



# Presenting a Hybrid Approach based on Two-stage Data Envelopment Analysis to Evaluating Organization Productivity

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Received 21 October 2013, Revised 15 February 2014, Accepted 10 March 2014

## Abstract

Measuring the performance of a production system has been an important task in management for purposes of control, planning, etc. Lord Kelvin said :“When you can measure what you are speaking about, and express it in numbers, you know something about it; but when you cannot measure it, when you cannot express it in numbers, your knowledge is of a meager and unsatisfactory kind.” Hence, management knowledge is measurable science and if we can’t measure each subject we can’t control it so management is impossible. We know, the major criteria performance is productivity and we should be able to measure it.

Data Envelopment Analysis (DEA), as an evaluation method, can estimate the relative efficiency of organizations systematically. The efficiency of an organization can be benchmarked by using DEA. DEA presents a model for evaluating the performance of a set of comparable decision making units (DMUs). In this paper we developed a new model for calculating productivity with Two-stage data envelopment analysis.

**Keywords:** Data Envelopment Analysis (DEA), productivity, measuring performance, Two-Stage DEA, effectiveness.

## 1. Introduction

Words of productivity and performance usually used in commercial and scientifically regions, although means of them really not defined. In really these words almost are baffling and usually used instead of efficiency and profitability.

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Nowadays organizations do not operate in insolation. They are part of a business environment that includes economic, legal, and other factors. They are usually part of an industry and are influenced by what is going on in that environment. Organizations are trying to improve their performance with time. For some, it is a challenge; for others it is a requirement for survival. Yet, for some it is the key to improved life, profitability, or reputation [5].

Measuring the performance of a production system has been an important task in management for purpose of control, planning, etc. Lord Kelvin said :“When you can measure what you are speaking about, and express it in numbers, you know something about it; but when you cannot measure it, when you cannot express it in numbers, your knowledge is of a meager and unsatisfactory kind.”

Hence, management knowledge is measurable science and if we cannot control it and management is impossible. We know, the major criterion performance is productivity and we should be able to measure it. Data Envelopment Analysis (DEA), as an evaluation method, can estimate the relative efficiency of organizations systematically. The efficiency of an organization can be benchmarked by using DEA. DEA present a model for evaluating the performance of a set of comparable Decision Making Units (DMUs).

In this paper we developed a new model for calculating productivity with Data Envelopment Analysis. Productivity introduced by Quesnay (1766), Littre (1889) used Productivity as power of production. But the inception of century 20 Productivity introduced as outputs of provided by a variety of inputs that used for production.

The paper is structured as follows: a summary of basic DEA model is given in section 2. The next section of the paper introduces efficiency, effectiveness and productivity in organizations. The other section addresses the modeling of productivity and using this in an organization. Finally, conclusions appear in the last section.

## 2. Data Envelopment Analysis (DEA)

DEA is a useful Data-oriented approach, used to evaluate relative performance of homogenous Decision-Making Units (DMUs) by using inputs and outputs of each DMU. The performance of each DMU, is referred to by efficiency score. In fact, based on input-output data, and some preliminary assumptions is built a Production Possibility Set (PPS) and efficient frontier. If a DMU lies on efficient frontier, it is efficient, otherwise it is inefficient. Also DEA provides efficiency scores and reference units for inefficient DMUs, to enable them for being efficient. Suppose we have  $n$  DMUs ( $DMU_j$ ;  $j=1,...,n$ ) use the same number,  $m$ , of inputs ( $x_{ij}$ ;  $i=1,...,m$ ) to produce the same number,  $s$ , of outputs ( $y_{rj}$ ;  $r=1,...,s$ ). see. Fig. 1.

