



A Russell Measure for Modeling Environmental Performance

H.Zare Haghghi ^{a*}, M.Rostamy-Malkhalifeh^a

(a)Department of Mathematics, Science and Research Branch, Islamic Azad University, Tehran, Iran

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Abstract

Data Envelopment Analysis (DEA) has been long employed as a popular methodology to evaluate the performance of various production activities with multiple inputs and outputs. However, an important issue is that the production process in the real world inevitably generates undesirable outputs (like wastes and pollutants) along with desirable outputs. Therefore, the undesirable outputs should be included into the environmental performance evaluation. This study surveys the two technologies which is available in the DEA literature for modelling environmental performance under weak disposability assumption of good and bad outputs. Then, it attempts to present a Russell measure that incorporates both desirable and undesirable outputs. To illustrate the use of the proposed method, an empirical application corresponding to 31 administrative regions of China is provided and interpreted.

Key words: Data Envelopment Analysis, Environmental performance, Undesirable output.

1. Introduction

Data Envelopment Analysis (DEA) has been long employed as a popular methodology to evaluate the performance of various production activities with multiple inputs and outputs. However, an important issue is that the production process in the real world inevitably generates undesirable outputs (like pollutants) along with desirable outputs. Nowadays, global warming, climate change, an increased emission of CO_2 in the air, and water pollution are major problems all over the world. These worldwide problems demonstrate the importance of incorporating undesirable outputs into performance assessment.

* Corresponding author: E-mail: zarehaghghi.srbiau@gmail.com

In recent years, many researchers try to model undesirable outputs within the DEA framework. Dealing with this topic, Fare et al. [3] proposed an approach in 1989. They replaced the disposability of outputs with weak disposability. Later, the study in this direction extended by scholars such as: Fare et al. [4, 6], Scheel [10], Hailu and Veeman [12], Kuosmanen [8], and Zhou et al. [15].

Moreover, a common treatment of undesirable outputs is to regard them as inputs. Some of the works in this way include: Dyson et al. [2], Seiford and Zhu [11], Dyckhoff and Allen [1], and Sueyoshi and Goto [13]. Nevertheless, this routine causes to be concerned with two problems. First: the free disposability principle between inputs and bad outputs implies that a finite amount of input can produce an infinite amount of undesirable outputs, while this is physically impossible [5]. Second: the free disposability principle does not recognize the relation between the desirable and undesirable outputs.

Consequently, the undesirable outputs should model as outputs and the link between desirable and undesirable outputs should be taken into account in performance evaluation. Shephard [12] was the first to introduce the weak disposability principle between good and bad outputs. Based on this principle, he presented a technology dealing with both good and bad outputs. Later, Kuosmanen [8] followed this way and offered another technology that was more flexible. Then, Kuosmanen and Podinovski [9] examined that the Shephard technology suffers from some drawbacks, and its serious drawback is that it is not convex. In addition, they demonstrated that the Kuosmanen technology is the only correct technology suitable for modelling undesirable outputs under weak disposability [9].

In this paper, we review the two mentioned technology and the corresponding axioms. Then, employing the Kuosmanen technology, we attempt to present a Russell measure that incorporates both desirable and undesirable outputs.

The rest of this paper is arranged as follows: In section 2, we survey the two technologies developed by Shephard [12] and Kuosmanen [8] for modeling undesirable outputs. Section 3 focuses on Kuosmanen technology and provides a Russell measure for evaluating the environmental performance of DMUs. In section 4, the results of the proposed measure are presented and interpreted, regarding an empirical application corresponding to 31 administrative regions of China. Summary and conclusions of the study are provided in section 5.

2. Preliminaries

In this paper, it is supposed that there are n observed DMUs (Decision Making Units) and the j th DMU, $j \in \{1, \dots, n\}$, is determined by the vector (x_j, g_j, b_j) , where $x_j = (x_{1j}, x_{2j}, \dots, x_{mj}) \in R^m, x_j \geq 0, x_j \neq 0$ is the vector of inputs, $g_j = (g_{1j}, g_{2j}, \dots, g_{sj}) \in R^s, g_j \geq 0, g_j \neq 0$ is the vector of desirable (good) outputs and $b_j = (b_{1j}, b_{2j}, \dots, b_{hj}) \in R^h, b_j \geq 0, b_j \neq 0$ is the vector of undesirable (bad) outputs. The production technology is characterized by the set $T = \{(x, g, b) \mid x \text{ can produce } (g, b)\}$. Consider the following principles which have been introduced in the DEA literature [8, 12] for incorporating undesirable factors into production technology:

- A1 Strong (free) disposability of inputs and good outputs. If $(x, g, b) \in T, 0 \leq g' \leq g$ and $x' \geq x$, then $(x', g', b) \in T$.

