Detect and Eliminate Congestion of the Intermediate Products in Supply Chain

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Abstract

Data envelopment analysis (DEA) is a nonparametric technique that includes models to evaluate the relative efficiency of Decision Making Units (DMUs). It has the ability to separate efficient units and inefficient units. One of the applications of this mathematical technique is evaluating performance of supply chain. According to such as the inefficiency factors of one DMU, is existence congestion in its inputs and with regard to the economic concept of congestion that is widely visible in the most phenomenon, therefore, cannot ignore its presence in supply chain. In this paper, at first a network DEA model is noted to evaluate the performance of supply chain. Then according to the important role of intermediate products in supply chain, the existence of congestion in this product is investigated, finally, an Inverse DEA model is introduced in order to overcome the problems caused by congestion. So that the amount required input for providing the desired intermediate product can be estimated by that. In this case, no congestion will occur in intermediate products. Finally for improving the efficiency of this unit with new intermediate products an inverse DEA model is proposed.

Keywords: Data envelopment analysis (DEA), Inverse DEA, supply chain, congestion.

1. Introduction

Data envelopment analysis (DEA) is a mathematical method to calculate the relative efficiency of a set of Decision Making Units (DMUs) with multiple inputs and outputs. It has been widely used to evaluate the relative performance of a set of production. This method was first developed by Charnes

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et al. [1], in 1978. By using this method, the possibility of identifying the strength and weakness points of the unit can be obtained for management.

According to the complex structure of supply chain, an appropriate performance measurement should be designed that considers the network structure of this. The classical DEA models treat with each unit as a "black box", it means that consider primary inputs and final outputs. Therefore, this cannot be useful to assess organizations such as supply chain which has network structure, so the researchers went towards the network DEA model.

Fara and Grosskopf [6] developed one network DEA method for modeling multi-step process, and Golany et al. [9] also presented a model for measuring the efficiency of Seri multiple system. Recently, congestion economic concept has been discussed in many of studies conducted in the field of DEA, congestion is known as inefficiency among researchers, but this inefficiency is different with pre-known technical inefficiency concept. Researches of this concept for the first time began by Fara and Svensson [8], and then Fara et al. [7] presented a method to identify density by DEA models. This method, called FGL, is the first represented method to assess congestion in DEA that can only determine the presence or lack of presence of congestion and does not show the amount of this in each input, the method of BCSW presented by Cooper et al. [5] for the first time and then Brockett applied it on real data in1998 to review interactions between employment and output in the Chinese products to observe that whether the employment should increase or decrease. Single model method of Cooper [3] is another method to calculate the congestion. In this method, Cooper could provide a new method while benefit from BCSW method and combing its two stages in one model. Jahanshahloo and Khodabakhshi [11] provided a model to calculate the congestion which was presented as two models and needed to solve three linear programming problems.

What is considered in this paper is the presence of congestion in the intermediate products of one supply chain. These products are the outputs of supplier section which enters to productive section as input. Due to that always achieving higher benefits and lower cost is of concerns of a productive organization, therefore identifying congestion and eliminating a unit with density is considered by many managers because it leads to decrease cost and increase the amount of output. In the following we have, in Section 2 of this paper at first some descriptions in the field of supply chain is expressed, then a model is provided in order to assess the performance of supply chain. In section 3, the congestion phenomenon is introduced and a model is expressed to identity amount of congestion and an Inverse DEA model is used to estimate required input then for improving the efficiency of this unit an inverse DEA model is introduced. Numerical examples is given in order to better understand of executive procedure in section 4 and   finally the conclusion of this paper is expressed in section 5 concludes.

2. Supply Chain

Yan and Chen [2] provided three different network DEA models for evaluating performance supply chain under the concepts of centralized, decentralized and mixed in 2011. We are using the decentralized control model in this paper. In this mechanism, each part of supply chain is controlled by one unique decision maker. Assume that we have two-stage supply chain according to Figure 1. In this supply chain there is a supplier and two manufacturers. And the efficiency of DMU can be examined by model 1.
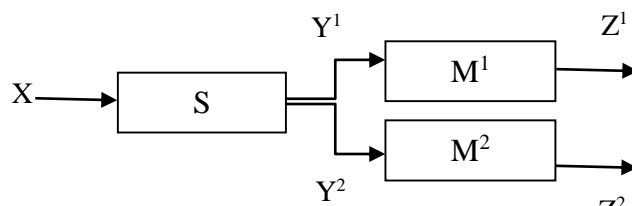