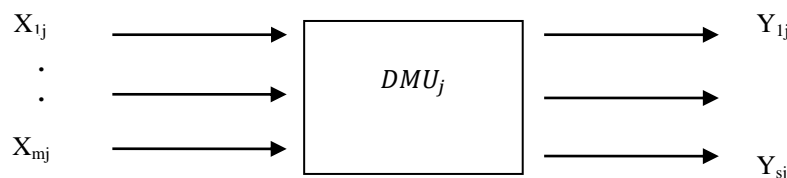


Fig. 1. Illustration of the  $DMU_j$  and its inputs, outputs

In DEA models, the maximum of relative ratio of weighted sum of outputs to that of inputs is regarded as the efficiency, by definition it follows that:

Efficiency (  $e_j$  ) =  $\frac{\sum_{r=1}^s u_r y_{rj}}{\sum_{i=1}^m v_i x_{ij}}$ ,  $j=1, \dots, n$  (1). Where  $u_r$  ( $r=1, \dots, s$ ) and  $v_i$  ( $i=1, \dots, m$ ) are vectors of multipliers for outputs and inputs, respectively. The problem arises, the multipliers  $u_r$  and  $v_i$  are unknown. Charnes, Cooper, Rohdes (1978), proposed CCR model for evaluation efficiency of  $DMU_p$  can be represented by model (2):

$$\begin{aligned} \max \quad & ep = \frac{\sum_{r=1}^s u_r y_{rp}}{\sum_{i=1}^m v_i x_{ip}} \\ \text{s.t.} \quad & \frac{\sum_{r=1}^s u_r y_{rj}}{\sum_{i=1}^m v_i x_{ij}} \leq 1 \quad ; j=1, \dots, n \\ & v_i \geq 0 \quad ; i = 1, \dots, m \\ & u_r \geq 0 \quad ; r = 1, \dots, s \end{aligned} \quad (2)$$

By imposing variable alterations, nonlinear CCR model, model (2) is finally transformed into the following linear programming, model (3):

$$\begin{aligned} \text{Max} \quad & e_p = \sum_{r=1}^s u_r y_{rp} \\ \text{s.t.} \quad & \sum_{r=1}^s u_r y_{rj} - \sum_{i=1}^m v_i x_{ij} \leq 0 \quad ; j=1, \dots, n \\ & \sum_{i=1}^m v_i x_{ij} = 1 \quad ; j=1, \dots, n \\ & v_i \geq 0 \quad ; i = 1, \dots, m \\ & u_r \geq 0 \quad ; r = 1, \dots, s \end{aligned} \quad (3)$$

DEA models no assumptions concerning the internal operations of a DMU rather, DEA treats each DMU as a “Black Box” by considering only the inputs consumed and outputs produced by each DMU, see Färe and Grosskopf [6]. Färe and Grosskopf developed Network DEA, to enable DEA models, deal with inside black box, see Fig. 2.

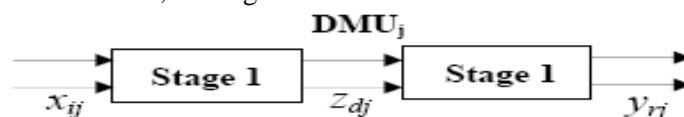


Fig. 2. Illustration inputs, outputs in two-stage DEA

In Fig. 2.  $DMU_j$  ( $j = 1, \dots, n$ ) consists of two stages, suppose first stage have,  $m$ , inputs ( $x_{ij}$ ;  $i=1, \dots, m$ ) and,  $d$ , outputs ( $z_{dj}$ ;  $d=1, \dots, D$ ) and second stage have,  $s$ , outputs ( $y_{rj}$ ;  $r=1, \dots, s$ ). The outputs of first stage is inputs of second stage, hence vectors of multipliers for outputs of first stage ( $\eta_d^1$ ), are equal vectors of multipliers for inputs of second layer ( $\eta_d^2$ ), (i.e.  $\eta_d^1 = \eta_d^2 = \eta_d$ ). With some alterations, model (3) transformed into the following linear programming, model (4).

$$\begin{aligned}
\text{Max } e_p &= \sum_{r=1}^s u_r y_{rp} \\
\text{s.t. } \sum_{i=1}^m \mu_i x_{ip} &= 1 \\
\sum_{r=1}^s u_r y_{rj} - \sum_{i=1}^m \mu_i x_{ij} &\leq 0 \\
\sum_{r=1}^s u_r y_{rj} - \sum_{i=1}^m \mu_i x_{ij} &\leq 0; j=1, \dots, n \\
\sum_{d=1}^D \eta_d z_{dj} - \sum_{i=1}^m v_i x_{ij} &\leq 0; j=1, \dots, n \\
\sum_{d=1}^D \eta_d z_{dj} - \sum_{i=1}^m v_i x_{ij} &\leq 0; j=1, \dots, n \\
u_r &\geq 0; r=1, \dots, s \\
v_i &\geq 0; i=1, \dots, m \\
\eta_d &\geq 0; d=1, \dots, D
\end{aligned} \tag{4}$$

