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# Basic ratio-based DEA models

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**Abstract**

In this paper, different input-oriented ratio-based DEA (DEA-R-I) models are proposed. By presenting the envelopment-additive-enhanced Russell model based on the DEA-R-I form, each decision making unit is evaluated and its efficiency score is calculated. Also, by presenting the central resource allocation model based on the DEA-R-I form, a suitable benchmark for all DMUs is proposed by solving a linear programming problem. Finally, the efficiency scores obtained for the data of 20 commercial bank branches by each of the proposed models are compared.

*Keywords* : DEA, DEA-R, Efficiency.

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## 1 Introduction

Data Envelopment Analysis (DEA), by Charnes et al. [2], is a method for evaluating the relative efficiency of comparable entities referred to as Decision Making Units (DMUs). DEA forms a production possibility frontier, or an efficient surface. If a DMU lies on the surface, i.e., there is no other DMU that can either produce the same outputs by consuming less inputs (input-oriented DEA) or produce more outputs by consuming the same amount of inputs (output-oriented DEA), it is referred to as an efficient unit, otherwise inefficient. DEA also provides efficiency scores and reference units for inefficient DMUs.The efficiency score tells the percentage by which a DMU should decrease its inputs (input-oriented DEA) or increase its outputs (output oriented DEA) in order to become efficient. Tracey and Chen [11] first proposed that weight hypothesis may lead to underestimation from additional weight restrictions. However, CCR, based on, implies inherent weight restrictions, and has never been extensively discussed. Such restrictions may lead to a failure to represent the relations of certain weights, thus, output-oriented DEA-R was developed to address such problems ([7]). Since unnecessary and unreasonable weight hypothesis would cause CCR to underestimate the efficiency of a DMU, an input-oriented DEA-R model was developed.

The DEA-R-I model was presented by Despic et al. [7] with weights restrictions. Later, Wei et al. worked on the subject in [3, 4]. In the present paper, basic DEA models such as the envelopment, additive, and enhanced Russell models will be proposed based on the DEA-R-I form. In section 2, a review of DEA is provided. In section 3, basic DEA-R-I models are proposed. Section 4 contains an application of the proposed model by a numerical example for 20 bank branches. Final Section 5 contains conclusions and starting points for future research.

## 2 Data Envelopment Analysis

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Consider $n$ $DMUs$ with $m$ inputs and $s$ outputs. The input and output vectors of $DMU_j$ $(j=1,\ldots,n)$ are $X_j=(x_{1j},\ldots,x_{mj})^t, Y_j=(y_{1j},\ldots,y_{sj})^t$ where $X_j \geq 0, X_j \neq 0, Y_j \geq 0, Y_j \neq 0.$

By using the variable returns to scale, convexity, and possibility postulates, the non-empty production possibility set (PPS) is defined as follows:

$$T_c = \left\{(X,Y) : X \geq \sum_{j=1}^{n} \lambda_j X_j, Y \leq \sum_{j=1}^{n} \lambda_j Y_j, \lambda_j \geq 0, j=1\ldots,n\right\}$$

By the above definition, the CCR model proposed by Charnes et al. [2] is as follows:

$$\begin{aligned} Min \quad & \theta - \varepsilon\left[\sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+\right] \\ S.t \quad & \sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- = \theta x_{ip}, \qquad i=1,\ldots,m \\ & \sum_{j=1}^{n} \lambda_j y_{rj} - s_r^+ = y_{rp}, \qquad r=1,\ldots,s \\ & \lambda_j \geq 0, \qquad j=1,\ldots,n \\ & s_i^- \geq 0, \qquad i=1,\ldots,m \\ & s_r^+ \geq 0, \qquad r=1,\ldots,s. \end{aligned} \tag{1}$$

Clearly, the evaluated $DMU_p$ is efficient if and only if $\theta^*=1$ and all slack variables in the optimal solution are zero in problem (1).

### 2.1 The DEA additive model

The additive model (*ADD*) maximizes the $L_1$ distance of the DMU under analysis to the projected point on the efficient frontier.

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$$
\begin{aligned}
Max \quad & \sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+ \\
s.t. \quad & \sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- = x_{ip}, \qquad i = 1,\dots,m \\
& \sum_{j=1}^{n} \lambda_j y_{rj} - s_r^+ = y_{rp}, \qquad r = 1,\dots,s \\
& \lambda_j \ge 0, \qquad j = 1,\dots,n \\
& s_i^- \ge 0, s_r^+ \ge 0, \qquad i = 1,\dots,m, \qquad r = 1,\dots,s.
\end{aligned}
\tag{2}
$$

Clearly, the evaluated $DMU_p$ is efficient if and only if $\theta^* = 1$ and all slack variables in the optimal solution are zero in problem (2).

