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# Complex-Valued Data Envelopment Analysis

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**Abstract**

Data Envelopment Analysis (DEA) is a nonparametric approach for measuring the relative efficiency of a decision making units consists of multiple inputs and outputs. In all standard DEA models semi positive real valued measures are assumed, while in some real cases inputs and outputs may take complex valued. The question is related to measuring efficiency in such cases. As far as we are aware, there is not any special formulation replying the complex-valued questions. The formulation developed in this paper enables one to estimate efficiency when the data are complex valued.

Keywords: Data Envelopment Analysis-Efficiency-Complex valued.

## 1. Introduction

Data Envelopment Analysis (DEA) based on the seminal work of Farrell (1957) was developed to measure the technical efficiency of individual decision making units (DMUs) in multiple inputs and output settings in similar operational environment. Standard DEA models, originally

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presented by Charnes et.al (1978) and Banker et.al (1984) do take semi positive real inputs and outputs. Both radial and non- radial models assume real valued inputs/outputs. However, in some real occasions, in which some inputs and outputs take only integer values, Lozano et.al (2006) and Kuosmanen et.al (2009) address the question and develops the axiomatic foundation for DEA in this case. The applicability of DEA, however, depends on the underlying production process. It is essential that all real and integer valued are subsets of complex numbers. The question is that what happens if the input and output factors are complex. The need to deal with complex-valued data in DEA occurs because real- valued data set are contained in complex data set. On the other hand, complex valued numbers are frequently used in engineering especially in electronic engineering. Therefore, modeling these numbers has attracted renewed interest in the recent DEA literature. The question is how we should formulate the DEA models for these quantities. The purpose of the paper is to fill the gap by developing the standard DEA models for these measures. The rest of the paper is organized as follows: the next section reviews the complex numbers and in the following section modeling complex-valued numbers through applying the standard radial models are introduced. In section 3 an empirical example illustrates the approach. Section 4 presents our concluding remarks and suggests avenues for future research.



**2. Technical Work Preparation**

In DEA literature, each observed DMU is characterized by a pair of non -negative input and output vectors $(X_j, Y_j) \in R_+^{m+s}, j = 1,\dots,n.$ According to the postulates presented by Charnes et.al (1978), the CCR input efficiency measure can be computed by solving the following problem:

$$
\begin{aligned}
& Min \ \theta \\
& s.t. \\
& \sum_{j=1}^{n} \lambda_j x_{ij} \le \theta x_{io} \quad , \ i = 1,\dots,m \\
& \sum_{j=1}^{n} \lambda_j y_{rj} \ge y_{ro} \quad , \ r = 1,\dots,s \\
& \lambda_j \ge 0 \quad j = 1,\dots,n.
\end{aligned}
$$

Now we turn to efficiency estimation of DMUs in presence of complex valued quantities. That is, the input and outputs vectors are presented as $(Z_j, W_j)$. In which $z_j = x_j + iy_j$ and $w_j = u_j + iv_j$, $j = 1,\dots,n.$ It is worth noting that all efficiency measures assume continuous real valued data. For dealing with complex numbers in a classical format of efficiency measurement, the norm or magnitude of each complex elements in input and output matrix can be used. As we are aware the norm of each complex number in format $z = x + iy$ can be computed as $|z| = \sqrt{x^2 + y^2}$. Motivating by this real quantity, a real input and output matrix can be stated. Representing these real valued matrix may assure that the axiomatic foundation of standard DEA models, both radial and non-radial are satisfied. Hence, examining the complex numbers through their real valued norm, the CCR measure gauges radial distance to the monotonic hull of the production possibility set. This arranging preserve the usual interpretation of the Farrell measure as a downward scaling potential in inputs as the given output levels. Also guarantees that the units with the score one are efficient. Consequently, the modified matrix are used in CCR input oriented in order to measure the efficiency score in presence of complex-valued numbers. It should be noted that applying the magnitude of each element dare to need a complex calculation. However, this modification is time consuming especially in large-scale problems, but coincides the constant return to scale axioms. Hence, we restrict attention only to data set and applying the CCR measure directly to modified data set. The application of the next section demonstrates that how the efficiency is computed when all inputs and outputs are complex.

**3. An Illustrative Example**

In this section we illustrate the proposed approach for dealing with complex numbers by calculating the magnitude of each element in the matrix. Imagine that we have twelve DMUs. Each unit consumes one input to produce one output. The data set are complex numbers.

Table 1
complex data set

| DMU | INPUT | OUTPUT |
|---|---|---|
| 1 | 5+2i | -1-5i |
| 2 | -1i | -1i |
| 3 | -7i | 7i |
| 4 | 3+2i | √3+2i |
| 5 | 3-1i | √3i |
| 6 | 6-i | 6+2i |
| 7 | -2i | 1-2i |
| 8 | 1+5i | 1+√5i |
| 9 | 1-√3i | 1-√3i |
| 10 | -1+i | -1+i |
| 11 | 7i | -7i |
| 12 | 1+i | -1-i |

As before mentioned, for applying standard DEA models such as CCR or BCC, real valued data are required. Hence, the magnitude of each element is computed then the alternative models can be applied. Table 2 shows the CCR efficiency scores for data set in Table 1. What's more, the norm of each input and output element as real- valued examination were used in efficiency measurement.

Table 2
efficiency score

| DMU | CCR Input Efficiency Score | CCR Output Efficiency Score |
|---|---|---|
| 1 | 0.39 | 0.39 |
| 2 | 0.42 | 0.42 |
| 3 | 0.42 | 0.42 |
| 4 | 0.31 | 0.31 |
| 5 | 0.36 | 0.36 |
| 6 | 1 | 1 |
| 7 | 0.46 | 0.46 |
| 8 | 0.20 | 0.20 |
| 9 | 0.42 | 0.42 |
| 10 | 0.42 | 0.42 |
| 11 | 0.42 | 0.42 |
| 12 | 0.42 | 0.42 |

As Table (2) presents the only efficient unit is #6. As we expect CCR efficiency score in both orientation are same. One more point to be noted is the approximation of each complex element through the real valued quantity. However, this approximation could suggest an avenue for handling complex data but it seems that we need further investigation in this field.

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## تحلیل پوششی داده ها در حضور داده های مختلط

**چکیده**

تحلیل پوششی داده‌ها تکنیک غیر پارامتری و مبتنی بر برنامه‌ریزی خطی برای سنجش کارایی واحدهای تصمیم‌گیرنده با ورودی‌ها و خروجی‌های چندگانه است. در تمام مدل‌های استاندارد تحلیل پوششی داده‌ها ورودی‌ها و خروجی‌ها نیمه مثبت و حقیقی مقدار فرض شده اند. حال سوالی که مطرح می‌شود این است که اگر داده‌های ورودی و خروجی به صورت اعداد مختلط باشند کارایی واحدها چگونه سنجیده می‌گردد. در این مقاله در تلاش برای برآورد کارایی واحدهای تصمیم گیرنده در حضور داده‌های مختلط هستیم .