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Research Article

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A Modified Directional Distance Formulation of DEA with Malmquist Index to Assess Bankruptcy

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Abstract

Bankruptcy in the same amount of time and history is very rampant and therefore the vision of the future can be prevented. Using data envelopment analysis (DEA) and malmquist index can precise evaluating of the performances of many different kinds of decision making units (DMU) such as hospitals, universities, business firms, etc. In this paper, we will modify directional distance formulation of DEA to assess bankruptcy with using malmquist index. This method is the most general non-radial directional distance model. The method measures worst relative efficiency within the interval of zero to one for various period times. Model locates worst performing DMUs and determines an inefficient frontier model simultaneously with decrease of output and increase of input. By using malmquist index we measure the productivity for various years .This study introduces a precise and comprehensive bankruptcy measure that could be used as an early warning system for bankruptcy assessment.

Keywords: Data Envelopment Analysis, Directional Distance function, Bankruptcy, Productivity, Malmquist.

1. Introduction

Bankruptcy in the same amount of time and history is very rampant. This occurs when the bankruptcy statutory which the company sold assets to pay debts Wimboh [32]. So the vision of the future can be prevented. Forecasting is an issue that has preoccupied the human mind to itself. Particularly can be

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said which one of the most important tasks science in various context has effort for discover relation meddle various phenomenon for forecasting of the future Tsaur [27].

Because the failure of some companies to continue their outward life therefore they will be indirectly lead to wasting resources. Newton [19] tell the lack of teaching, experience, and inefficiency management for remaining competitive in the business, which makes trouble. Many studies have been done on the decision to invest one thing that can help investment decision making is appropriate tools and models to assess the financial condition and state organizations.

One of the tools used for the decision to invest in a company are bankruptcy prediction models which indicated in two methods .The first method is the probability of bankruptcy by a group of financial ratios have been estimated by expert to be combined. In the second method, the probability of bankruptcy through changes in market risk as the variance rate of return for a share of systematic risk occurs. In the periods, a company has good output. But this growth is not constant.When productivity is important in the company then the company can hope to profit alirezaie [4]. Sten (1953) discovered index value and by using this index evaluated companies and institutions. Cave D.W, and L.R.Christensen [7] discovered how distance function could be used by productivity change in the malmquist productivity index. The feature of this index is that this index is decomposed into sub-sections. Fare [8] showed that the Malmquist productivity index is decomposed into technical change and technical efficiency change. Wang [30] by using the Malmquist index studied Capital Management in Pharmaceutical Industry in Taiwan. The results showed that seven companies had good worked and have better management and five other companies do not have good management. Abdul Rahman et al. [1] by using malmquist productivity index studied performance in the Sugar Industry. The total productivity growth measure using by panel data (Panels) of 20 companies in Karachi Stock Exchange during 1998- 2007. The results showed that the growth of the sugar industry 8% improved while technical efficiency change management declined 8%. Basti and Akin [5] studied performance productivity in familiar companies and unfamiliar companies in Turkey during the years 2003-2007 using by malmquist productivity index. The results showed that the differences have not between familiar and unfamiliar companies. Average productivity of both companies during the period except year 2006 has been reduced. Abdul Rahman et al [2] studied performance growth productivity in production industry using by malmquist index during a 10-year period in exchange. The results showed that total productivity have complex process. According to these results, the cement industry, oil and gas situation were relatively permanent. Most of the industries have much progress in technical productivity, but a few industries, with technological change on productivity growth had a negative impact. Diaz and Sanchez [12] showed the performance of small and medium Spanish companies during the years 1995- 2001.By using stochastic frontier in production function calculated the performance. The results showed that small and medium companies had the most productivity, and small companies can get out of economic problems easily. Sten Throe et al [25] by using malmquist index studied on the financial statements in American company.

Researchers combined the knowledge of the malmquist productivity index with DEA. Rezitis [21] showed this index in methods non-parametric DEA is important. This index using by of distance function measure the productivity change between two different times.

Fare et al. [8] developed a Malmquist productivity index. He using by opinions Farrel [16] developed Malmquist productivity index for each unit of output and production inputs.

In this paper, a directional distance formulation of DEA by Shetty et al [24] to predict the bankruptcy was developed, by using Malmquist productivity index which will be modified. Here a model to measure the worst relative performance within the range of zero to one. This is contrary to the best relative performance in data envelopment analysis. By using the Malmquist productivity index to measure productivity, the study introduces a precise and comprehensive bankruptcy measure that could be used as an early warning system for bankruptcy assessment.

2. Methodology

Using data envelopment analysis (DEA) can precisely evaluate the performances of many different kinds of decision making units (DMU) such as hospitals, universities, business firms, etc. In addition, DEA has more profit when other approaches have complexity (often-unknown) nature of the relations between the multiple inputs and multiple outputs involved in DMUs Cooper et al. [10].

2.1. Data envelopment analysis

Here, we briefly describe DEA formulation of Fare et al. (1994) Malmquist productivity index can be calculated from the distance below, or other similar functions.