We evaluate the efficiency of the entire DMU by solving model (4). Then by solving a DEA model for each stage obtain its efficiency score, of course by equation  $e_1 * e_2 = e_p$  ( $e_1$ , efficiency score of 1-stage,  $e_2$ , efficiency score of 2-stage,  $e_p$ , efficiency score of entire DMU can calculate efficiency score each stage).

### 3. Efficiency, Effectiveness and Productivity

#### 3.1 Productivity

Productivity first used by Quesnay in agricultural journal about two century ago. The Productivity has been successfully applied two a wide range of organizations, industries, especially in economical industries. Productivity is the most important advantage in competition. Organization can improve their competitive functions by increasing their productivity and effectiveness. There are lots of definitions for conception of productivity, see table (1).

Table 2. Definitions of productivity

The power of production.	Littre (1883)
The degree to which the actual outputs of the system corresponds to its desired or planned outputs.	Bernolak (1970)
Ratio of input to output.	Choe et, al (1988)
Ratio of total income to sum cost and profit.	Fisher (1990)
Ratio of outputs (preferable goods, services) to inputs (assets, material, labour, land, machinery)	Mills et, al (1993)
Productivity is defined as the product that produced by raw material, capital, technology.	Productivity center of japan (1991)

Ratio of actual outputs to expected resources.	Sink et, al (1996)
The capability of supplying requirements, preferences and demands by consumption at least resources.	Merzeng et, al (2001)

### 3.2. The relationship between productivity, effectiveness, and efficiency

The public in general would have different ideas regarding “efficiency, effectiveness and productivity” because these concepts have apparently different meanings and purposes [13]. In descriptive terms, efficiency is doing things right and effectiveness, is doing the right things. In simple ratios, efficiency measures how good performance is compared to some measure of what machinery, labour, etc, should do (efficiency is ratio actual outputs to standard outputs). Effectiveness, measures how well we turn resources into quality sets of our planned outputs (effectiveness is ratio standard outputs to resources required). Productivity is a combination of efficiency and effectiveness, productivity appears to be very much similar to efficiency but there is a fine difference, productivity is defined as output per unit of input, efficiency measures outputs solely in terms of the inputs employed, see Fig3 .

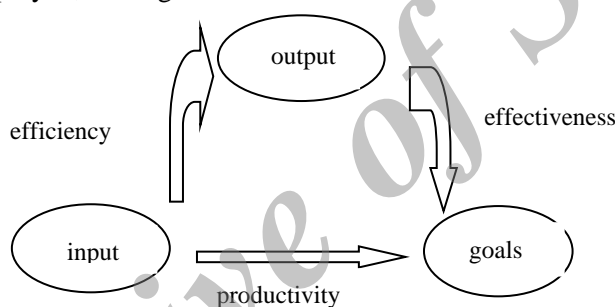


Fig. 3. Relationship between, efficiency, effectiveness, productivity

### 3.3. The Importance of Measuring Productivity

Continuing economic activities of firms without awareness of the status quo and the firm situation among the rivals, in spite of spending considerable costs to improve productivity, if not impossible but very difficult, and may lead to crises such as wastage, increased duplications as well as loss of sale market. One of the ways for releasing from this problem is to use productivity cycle. Productivity cycle is a process base on which measurement and productivity plans in each organization are made. A productivity cycle consists of the four following stages acting as a closed cycle.

1. Productivity measurement
2. Evaluation and analysis of indices
3. Productivity planning
4. Productivity improvement (doing improvement activities)

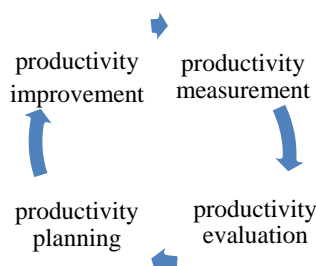


Fig. 4. Productivity Cycle

Undoubtedly, a powerful device for measuring productivity of the organization compared to rival organizations is Data Envelopment Analysis (DEA).