A2 Weak disposability of good and bad outputs. If $(x, g, b) \in T, 0 \leq \theta \leq 1$, then $(x, \theta g, \theta b) \in T$.

A3 T is convex.

Axiom (A2) recognizes the relation between good and bad outputs, because the pollutants can be reduced in proportion to the reduction of good outputs. The multiplier θ used in this axiom is pointed out as the abatement factor [8].

Shephard [12] applied a single abatement factor to model weak disposability and presented the following technology T_S :

$$T_S = \left\{ (x, g, b) \mid \sum_{j=1}^n \lambda_j x_j \leq x, \sum_{j=1}^n \theta \lambda_j g_j \geq g, \sum_{j=1}^n \theta \lambda_j b_j = b, \sum_{j=1}^n \lambda_j = 1, \lambda_j \geq 0, (j = 1, \dots, n), 0 \leq \theta \leq 1 \right\} \quad (1)$$

Note that variables $\lambda_1, \lambda_2, \dots, \lambda_n$ are the structural variables.

Later, Kuosmanen [8] examined an alternative approach to deal with axiom (A2). He argued that the correct minimum extrapolation technology necessitates n distinctive abatement factors. Therefore, he employed distinctive abatement factors $\theta_j, (j = 1, \dots, n)$ corresponding to each observed firm, and developed the following technology T_K :

$$T_K = \left\{ (x, g, b) \mid \sum_{j=1}^n \lambda_j x_j \leq x, \sum_{j=1}^n \theta_j \lambda_j g_j \geq g, \sum_{j=1}^n \theta_j \lambda_j b_j = b, \sum_{j=1}^n \lambda_j = 1, \lambda_j \geq 0, (j = 1, \dots, n), 0 \leq \theta_j \leq 1, (j = 1, \dots, n) \right\} \quad (2)$$

Subsequently, Kuosmanen and Podinovski [9] claimed that the Kuosmanen approach is more adaptable with respect to the choice of abatement factors. Moreover, using a simple numerical example, they showed that the Shephard technology is not convex; rather the Kuosmanen technology is convex. Furthermore, they proved that T_K is indeed the correct technology that satisfies the minimum extrapolation principle of DEA under the mentioned axioms of (A1), (A2) and (A3).

Additionally, it should be noted that T_S and T_K are both nonlinear, since θ and θ_j are multiplied with λ_j . Nevertheless, Kuosmanen [8] stated that T_K can be linearized as follows:

$$\lambda_j = \underbrace{\theta_j \lambda_j}_{\eta_j} + \underbrace{(1 - \theta_j) \lambda_j}_{\mu_j}, \quad (j = 1, \dots, n).$$

$$T_K = \left\{ (x, g, b) \mid \sum_{j=1}^n (\eta_j + \mu_j) x_j \leq x, \sum_{j=1}^n \eta_j g_j \geq g, \sum_{j=1}^n \eta_j b_j = b, \sum_{j=1}^n (\eta_j + \mu_j) = 1, \eta_j \geq 0, \mu_j \geq 0, (j = 1, \dots, n) \right\} \quad (3)$$

However, Kuosmanen and Podinovski emphasized that T_S cannot be linearized by the above approach, because it is not convex [9].

3. The proposed RUSSELL measure

This section attempts to present a Russell measure that incorporates both desirable and undesirable outputs. For this purpose, the Kuosmanen technology is employed, because its usefulness was clarified in the previous sections. Here, the Russell measure in the presence of undesirable outputs is denoted by *ERM* (Environmental Russell Measure). The proposed model is as follows:

$$ERM = Min \frac{1}{m+h} \left(\sum_{i=1}^m \theta_i + \sum_{f=1}^h \gamma_f \right) \frac{1}{s} \sum_{r=1}^s \varphi_r$$

$$s.t. \begin{bmatrix} \theta_1 x_{1k} \\ \vdots \\ \theta_m x_{mk} \\ \varphi_1 g_{1k} \\ \vdots \\ \varphi_s g_{sk} \\ \gamma_1 b_{1k} \\ \vdots \\ \gamma_h b_{hk} \end{bmatrix} \in T_K \quad (4)$$