$D\,(X_0, Y_o) = \inf\{\theta / (\theta X_0, Y_o)\} \quad \epsilon PPS$

Efficiency frontier using DEA techniques for decision-making units is specified. For calculate the Malmquist productivity index first describe linear programming by Charnes et al. [9] in the nature of input between two various time.

$$O \,\epsilon Q = \{1, 2\ldots n\}$$

$$\begin{aligned} & D_o^t(X_o^t, Y_o^t) = \text{Min } \theta \\ \text{s.t} \quad & \sum_{j=1}^{n} \lambda_j x_{ij}^t \leq \theta x_{io}^t \qquad i=1, 2\ldots m \qquad (M_1) \\ & \sum_{j=1}^{n} \lambda_j y_{rj}^t \geq y_{ro}^t \qquad r=1, 2\ldots s \\ & \lambda_j \geq 0 \qquad j=1, 2\ldots n \end{aligned}$$

Assuming that there are a set of 'n' DMUs, each operating with 'm' inputs and 's' outputs. All data are assumed non-negative but at least one component of every input and output vector is positive and is given by

$x_{ij}\geq 0 \quad x_j\neq 0 \quad y_{rj}\geq 0 \quad y_j \geq 0 \quad y_j \neq 0 \quad$ for j=1, 2… n

x_{io}^t , y_{ro}^t are the input and output of DMU_o at time (t). The performance $D_o^t(X_o^t, Y_o^t) = \theta^*$ shows how much input DMU_o can be decreased until the output is produced. Instead of t in the above model, if we replace t+1, this DMU_o has Technical efficiency at the time t+1. The $D_o^t(X_o^{t+1}, Y_o^{t+1})$ measures DMU_o distance in time t+1 with frontier t. linear programming is as follows:

$$
\begin{aligned}
& D_o^t(X_o^{t+1}, Y_o^{t+1}) = \text{Min } \theta \\
\text{s.t} \quad & \textstyle\sum_{j=1}^{n} \lambda_j x_{ij}^t \leq \theta\, x_{io}^{t+1} && i=1, 2 \ldots m \\
& \textstyle\sum_{j=1}^{n} \lambda_j y_{rj}^t \geq y_{ro}^{t+1} && r=1, 2 \ldots s \\
& \lambda_j \geq 0 && j=1, 2 \ldots n
\end{aligned}
$$

The same can be measure $D_o^{t+1}(X_o^t, Y_o^t)$ for DMU_o with coordinates t, relative to t+1.

$$
\begin{aligned}
& D_o^{t+1}(X_o^t, Y_o^t) = \text{Min } \theta \\
\text{s.t} \quad & \textstyle\sum_{j=1}^{n} \lambda_j x_{ij}^t \leq \theta x_{io}^{t+1} && i=1, 2 \ldots m \\
& \textstyle\sum_{j=1}^{n} \lambda_j y_{rj}^t \geq y_{ro}^{t+1} && r=1, 2 \ldots s \\
& \lambda_j \geq 0 && j=1, 2 \ldots n
\end{aligned}
$$

Far et al. (1994) assumed that $D_o^{t+1}(X_o^{t+1}, Y_o^{t+1})$ and $D_o^t(X_o^t, Y_o^t)$ when will be efficient if both of them are equal to one. The change in relative performance was defined as $TEC_o = \frac{D_o^{t+1}(X_o^{t+1}, Y_o^{t+1})}{D_o^t(X_o^t, Y_o^t)}$. The rate of technological change between two times t and t +1 as geometric composition is expressed:

$FS_o = [\frac{D_o^t(X_o^{t+1}, Y_o^{t+1})}{D_o^{t+1}(X_o^{t+1}, Y_o^{t+1})} \cdot \frac{D_o^t(X_o^t, Y_o^t)}{D_o^{t+1}(X_o^t, Y_o^t)}]^{1/2}$. Three cases occur:

1) $FS_o > 1$ moving frontier is positive or development observed.

2) $FS_o < 1$ moving frontier is negative.

3) $FS_o = 1$ the frontier does not change.

Malmquist productivity index obtained with the product of changes in efficiency and technology changes:

$M_o = [\frac{D_o^t(X_o^{t+1}, Y_o^{t+1})}{D_o^t(X_o^t, Y_o^t)} \cdot \frac{D_o^{t+1}(X_o^{t+1}, Y_o^{t+1})}{D_o^{t+1}(X_o^t, Y_o^t)}]^{1/2}$. This value is expressed as a convex combination of geometric. Because the shortest weaknesses specify in efficiency, and the shortest change affects in efficiency Malmquist productivity index.

1) $M_o > 1$ so Productivity will be increased.

2) $M_o < 1$ so Productivity will be decreased.

3) $M_o = 1$ so productivity is constant.