Figure 1.A Two-Stage Supply Chain with one supplier and two manufacturers

In the following model, the first, the fifth and the eighth inequalities related to the initial inputs at given of final outputs, and the second, the third, the fourth, and the seventh inequalities are associated with intermediate products.

$$Min \quad \theta_{decentral}$$
$$s.t$$
$$\sum_{j=1}^{n} \lambda_j^1 X_j \leq \theta_{decentral} X_o$$
$$\sum_{j=1}^{n} \lambda_j^1 Y_j^1 \geq \sum_{j=1}^{n} \lambda_j^2 Y_j^1 \qquad (1)$$
$$\sum_{j=1}^{n} \lambda_j^1 Y_j^2 \geq \sum_{j=1}^{n} \lambda_j^3 Y_j^2$$
$$\sum_{j=1}^{n} \lambda_j^2 Y_j^1 \leq Y_o^1$$
$$\sum_{j=1}^{n} \lambda_j^2 Z_j^1 \geq Z_o^1$$
$$\sum_{j=1}^{n} \lambda_j^2 = 1$$
$$\sum_{j=1}^{n} \lambda_j^3 Y_j^2 \leq Y_o^2$$
$$\sum_{j=1}^{n} \lambda_j^3 Z_j^2 \geq Z_o^2$$
$$\lambda_j^1, \lambda_j^2, \lambda_j^3 \geq 0, j = 1,...,n$$

Definition 1: If, in the above model, the objective function is equal to one, then supply chain will be efficient.

3. Congestion

So far we have dealt with situations in which they were allowed to increase inputs, for example as we see in T_c. Unlimited input of $X \geq \lambda X$ is used to produce output of Y, but there are some cases where an increase in one or more input is caused to deterioration of one or more outputs. Such situations are called congestion. To deal with such situations we should considered the product possibility set as follows:

$$P_{convex} = \{(X,Y)\big| X = \sum \lambda_j X_j, Y \leq \sum \lambda_j Y_j, \sum \lambda_j = 1, \lambda_j \geq 0\}$$

Indeed, congestion is considered as a separate component of inefficacy. Consider the following definitions to further evaluation of this difference.

Definition 2: Technical inefficiency exists when we can improve some inputs and outputs without worsening any other inputs or outputs.

Definition 3: The ideal of congestion refers to the state in the economy that the reduction of some inputs increases outputs without worsening of others.

When we observe the congestion in the intermediate product, at first we identify component inputs vector of manufacturer which has congestion by one of the available models and determine the amount of congestion. Now by reduction of this amount from desired component, we will reach to the desired inputs vector of manufacturer (it is the outputs vector of supplier) which has no congestion. If we assume that C_r is the congestion in vector Y^1, we will have:

$$\beta_r = Y_r^1 - C_r$$

Here β_r vector, is the desired output vector. (Note that desired output is the same input without congestion). Now by using this following model, we determine required input to provide this output vector. So, the X input vector will change to α vector. We have the following inverse DEA model by using [10].

$$Min \quad \alpha_o = (\alpha_{1o}, ..., \alpha_{mo})$$

$$s.t$$

$$\sum_{j=1}^{n} \lambda_j^1 X_j \le \theta_{decentral}^* \alpha_o$$

$$\sum_{j=1}^{n} \lambda_j^1 \beta_j \ge \sum_{j=1}^{n} \lambda_j^2 \beta_j \qquad (2)$$

$$\sum_{j=1}^{n} \lambda_j^1 Y_j^2 \ge \sum_{j=1}^{n} \lambda_j^3 Y_j^2$$

$$\sum_{j=1}^{n} \lambda_j^2 \beta_j \le \beta_o$$

$$\sum_{j=1}^{n} \lambda_j^2 Z_j^1 \ge Z_o^1$$

$$\sum_{j=1}^{n} \lambda_j^2 = 1$$

$$\sum_{j=1}^{n} \lambda_j^3 Y_j^2 \le Y_o^2$$

$$\sum_{j=1}^{n} \lambda_j^3 Z_j^2 \ge Z_o^2$$

$$\lambda_j^1, \lambda_j^2, \lambda_j^3 \ge 0, j = 1, ..., n$$

$$\alpha_i \ge 0, i = 1, ..., m$$

By solving above model we obtain at least initial input level of DMU. Note that this model for less value of X is feasible.

Therefore, if the α input vector produces β_r output vector, we will not see any congestion.

4. Numeric example

Consider one two-stage supply chain with one supplier and two producers as Figure 1. The data of this example is shown in Table 1 and involves n=7 of the DMU with two inputs, two intermediate products and two final outputs are seen.