### *2.2 Enhanced Russell model*

Koopmans [8] defined an input-output vector as technically efficient if and only if increasing any output or decreasing any input is possible only by decreasing some other output or increasing some other input, respectively. A nonradial Pareto-Koopmans measure of technical efficiency for $DMU_p$ can be computed as follows:

$$
\begin{aligned}
Min \quad & \frac{\frac{1}{m}\sum_{i=1}^{m}\theta_i}{\frac{1}{s}\sum_{r=1}^{s}\varphi_r} \\
s.t. \quad & \sum_{j=1}^{n} \lambda_j x_{ij} \le \theta_i x_{ip}, \qquad i = 1,\dots,m \\
& \sum_{j=1}^{n} \lambda_j y_{rj} \ge \varphi_r y_{rp}, \qquad r = 1,\dots,s \\
& \lambda_j \ge 0, \qquad j = 1,\dots,n \\
& \varphi_r \ge 1 \qquad r = 1,\dots,s \\
& \theta_i \le 1 \qquad i = 1,\dots,m
\end{aligned}
\tag{3}
$$

In model (3) we suppose that all inputs and outputs are of the same importance in determining the efficiency. That is to say, the priority in increasing or decreasing the outputs or the inputs, respectively, is the same for all inputs and outputs.

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## 3 Input-oriented DEA-R model

Despic et al. [7] proposed a new mathematical model for efficiency score, which combines the DEA methodology with the idea of ratio analysis.

$$\begin{aligned}
&Max \quad \Delta \\
&S.t. \quad \sum_{r=1}^{s}\sum_{i=1}^{m} w_{ir}\left(\frac{x_{ij}/y_{rj}}{x_{ip}/y_{rp}}\right) \geq \Delta, \qquad j=1,\ldots,n \qquad (4)\\
&\qquad \sum_{r=1}^{s}\sum_{i=1}^{m} w_{ir} = 1 \\
&\qquad w_{ir} \geq 0, \quad \Delta \geq 0 \quad i=1,\ldots,m \quad r=1,\ldots,s
\end{aligned}$$

Model (4) is a DEA-R-I model that calculates the DEA-R-I efficiency score of $DMU_p$.

$$\begin{aligned}
&Min \quad \theta_R \\
&st. \quad \sum_{j=1}^{n} \lambda_j \left(\frac{x_{ij}/y_{rj}}{x_{ip}/y_{rp}}\right) \leq \theta_R, \qquad i=1,\ldots,m \quad r=1,\ldots,s \quad (5)\\
&\qquad \sum_{j=1}^{n} \lambda_j \geq 1, \\
&\qquad \lambda_j \geq 0, \qquad j=1,\ldots,n.
\end{aligned}$$

---

**Definition 1.** $DMU_p$ is R-CCR-I–efficient (input-oriented CCR –R-efficient) if and only if $\theta_R^* = 1$.

Model (5) has the following properties.

1) The efficiency and super-efficiency scores obtained by this model are greater than or equal to those of the CCR model.
2) The efficiency scores of the model in the input and output orientations are not necessarily equal.
3) In a situation involving no weight restrictions, the input-target improvement strategy given by DEA-R-I is always better than the CCR-I model.
4) When DEA-R-I weights are concentrated on one output, the CCR-I efficiency and DEA-R-I efficiency are the same.
5) When DEA-R-I weights are not concentrated on multiple outputs, the CCR-I efficiency and DEA-R-I efficiency are the same, with the exception of, every times of corresponding output of targeted DMU. (See [3, 4, 5]).
   The two-phase model based on the DEA-R-I model, considering slack variables, is proposed as follows.

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$$\begin{aligned} Min \quad & \theta - \varepsilon\left( \sum_{r=1}^{s}\sum_{i=1}^{m} S_{ir} \right) \\ s.t. \quad & \sum_{j=1}^{n} \lambda_j \left( \frac{x_{ij} / y_{rj}}{\frac{x_{ip}}{y_{rp}}} \right) + S_{ir} = \theta, \qquad i = 1,\dots,m \qquad r = 1,\dots,s \\ & \sum_{j=1}^{n} \lambda_j \geq 1, \\ & S_{ir} \geq 0 \qquad i = 1,\dots,m \qquad r = 1,\dots,s \\ & \lambda_j \geq 0, \qquad j = 1,\dots,n. \end{aligned} \tag{6}$$

The additive model based on the DEA-R-I model for distinguishing between efficient and inefficient units is suggested as follows.