$$0 \leq \theta_i \leq 1, \quad (i = 1, \dots, m),$$

$$\varphi_r \geq 1, \quad (r = 1, \dots, s),$$

$$0 \leq \gamma_f \leq 1, \quad (f = 1, \dots, h).$$

Here, k indicates the DMU under evaluation. The constraints $\theta_i \leq 1$, $\varphi_r \geq 1$ and $\gamma_f \leq 1$ are included into model (4) to see whether a DMU can be found to dominate DMU_k . In this model, if $x_{ik} = 0$ ($b_{fk} = 0$), then the term θ_i (γ_f) is deleted from the objective function. Moreover, if $g_{rk} = 0$, then it is replaced by a very small positive number which serves as a penalty. Using T_K , the outcome model is:

$$\begin{aligned}
 ERM = \text{Min} \quad & \frac{1}{m+h} \left(\sum_{i=1}^m \theta_i + \sum_{f=1}^h \gamma_f \right) \\
 & \frac{1}{s} \sum_{r=1}^s \varphi_r \\
 \text{s.t.} \quad & \sum_{j=1}^n (\eta_j + \mu_j) x_{ij} \leq \theta_i x_{ik} \quad (i = 1, \dots, m), \\
 & \sum_{j=1}^n \eta_j g_{rj} \geq \varphi_r g_{rk} \quad (r = 1, \dots, s), \\
 & \sum_{j=1}^n \eta_j b_{fj} = \gamma_f b_{fk} \quad (f = 1, \dots, h), \\
 & \sum_{j=1}^n (\eta_j + \mu_j) = 1, \eta_j \geq 0, \mu_j \geq 0 \quad (j = 1, \dots, n), \\
 & 0 \leq \theta_i \leq 1 \quad (i = 1, \dots, m), \\
 & \varphi_r \geq 1 \quad (r = 1, \dots, s), \\
 & 0 \leq \gamma_f \leq 1 \quad (f = 1, \dots, h).
 \end{aligned} \tag{5}$$

The *ERM* measure incorporates all inefficiencies that the model can identify. Simply, it can be verified that all of the constraints of the first, second and third groups are binding on optimality. In the following theorem, we demonstrate that *ERM* lies between zero and unity.

Theorem 1. $0 < ERM \leq 1$.

Proof. Since $\theta_i = 1 (\forall i)$, $\varphi_r = 1 (\forall r)$, $\gamma_f = 1 (\forall f)$, $\mu_j = 0 (\forall j)$, $\eta_j = 0 (\forall j \neq k)$, and $\eta_k = 1$ is a feasible solution to model (5) with unity objective function value, then $ERM \leq 1$. Moreover, $\theta_i \geq 0 (\forall i)$, $\varphi_r \geq 1 (\forall r)$ and $\gamma_f \geq 0 (\forall f)$, therefore $ERM \geq 0$. Now, we only need to prove that $ERM \neq 0$. Suppose that $ERM=0$. This implies that $\theta_i = 0 (\forall i)$ and $\gamma_f = 0 (\forall f)$. Consequently, the undesirable constraints yield that $\eta_j = 0 (\forall j)$, and therefore, the desirable constraints lead to $\varphi_r g_{rk} \leq 0 (\forall r)$. At last, it is inferred that $\varphi_r g_{rk} = 0 (\forall r)$, while this is a contradiction. Consequently $ERM \neq 0$. ■

Although model (5) has a fractional objective function, however, it can be converted to a linear model, using the Charnes-Cooper transformation. Letting $\frac{1}{s} \sum_{r=1}^s \varphi_r = \frac{1}{t}$, and multiplying each constrain by t , we then have:

$$\begin{aligned}
ERM = \text{Min} \quad & \frac{1}{m+h} \left(\sum_{i=1}^m t \theta_i + \sum_{f=1}^h t \gamma_f \right) \\
\text{s.t.} \quad & \sum_{j=1}^n t(\eta_j + \mu_j) x_{ij} \leq t \theta_i x_{ik} \quad (i = 1, \dots, m), \\
& \sum_{j=1}^n t \eta_j g_{rj} \geq t \varphi_r g_{rk} \quad (r = 1, \dots, s), \\
& \sum_{j=1}^n t \eta_j b_{fj} = t \gamma_f b_{fk} \quad (f = 1, \dots, h), \\
& \sum_{j=1}^n t(\eta_j + \mu_j) = t, t \eta_j \geq 0, t \mu_j \geq 0 \quad (j = 1, \dots, n), \\
& 0 \leq t \theta_i \leq t \quad (i = 1, \dots, m), \\
& t \varphi_r \geq t \quad (r = 1, \dots, s), \\
& 0 \leq t \gamma_f \leq t \quad (f = 1, \dots, h), \\
& \sum_{r=1}^s t \varphi_r = s, t \geq 0.
\end{aligned} \tag{6}$$