#### 4. The Factors Affecting Increased Productivity

Production is a social, complicated, and continuous process. Internal relationships between human resources, capital and organizational environment, and general coordination and incorporation of them together is very important in production process. Increased productivity depends on the fact that the manager can recognize successfully the factors of productive-social system. Among several factors affecting productivity, two groups have the main role.

1. Internal factors
2. External factors

Internal or intrinsic factors can be controlled by the individual or the manager of the firm, while external or extrinsic factors are out of the control of the individual or the firm. To improve the organization performance, external factors which affect management and efficiency of the firm, must be considered along with internal factors. Hence, the first management step taken towards increasing productivity is to identify the range of problems in terms of two groups of internal and external factors, and the second step is to identify the factors can be controlled. Fig. 5 shows the factors affecting productivity.

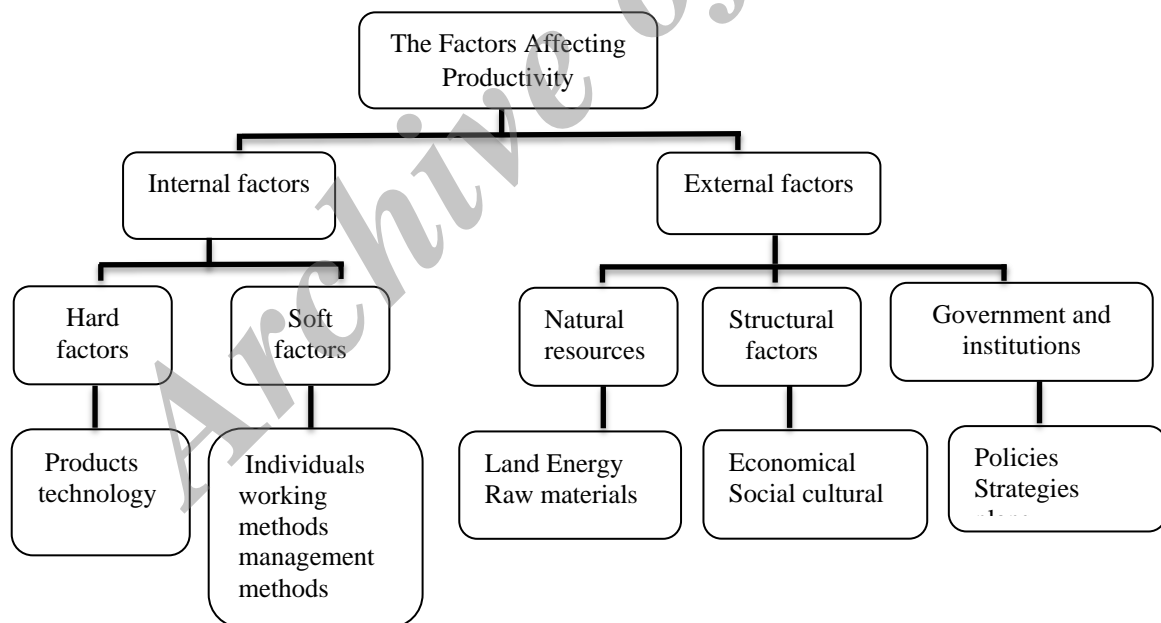


Fig. 5. The Factors Affecting Productivity

Now, each of the internal and external factors are discussed:

##### A. Internal factors:

1. Soft factors: these factors include individuals, organizational systems, working methods, and management methods, that are the main source of improvement and technology.
2. Hard factors: the factors which can not easily changed within a short time, and include the firm's products, production technology, machinery and equipment.