$$\begin{aligned} Max \quad & \sum_{r=1}^{s}\sum_{i=1}^{m} S_{ir} \\ s.t. \quad & \sum_{j=1}^{n} \lambda_j \left( \frac{x_{ij} / y_{rj}}{\frac{x_{ip}}{y_{rp}}} \right) + S_{ir} = 1, \qquad i = 1,\dots,m \qquad r = 1,\dots,s \\ & \sum_{j=1}^{n} \lambda_j \geq 1, \\ & S_{ir} \geq 0 \qquad i = 1,\dots,m \qquad r = 1,\dots,s \\ & \lambda_j \geq 0, \qquad j = 1,\dots,n. \end{aligned} \tag{7}$$

The enhanced Russell model based on the DEA-R-I model is suggested as follows.

$$\begin{aligned} Min \quad & \frac{1}{ms} \sum_{r=1}^{s}\sum_{i=1}^{m} \theta_{ir} \\ s.t. \quad & \sum_{j=1}^{n} \lambda_j \left( \frac{x_{ij} / y_{rj}}{\frac{x_{ip}}{y_{rp}}} \right) \leq \theta_{ir}, \qquad i = 1,\dots,m \qquad r = 1,\dots,s \\ & \sum_{j=1}^{n} \lambda_j \geq 1, \\ & \theta_{ir} \leq 1 \qquad i = 1,\dots,m \qquad r = 1,\dots,s \\ & \lambda_j \geq 0, \qquad j = 1,\dots,n. \end{aligned} \tag{8}$$

The central resource allocation model based on the DEA-R-I model is suggested as follows.

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$$\begin{aligned}
&Min \quad \theta \\
&s.t. \quad \sum_{k=1}^{n} \sum_{j=1}^{n} \lambda_{kj} \left( \frac{x_{ij}/y_{rj}}{\dfrac{\sum_{j=1}^{n} x_{ij}}{\sum_{j=1}^{n} y_{rj}}} \right) \le \theta, \qquad i=1,\dots,m \qquad r=1,\dots,s \qquad (9) \\
&\qquad \sum_{j=1}^{n} \lambda_{kj} \ge 1, \qquad\qquad k=1,\dots,n \\
&\qquad \lambda_{kj} \ge 0, \qquad\qquad k=1,\dots,n,\ j=1,\dots,n.
\end{aligned}$$

## 4 An application

We consider the data of 20 bank branches with three inputs and eight outputs, in the following table:

Table 1.

Outputs and Inputs of bank branches.

| Inputs: | Outputs: |
|---|---|
| 1. Interest paid | 1. Interest-free saving account |
| 2. Level of education of the staff | 2. Current account |
| 3. Demand | 3. Short-term |
| | 4. long-term |
| | 5. Other |
| | 6. Loans |
| | 7. Interest received |
| | 8. Banking fees |

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Table 2.

Outputs of bank branches.

| DMUs | $O_1$ | $O_2$ | $O_3$ | $O_4$ | $O_5$ | $O_6$ | $O_7$ | $O_8$ |
|---|---|---|---|---|---|---|---|---|
| 1 | 709874 | 2610439 | 172126 | 342833 | 228993 | 2761590 | 59161.51 | 9589.48 |
| 2 | 73560 | 317617 | 2326 | 26234 | 85324 | 252120 | 3504.68 | 180.38 |
| 3 | 435248 | 489666 | 6506 | 79194 | 824241 | 2043899 | 5817.55 | 760.9 |
| 4 | 29165 | 971640 | 1559 | 46160 | 354722 | 452931 | 622.48 | 244.75 |
| 5 | 216148 | 158130 | 845 | 90278 | 12256 | 552673 | 49570.82 | 20.7 |
| 6 | 514881 | 508735 | 103257 | 69604 | 396716 | 1751591 | 34115.58 | 1234.52 |
| 7 | 15676 | 172947 | 822 | 7373 | 287183 | 537567 | 1119.32 | 143.02 |
| 8 | 155799 | 269617 | 5451 | 83400 | 71509 | 1534505 | 10163.75 | 1868.82 |
| 9 | 13707 | 163397 | 5759 | 86775 | 135464 | 4829312 | 13520.68 | 1024.48 |
| 10 | 666679 | 242971 | 2319 | 5148 | 16992 | 2949072 | 21773.84 | 22.1 |
| 11 | 106682 | 408506 | 2439 | 173299 | 20643 | 1808353 | 2434.86 | 1193.46 |
| 12 | 190455 | 174657 | 9402 | 110027 | 69802 | 510656 | 2594.17 | 531.59 |
| 13 | 337298 | 293580 | 12340 | 148121 | 24997 | 768622 | 1946.46 | 94.83 |
| 14 | 133062 | 131262 | 15723 | 68920 | 150625 | 422274 | 422.05 | 461.59 |
| 15 | 152552 | 189936 | 11778 | 41074 | 420668 | 729915 | 2269.94 | 376.75 |
| 16 | 245412 | 106089 | 3651 | 12176 | 118734 | 448984 | 1200.53 | 102.69 |
| 17 | 392104 | 646367 | 6241 | 30690 | 375109 | 2144785 | 10428.37 | 295.05 |
| 18 | 27880 | 196292 | 2034 | 6021 | 6122 | 28007 | 365.28 | 144.75 |
| 19 | 204028 | 502730 | 3511 | 38927 | 1100502 | 1977217 | 8529.12 | 731.69 |
| 20 | 52156 | 71458 | 12850 | 14869 | 15344 | 277254 | 2761.24 | 162.163 |