Let $t \eta_j = \eta'_j, t \mu_j = \mu'_j, t \theta_i = \theta'_i, t \varphi_r = \varphi'_r$ and $t \gamma_f = \gamma'_f$, we therefore achieve the following linear problem:

$$\begin{aligned}
ERM = \text{Min} \quad & \frac{1}{m+h} \left(\sum_{i=1}^m \theta'_i + \sum_{f=1}^h \gamma'_f \right) \\
\text{s.t.} \quad & \sum_{j=1}^n (\eta'_j + \mu'_j) x_{ij} \leq \theta'_i x_{ik} \quad (i = 1, \dots, m), \\
& \sum_{j=1}^n \eta'_j g_{rj} \geq \varphi'_r g_{rk} \quad (r = 1, \dots, s), \\
& \sum_{j=1}^n \eta'_j b_{fj} = \gamma'_f b_{fk} \quad (f = 1, \dots, h), \\
& \sum_{j=1}^n (\eta'_j + \mu'_j) = t, \eta'_j \geq 0, \mu'_j \geq 0 \quad (j = 1, \dots, n), \\
& 0 \leq \theta'_i \leq t \quad (i = 1, \dots, m), \\
& \varphi'_r \geq t \quad (r = 1, \dots, s), \\
& 0 \leq \gamma'_f \leq t \quad (f = 1, \dots, h), \\
& \sum_{r=1}^s \varphi'_r = s, t \geq 0.
\end{aligned} \tag{7}$$

4. Numerical example

In this section, we apply the proposed method for assessing the environmental performance of 31 administrative regions of China. See the data set in table 1 which is adopted from Wu et al. article [14]. These data have two inputs: the total investment in the fixed assets of industry (TIFA) and the electricity consumption by industry (EC), one desirable output: the gross industrial output value (GIOV), and two undesirable outputs: the total volume of industrial waste gas emission (TWGE) and the total volume of waste water discharge (TWWD). For ease of comparison, we have named the industries D1 to D31 which is exhibited in the second column of table 1.

Table 1:
Data set of industry of Chia in 2010

District	DMU	TIFA	EC	GIOV	TWGE	TWWD
Anhui	D1	9121.829	1077.91	18732	17849	70971
Beijing	D2	4554.356	809.9	13699.84	4750	8198
Chongqing	D3	5049.258	626.44	9143.55	10943	45180
Fujian	D4	6534.803	1315.09	21901.23	13507	124168
Gansu	D5	2274.305	804.43	4882.62	6252	15352
Guangdong	D6	11903.36	4060.13	85824.64	24092	187031
Guangxi	D7	5166.135	993.24	9644.13	14520	165211
Guizhou	D8	2483.012	835.38	4206.37	10192	14130
Hainan	D9	903.8264	159.02	1381.25	1360	5782
Hebei	D10	11737.07	2691.52	31143.29	56324	114232
Heilongjiang	D11	5019.085	747.84	9535.15	10111	38921
Henen	D12	12868.24	2353.96	34995.53	22709	150406
Hubei	D13	7276.638	1330.44	21623.12	13865	94593
Hunan	D14	7374.157	1171.91	19008.83	14673	95605
Inner Mongolia	D15	6831.416	1536.83	13406.11	27488	39536
Jiangsu	D16	18977.92	3864.37	92056.48	31213	263760
Jiangxi	D17	6696.149	700.51	13883.06	9812	72526
Jilin	D18	6313.748	576.98	13098.35	8240	38656
Liaoning	D19	12480.94	1715.26	36219.42	26955	71521
Ningxia	D20	1193.702	546.77	1924.39	16324	21977
Qingghai	D21	789.5051	465.18	1481.99	3952	9031
Shaanxi	D22	5462.784	859.22	11199.84	13510	45487
Shandong	D23	17664.34	3298.46	83851.4	43837	208257
Shanghai	D24	4252.32	1295.87	30114.41	12969	36696
Shangxi	D25	4702.091	1460	12471.33	35190	49881
Sichuan	D26	9790.274	1549.03	23147.38	20107	93444
Tianjin	D27	4571.888	645.74	16751.82	7686	19680
Tibet	D28	306.567	20.41	62.22	16	736
Xinjiang	D29	2749.838	5341.9	5341.9	9310	25413
Yunnan	D30	4024.972	1004.07	6464.63	10978	30926
Zhejiang	D31	10246.41	2820.93	51394.2	20434	217426