B. Internal factors: external factors are the factors including the government's policies, political, economic, and social conditions, electricity, water, and ..., that affect the productivity of the firms, but the firms are not able to control them. These factors are divided into three general groups:

1. Structural factors: changes in the society's structure improve the organization's productivity. These factors include economic, social, and political changes. For example, economic growth, financial strength and stability, and per capita income affect industrial competition and productivity improvement.
2. Natural factors: natural resources, the most important examples of which can be the land, energy, and raw materials, have undeniable effect on productivity improvement.
3. Factors relating to the government and infrastructure: the government's policies, strategies and plans have important effect on the organizations' productivity. Most of the structural changes affecting productivity are caused by enacting the rules and regulations and by the governmental institutions' performance.

## 5. Modeling for Evaluation

The factors affecting productivity were fully explained in the section 3. Internal factors are the factors can be controlled by the management and external factors are out of the control of the management. The internal factors themselves are divided into hardware and software factors that are interdependent for production. However, their primacy and regency can be considered as shown in Fig. 6, that is, to properly produce a first-rate product, which is achieved by hardware factors, reliable software factors must be generated, meaning that without motivated staff and strong management, a high quality production cannot be possible, even if we have advanced equipment at our disposal. So, the main condition for a top production is efficient human resources along with suitable equipment.

As mentioned earlier, external factors affect productivity level, and this effect can be in the form of effect on soft factors and hard factors. Economic and or cultural conditions can affect the motivation of the employees. The role of non-monetary motivations along with monetary motivations is undeniable. So, the environmental factors can change the productivity of soft factors. The effect of the environmental factors on hard factors can be discussed as well. Many structural changes affecting productivity are due to enacting rules and regulations and to the governmental institutions' performance. In addition, the government's policies can have important effect on preparing equipment and, etc, Supplying energy and its limitations in different seasons can be considered as one of the most important factors changing productivity in industries. Price of the energy is of the factors in connection with the cost of product. In Fig. 6, the relationship between the said factors is shown.

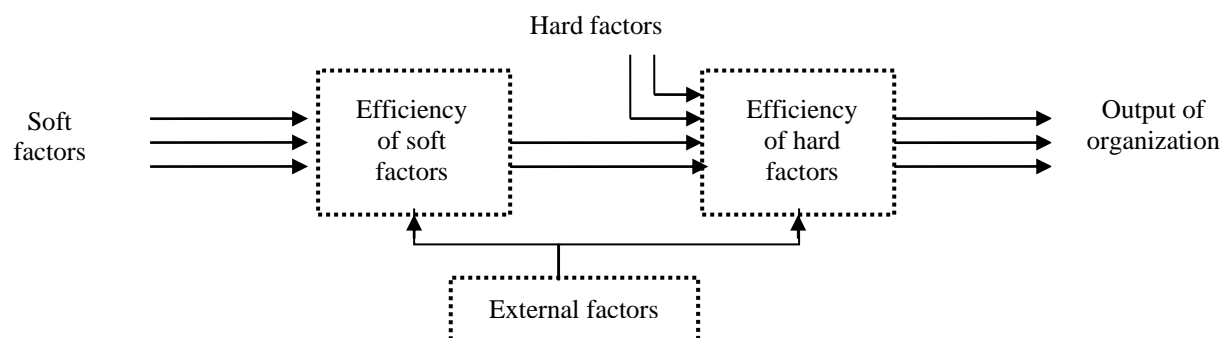


Fig. 6 Modeling Organizational

We accepted that external factors, hereafter referred to as "uncontrollable factors", have effect on productivity level of the areas of soft and hard factors. Now the question is "Have these uncontrollable factors simply negative effects?" In other words, "Do they move towards decreasing productivity?" There is no doubt that, in most of the cases, these factors as favorable factors will have positive effects on the organization. For example, stabilizing exchange rate is favorable to many organizations and unfavorable to many others. Hence, uncontrollable factors can be divided into two following groups:

- a. Uncontrollable factors with positive effects
- b. Uncontrollable factors with negative effects

The input of hard factors includes technology, equipment and ... known as controllable factors. However, we should accept that most of these factors may be fully uncontrollable under special conditions. For example, outdated machinery, incomplete raw materials and ... act to some extent as uncontrollable factors. These factors are called slightly controllable indices, if  $\alpha$  is the controllability rate of these factors.  $(1-\alpha)$  is defined as the rate of effects of these effects cannot be controlled by the manager. With making the said changes, Fig. (6) Will be shown as Fig. (7).