Table 3

Inputs of bank branches.

| DMUs | $I_1$ | $I_2$ | $I_3$ |
|---|---|---|---|
| 1 | 12963.87 | 37.75 | 134529 |
| 2 | 1483.69 | 22.74 | 10244 |
| 3 | 11756.02 | 25.5 | 42668 |
| 4 | 866.86 | 20.94 | 44128 |
| 5 | 4545.92 | 14.43 | 13043 |
| 6 | 9139.03 | 18.86 | 87981 |
| 7 | 308.67 | 25.66 | 97763 |
| 8 | 3185.7 | 26.27 | 7629 |
| 9 | 832.02 | 21.91 | 430513 |
| 10 | 11589.09 | 16.75 | 7859 |
| 11 | 2886.74 | 21.2 | 1192 |
| 12 | 3880.46 | 23.67 | 53209 |
| 13 | 6269.71 | 21.85 | 27506 |
| 14 | 2616.9 | 21.56 | 17988 |
| 15 | 2600.23 | 37.5 | 50229 |
| 16 | 4257 | 24.5 | 32618 |
| 17 | 6179.39 | 25.72 | 41817 |
| 18 | 528.92 | 16.28 | 22262 |
| 19 | 3679.91 | 28.76 | 16795 |
| 20 | 916.42 | 30.4 | 30326 |

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Table 4

Efficiency score for commercial bank branches

| DMUs | Model (1) | Model (2) | Model (5) | Model (7) | Model (8) |
|---|---|---|---|---|---|
| 1 | 1.0000 | 0.0000 | 1.0000 | 0.0000 | 1.0000 |
| 2 | 1.0000 | 0.0000 | 1.0000 | 0.0000 | 1.0000 |
| 3 | 1.0000 | 0.0000 | 1.0000 | 0.0000 | 1.0000 |
| 4 | 1.0000 | 0.0000 | 1.0000 | 0.0000 | 1.0000 |
| 5 | 1.0000 | 0.0000 | 1.0000 | 0.0000 | 1.0000 |
| 6 | 1.0000 | 0.0000 | 1.0000 | 0.0000 | 1.0000 |
| 7 | 1.0000 | 0.0000 | 1.0000 | 0.0000 | 1.0000 |
| 8 | 1.0000 | 0.0000 | 1.0000 | 0.0000 | 1.0000 |
| 9 | 1.0000 | 0.0000 | 1.0000 | 0.0000 | 1.0000 |
| 10 | 1.0000 | 0.0000 | 1.0000 | 0.0000 | 1.0000 |
| 11 | 1.0000 | 0.0000 | 1.0000 | 0.0000 | 1.0000 |
| 12 | 0.931985 | 2.31811E+6 | 0.9886 | 10.7747 | 0.5511 |
| 13 | 1.0000 | 0.0000 | 1.0000 | 0.0000 | 1.0000 |
| 14 | 0.963303 | 0.0000 | 1.0000 | 0.0000 | 1.0000 |
| 15 | 1.0000 | 0.0000 | 1.0000 | 0.0000 | 1.0000 |
| 16 | 0.908524 | 2.27890E+6 | 0.9085 | 14.8423 | 0.3816 |
| 17 | 1.0000 | 0.0000 | 0.0000 | 0.0000 | 1.0000 |
| 18 | 0.973695 | 0.0000 | 1.0000 | 0.0000 | 1.0000 |
| 19 | 1.0000 | 0.0000 | 1.0000 | 0.0000 | 1.0000 |
| 20 | 1.0000 | 0.0000 | 1.0000 | 0.0000 | 1.0000 |

## 5 Conclusion

Regarding the fact that basic DEA models, such as the input-oriented envelopment form, the additive model, etc., play a fundamental role in many DEA contexts, using these models in the DEA-R-I form can be very important for comparing the efficiency –projection of DMUs and for their ranking. Further concentration on DEA-R-I models maintains the value of basic DEA models. It can also help to develop and extend different contexts in performance evaluation of DMUs. Ranking of DMUs and finding a suitable benchmark by using the proposed models can be considered for further studies.

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