Table 2:
Results of the proposed Russell measure

District	DMU	ERM	θ_1^*	θ_2^*	ϕ_1^*	γ_1^*	γ_2^*
Anhui	D1	0.455	0.392	1	1.333	0.602	0.429
Beijing	D2	1	1	1	1	1	1
Chongqing	D3	0.380	0.432	1	1.565	0.563	0.386
Fujian	D4	0.526	0.659	1	1.393	0.966	0.304
Gansu	D5	0.331	0.861	0.689	2.58	0.868	1
Guangdong	D6	1	1	1	1	1	1
Guangxi	D7	0.262	0.642	1	2.383	0.682	0.170
Guizhou	D8	0.257	0.735	0.612	2.757	0.490	1
Hainan	D9	0.375	0.797	0.969	2.286	1	0.666
Hebei	D10	0.364	0.374	0.500	1	0.234	0.346
Heilongjiang	D11	0.386	0.509	1	1.801	0.732	0.538
Henen	D12	0.495	0.383	0.653	1	0.614	0.332
Hubei	D13	0.517	0.584	0.974	1.393	0.935	0.388
Hunan	D14	0.467	0.525	1	1.430	0.798	0.347
Inner Mongolia	D15	0.319	0.622	0.843	2.246	0.472	0.928
Jiangsu	D16	1	1	1	1	1	1
Jiangxi	D17	0.505	0.360	1	1.157	0.705	0.270
Jilin	D18	0.604	0.321	1	1.003	0.687	0.414
Liaoning	D19	0.652	0.408	0.932	1	0.526	0.743
Ningxia	D20	0.150	1	0.562	3.518	0.179	0.375
Qingghai	D21	0.229	1	0.379	2.486	0.402	0.497
Shaanxi	D22	0.381	0.531	1	1.768	0.631	0.531
Shandong	D23	1	1	1	1	1	1
Shanghai	D24	1	1	1	1	1	1
Shangxi	D25	0.3	0.904	0.888	2.415	0.369	0.736
Sichuan	D26	0.444	0.434	0.837	1.301	0.645	0.393
Tianjin	D27	1	1	1	1	1	1
Tibet	D28	1	1	1	1	1	1
Xinjiang	D29	0.287	0.833	1	2.836	0.701	0.726
Yunnan	D30	0.255	0.832	1	3.593	0.911	0.915
Zhejiang	D31	0.697	0.815	1	1.158	0.817	0.596

Here, the GAMS (General Algebraic Modeling System) software is utilized for the computations. Table 2 displays the results of the environmental efficiency of the industries. The first column shows the amounts of the environmental Russell measure which is computed by model (7). D2, D6, D16, D23, D24, D27, and D28 are the industries which are identified as environmental efficient industries. Thus, we can conclude that these 7 industries pay attention to the reduction of their pollutants accompanying with improving their commercial targets.

The other columns in table 2 represent respectively the proportions of decreases in inputs, increase in desirable output and decreases in undesirable outputs. For example, the efficiency of D9 is 37.5% therefore; it can reach the efficient frontier by reducing its inputs in the proportions of 79.7% and 96.9%, increasing on an average of 22.86% its desirable output, preserving its first undesirable output, and decreasing an average of 66.6% its second undesirable output. It should be noticed that 3 industries (D10, D12 & D19) perform effectively in the desirable output and attain $\phi_1^* = 1$, but they do not manage successfully the undesirable outputs.

5. Summary and conclusion

Nowadays, global warming, climate change, an increased emission of CO_2 in the air, and water pollution are major problems all over the world. These worldwide problems indicate the importance of developing firms with less undesirable outputs.

In this paper, we surveyed the two technologies which are available in the DEA literature for modeling environmental performance under weak disposability assumption of desirable and undesirable outputs. Then, we attempt to present a Russell measure that incorporates both desirable and undesirable outputs.

To illustrate the use of the proposed method, we applied the proposed method for assessing the environmental performance of 31 administrative regions of China. 7 industries attained the full environmental efficiency measure via the proposed model. Thus, we concluded that these industries pay attention to the reduction of their pollutants accompanying with improving their commercial targets.

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