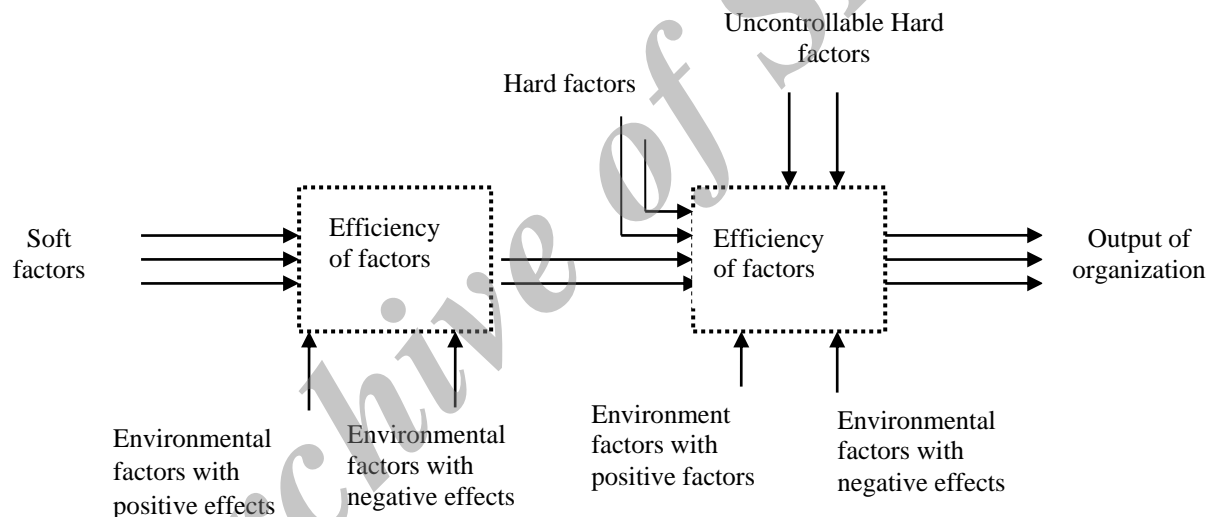


Fig. 7. Modeling Organizational Productivity

## 6. Mathematical Modeling with DEA

As mentioned earlier in the previous section, there are some input factors the controllability rate of which are determined by the managers of decision-making departments, that is, they have control over these indices to some extent. Assume that  $0 \leq \alpha_i \leq 1$  indicates the percentage of control of the  $i^{\text{th}}$  input.  $\alpha_i = 1$  means fully uncontrollable and  $\alpha_i = 0$  means fully uncontrollability of the related input. In addition, the external factors which were known as uncontrollable factors, are shown as  $ht_j$ . These uncontrollable indices can be existed both in the area of soft factors ( $t = 1, \dots, T$ ) and in the area of hard factors ( $t = T+1, \dots, T+K$ ), see Fig. 8.

