-

Alternatives	Criteria						
	C_1^c	C_2^c	C_3^c	C_4^c	C_5^c	C_6^c	C_1^o
S^1	0.0000	0.0207	0.0181	0.0000	0.0098	0.7015	[0,0.25]
S^2	0.0663	0.0243	0.0000	0.5637	0.0660	0.0000	[0,0.25]
S^3	0.5617	0.6357	0.6264	0.3561	0.6759	0.0423	[0.75,1]
S^4	0.3933	0.2280	0.1665	0.3587	0.2731	0.0108	[0.5,0.75]
S^5	0.0495	0.0772	0.0368	0.0084	0.0043	0.0072	[0.25,0.5]
S^6	0.1671	0.0400	0.0024	0.1653	0.0000	0.0354	[0.25,0.5]
S^7	0.3074	0.3621	0.4279	0.2655	0.0816	0.1116	[0.5,0.75]
S^8	0.2916	0.0000	0.0512	0.1055	0.0258	0.0033	[0.25,0.5]
S^+	0.5617	0.6357	0.6264	0.5637	0.6759	0.7015	1
Note that " $z \in [a, b]$ " represent " $a \leq z \leq b$ ".							

Table 6.4

Final distance performance and rankings

Alternatives	Criteria	
	$D(A^j)$	Ranking 4
S^1	0.9805	2
S^2	0.4371	6
S^3	0.9944	1
S^4	0.7307	3
S^5	0.3632	8
S^6	0.3827	7
S^7	0.6773	4
S^8	0.5574	5

7 Conclusions

Considering the fact that one of the most important processes performed in enterprises today is the evaluation, selection and continuous measurement of suppliers, in this paper a hybrid DEA-TOPSIS method was introduced for supplier selection. Since proposed supplier selection method uses DEA in selection process and considers the fact that in practice, a DM may often have ideal or anti-ideal alternatives (points) directly on consequences, rather than on normalized values, it is a flexible manner which evaluates and selects suppliers.

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