Table1. The input- output and intermediate product and the efficiency of units

DMU	X	Y^1	Y^2	Z^1	Z^2	θ^*
DMU 1	(20 ، 151)	(100 ، 90)	(152 ، 80)	(84 ، 65)	(53 ، 40)	1.00
DMU 2	(19 ، 131)	(150 ، 50)	(190 ، 100)	(68 ، 60)	(52 ، 61)	1.00
DMU 3	(25 ، 160)	(140 ، 55)	(250 ، 100)	(52 ، 41)	(56 ، 45)	0.78
DMU 4	(27 ، 168)	(180 ، 72)	(260 ، 147)	(58 ، 37)	(54 ، 73)	0.74

Table1.(continued)

DMU 5	(22 ، 158)	(94 ، 66)	(250 ، 120)	(50 ، 10)	(24 ، 63)	0.76
DMU 6	(55 ، 255)	(230 ، 90)	(200 ، 70)	(42 ، 40)	(76 ، 94)	0.50
DMU 7	(33 ، 235)	(95 ،88)	(150 ، 170)	(58 ، 46)	(82 ، 95)	0.94

The efficiency is calculated by the method of decentralized control has been brought as input nature. It is in the final column of above table. We see the amount of congestion in the inputs of manufacturer1 for DMU$_4$, DMU$_6$ by using the single method of Cooper. The amount of congestion of DMU_4 is

$(c_{14}, c_{24}) = (57.5, 0)$ and   for DMU_6 we   obtain $(c_{16}, c_{26}) = (114.25, 0)$.   We   achieve

$(\beta_{14}, \beta_{24}) = (122.5, 72)$ and $(\beta_{16}, \beta_{26}) = (115.75, 90)$ by decreasing the congestion of Y. To determine

the new initial inputs of DMU, by solving the corresponding linear programing problem we will have

$(\alpha_{14}, \alpha_{24}) = (24.08, 168)$ and $(\alpha_{16}, \alpha_{26}) = (43.277, 255)$. In this case, the efficiency of DMU

with (X, Y^1, Y^2, Z^1, Z^2) and $(\alpha, \beta, Y^2, Z^1, Z^2)$ is equal.

Now consider the efficiency of DMU is θ^* and we would like to improve it to $\bar{\theta}$. Thus we have to put

the $(\frac{\theta^*}{\bar{\theta}}\alpha, \beta, Y^2, Z^1, Z^2)$ instead of $(\alpha, \beta, Y^2, Z^1, Z^2)$.

5. Conclusion

In this paper the internal operations of a supply chain is considered. Due to that identification of congestion and elimination of that leads to the reduction of current cost and increase of outputs, so ignoring it will be a deniable work. So we paid attention to the presence of this phenomenon in the intermediate products of one supply chain in this paper. We used an inverse DEA model to eliminate that. Finally the improvement of efficiency of DMU is investigated.

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E. Mollaeian, M. Rostamy-Malkhalifeh

شناسایی و حذف تراکم از محصولات میانی یک زنجیره تامین

چکیده:

تحلیل پوششی داده ها (DEA) یک تکنیک ناپارامتری می باشد که شامل مدل هایی به منظور ارزیابی کارایی نسبی واحدهای تصمیم گیرنده (DMU) می باشد. همچنین می تواند واحدهای کارا و ناکارا از یکدیگر متمایز کند. یکی از کاربردهای این تکنیک ریاضیاتی ارزیابی عملکرد زنجیره های تامین می باشد. با توجه به آنکه یکی از عوامل ناکارایی یک DMU وجود تراکم در ورودی های آن می باشد و با توجه به مفهوم اقتصادی تراکم که در اکثر پدیدها قابل رویت می باشد، نمی توان از وجود آن در زنجیره های تامین نیز چشم پوشی کرد. در این مقاله، در ابتدا یک مدل DEA شبکه ای برای ارزیابی عملکرد زنجیره تامین در نظر گرفته شده است. سپس با توجه به نقش مهم محصولات میانی در زنجیره تامین، به وجود تراکم در محصولات میانی بحث شده است. در نهایت یک مدل DEA معکوس برای غلبه بر این مشکل معرفی شده است. بنابراین مقدار ورودی مورد نیاز برای تولید محصولات میانی مطلوب با استفاده از آن تخمین زده می شود. به این ترتیب هیچ تراکمی در محصولات میانی به وقوع نمی پیوندند. نهایتا به منظور بهبود کارایی این واحد با محصولات میانی جدید یک مدل DEA معکوس ارائه شده است.