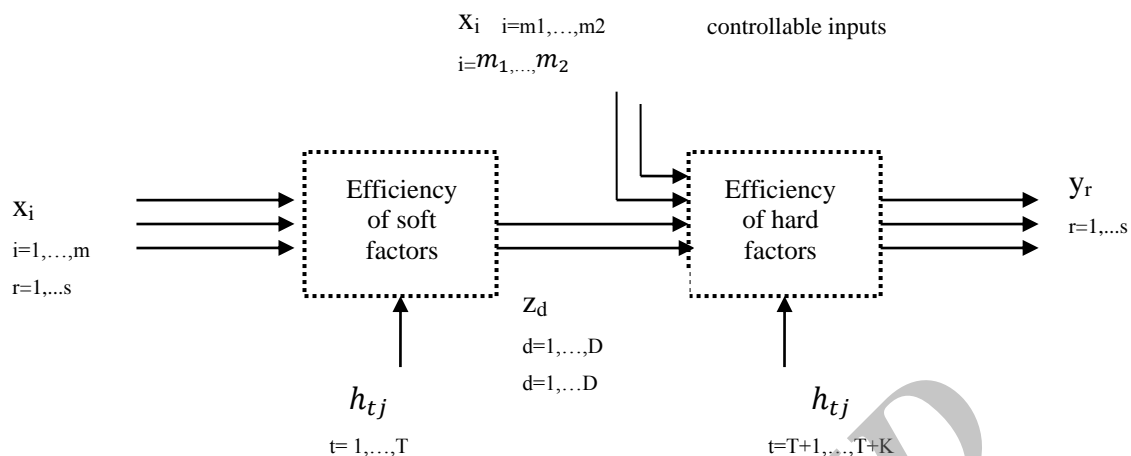


Fig. 8. Mathematical modeling Organizational Productivity

Multiple simplified model for evaluation of DMUp will be as below, model (5)

$$\begin{aligned}
 \text{Max } e_p &= \sum_{r=1}^s \mu_r y_{rp} \\
 \text{s.t. } \sum_{i=1}^m \mu_i x_{ip} &= 1 \\
 \sum_{i=1}^m u_r y_{ip} - \sum_{i=1}^{m_2} \eta_i x_{ij} - \sum_{i=m_1+1}^{m_2} \left[ v_i (x_{ij} - (1-\alpha_i) x_{ip}) - \mu_i \sum_{t=1}^T (h_{tj} - h_{tp}) \right] &\leq 0 ; j=1, \dots, n \\
 \sum_{d=1}^D \eta_d z_{dj} - \sum_{i=1}^{m_1} v_i x_{ij} - \sum_{\substack{t=1 \\ t \in ND}}^T \mu_T (h_{tj} - h_{tp}) &\leq 0 ; j=1, \dots, n \\
 \sum_{r=1}^s u_r y_{rj} - \sum_{i=1}^{m_2} \left[ v_i (x_{ij} - (1-\alpha_i) x_{ip}) \right] - \sum_{\substack{t=1 \\ t \in ND}}^T \mu_T (h_{tj} - h_{tp}) &\leq 0 ; j=1, \dots, n \\
 u_r &\geq \varepsilon ; r=1, \dots, s
 \end{aligned}$$

In model (5), ND stands for uncontrollable indices. However, the said dual model can be easily written so that envelopment form of the model is obtained.

## 7. Numerical Analysis of Productivity Using the Provided Model

In this section, the organization's productivity is studied in terms of function, and the practical results from this study are shown in tables. For ten organizations with the following inputs and outputs, the productivity rates proportional to the provided model are calculated using Gams software.

Table 2. Input and Output Values

X2	X3	X4	Z1	Z2	X5	$\alpha_i$	X6	h1	o1	o2	o3
84	345	4325	52	68	52	0.9	10	0.2	53	0.98	13
350	7695	8761	42	95	621	0.75	32	0.1	1095	0.732	97
325	7341	8643	60	15	625	0.6	42	0.15	1468	0.653	81
352	5067	6954	60	85	125	0.7	22	0.8	5404	0.6	135
209	822	6308	57	73	632	0.75	58	0.7	102	0.9	31
240	3956	6260	96	14	542	0.9	75	0.14	754	0.72	46
377	3321	4392	97	28	97	0.7	28	0.2	863	0.823	66
360	188	3214	140	44	125	0.65	65	0.9	325	0.496	38
346	637	2987	84	65	254	0.75	65	0.15	168	0.822	34
338	786	5763	69	73	425	0.85	73	0.85	88	0.866	24

In Table 2., the indices of X1 to X4 are the input indices of soft factors, and the indices of X5 and X6 the input indices of hard factors, among which the index of X5 is a slightly controllable one the controllability rate of which is determined by the value of  $\alpha_i$ . The indices of Z1 and Z2 are the output indices of the first class (soft factors) which are considered as the input indices of the second class (hard factors). Finally, o1 to o3 constitute the output indices of the second class (hard factors). According to Table 3, we will have:

Table 3. Indices of the Provided Model

Indices	X1	X2	X3	X4	Z1	Z2	X5	$\alpha_i$	X6
Index Situation	Input of soft area	Input of soft area	Input of soft area	Input of soft area	Output of soft area, considered as input of hard area	Output of soft area, considered as input of hard area	Input of hard factor, uncontrollable to some extent	Controllability rate of index	Input of hard factor
Type of Index	Service speed	Advanced services	Specialty of employees	Skill of employees	Customer attraction rate	Customer satisfaction	Customer attraction rate		Competitive pricing

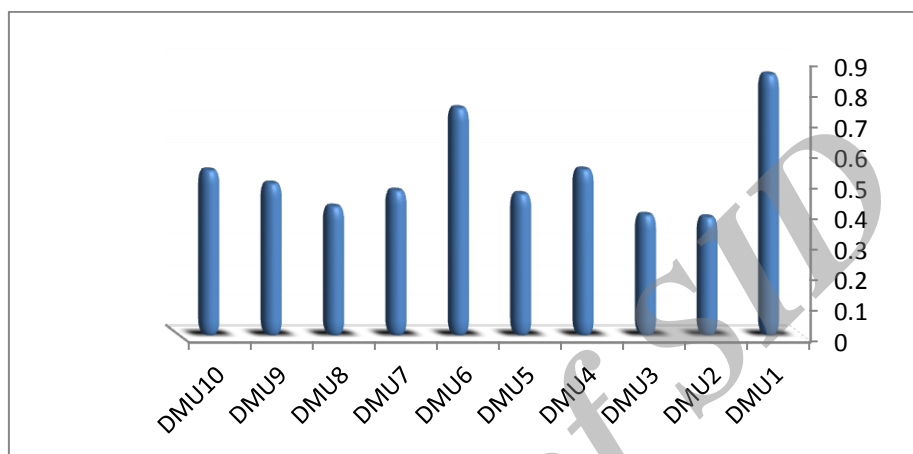
Indices	h1	o1	o2	o3
Index Situation	Input of external factor in hard area	Output of hard area	Output of hard area	Output of hard area
Type of Index	Economic index, District 6 <sup>†</sup>	Electronic services	Return on capital	Cost to income

The values of productivity of the organizations are shown, after calculation, in Table (4) as Graph (1).

<sup>†</sup> This index shows economic strength of the people in the area in which a bank branch is established.

Table 4. Measuring the Organizations' Productivity

DMU	1	2	3	4	5	6	7	8	9	10
Productivity value	0.852	0.386	0.394	0.542	0.462	0.742	0.473	0.421	0.496	0.539



Graph 1. The Organizations' Productivity Value

As shown in Table 4, productivity of no organization is equal to 1, while if model (3) was used, certainly one of the units is 1, because of paying attention to in-house structures so that in this model two groups of soft and hard factors are studied as a two-classes structure, and the inefficiency in at least one of the two soft and hard areas led to total efficiency of the organization not to become 1. However, the efficiency level can be easily analyzed and calculated in each of the two soft and hard area.

The organizations Nos. 8, 2, and 3 have the lowest productivity, indicating the highest problems these organizations face. The other organizations have better performance in terms of the score obtained. Which having the efficiency of 0.852, Dept. No. 1 has the best organizational performance, indicating that the effect of uncontrollable factors on total organization has been minimized.

There is no doubt that organizational analysis will be useful when we have suitable information about the weaknesses and strengths of the organization at our disposal. The provided model, due to separation of the indices in the organizational performance areas, specifies easily the weaknesses of the organization.

## 8. Conclusion

Undoubtedly, moving towards productivity is no longer an option but an obligatory necessity and an inevitable action, and only through improving productivity and using the available facilities as best as possible, the objectives of the plan will be achieved.

To inform of the productivity growth level, it must be measured in terms of different indices and in certain periods. By measuring the productivity indices, it can be found how the attempts towards productivity were useful. Productivity measurement also must be made in accordance with a suitable system, otherwise the results may not be reliable.

In this paper, it was attempted to pave the way for improving productivity through providing a model for evaluating productivity and identifying the weaknesses and strengths of the organization. The model provided in an organization with ten decision making departments was implemented, as well as the results obtained from the study indicate that to implement a continuous productivity improvement, we must be able to have a productivity evaluation system at our disposal, that includes all factors affecting the organization, and by measuring its value, the way for improvement will be paved in future.

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