2.2. Directional distance formulation of DEA with Malmquist index:

Directional vector for directional distance function evaluated is very important. Using the VRS DEA formulation for the directional distance function developed by Chambers et al. [8] describe formulation Malmquist directional distance function. To arbitrary, choose two of the limitations of linear programming problems, (2b) and (2c):

$$\begin{aligned} &\text{Max} \quad \beta \\ \text{s.t} \quad &\textstyle\sum_{j=1}^{n} \lambda_j y_{rj}^{t} - \beta g_{y_{ro}} \ge y_{ro}^{t} && (2b) \\ &\textstyle\sum_{j=1}^{n} \lambda_j x_{ij}^{t} + \beta g_{x_{io}} \le x_{io}^{t} && (2c) \\ &\textstyle\sum_{j=1}^{n} \lambda_j = 1 \\ &\lambda_j \ge 0 \quad j = 1, 2, \dots, n \\ &g_{y_{ro}} g_{x_{io}} \ge 0 \qquad \beta \ge 0 \end{aligned}$$

$$\begin{aligned} &\text{Max} \quad \beta \\ \text{s.t} \quad &\textstyle\sum_{j=1}^{n} \lambda_j y_{rj}^{t+1} - \beta g_{y_{ro}} \ge y_{ro}^{t+1} && (M_2) \\ &\textstyle\sum_{j=1}^{n} \lambda_j x_{ij}^{t+1} + \beta g_{x_{io}} \le x_{io}^{t+1} \\ &\textstyle\sum_{j=1}^{n} \lambda_j = 1 \\ &\lambda_j \ge 0 \quad j = 1,2, \dots, n \\ &g_{y_{ro}} g_{x_{io}} \ge 0 \quad \beta \ge 0 \end{aligned}$$

$$\begin{aligned} &\text{Max} \quad \beta \\ \text{s.t} \quad &\textstyle\sum_{j=1}^{n} \lambda_j y_{rj}^{t+1} - \beta g_{y_{ro}} \ge y_{ro}^{t} \\ &\textstyle\sum_{j=1}^{n} \lambda_j x_{ij}^{t+1} + \beta g_{x_{io}} \le x_{io}^{t} \\ &\textstyle\sum_{j=1}^{n} \lambda_j = 1 \\ &\lambda_j \ge 0 \quad j = 1,2 \dots, n \\ &g_{y_{ro}} g_{x_{io}} \ge 0 \quad \beta \ge 0 \end{aligned}$$

$$
\begin{aligned}
& \text{Max} \quad \beta \\
\text{s.t} \quad & \sum_{j=1}^{n} \lambda_j y_{rj}^{t} - \beta g_{y_{ro}} \geq y_{ro}^{t+1} \\
& \sum_{j=1}^{n} \lambda_j x_{ij}^{t} + \beta g_{x_{io}} \leq x_{io}^{t+1} \\
& \sum_{j=1}^{n} \lambda_j = 1 \\
& \lambda_j \geq 0 \quad j = 1,2,\dots,n \\
& g_{y_{ro}} g_{x_{io}} \geq 0 \quad \beta \geq 0
\end{aligned}
$$

The factor β is measure of technical inefficiency and its efficiency is (β-1). At the time, this model being to increase output and decrease input. Input- oriented model and output-oriented model can be obtained, if setting directional vector g_{y_o} or g_{x_o} equal to zero respectively in Eqs. (2b) and (2c)

2.3. A modified directional distance formulation of DEA to assess bankruptcy with Malmquist index

One of the method bankruptcy assessment developed using by the additive DEA. Premachandra et al. [20] by using DEA additive model take a set of financial ratios as output variables, so lower value of a ratio indicates better progress of the firm and, another set of financial ratios as input variables that higher value of these ratios indicate better progress of the firm. The method has disadvantages, such as, input and output variables are in reverse order to conventional DEA assessment and, this would generally lead to different results and while estimating efficient input–output levels for non-default units, they do not provide any measure of bankruptcy and, the results are units dependent.

Therefore, this paper develop a model to measure worst relative efficiency in the range of zero to one that locate worst performance of DMUs and determine an inefficiency frontier. This approach is contrary to the best relative efficiencies of conventional DEA models and directional distance formulation of DEA. In this model have increase inputs and decrease outputs. White [31], believe that competition move to markets toward a state of equilibrium, thus the firms will be to remain which they with average costs production little. While inefficient firms using by an old technologies and producing in excess supply are eliminated. In this study, the inputs will be increase and the outputs are decrease therefore this is unfavorable situation, and bankruptcy score should be assessed in the direction of anti-ideal DMU. Wang and Yang [29] called this DMU as anti-ideal DMU. Therefore denote inputs and outputs of this anti-ideal DMU as $(x_i^{max}),\dots i = 1,2\dots m$, $(y_r^{min}),\dots r = 1,2\dots s$ are the maximum inputs and minimum outputs among all the DMUs, i.e.

$x_i^{max}=\max(x_{ij})\dots \quad i=1, 2\dots m$ (E1)

$y_r^{min} = \min(y_{rj})\dots \quad r=1,2,\dots,s \quad j= 1,2,\dots,n$

Let DMU_o be one of the DMUs, this is to be evaluated for bankruptcy assessment. The corresponding input-output bundle (X_o,Y_o) and a directional input and output bundle (g_x,g_y) are used in development of bankruptcy assessment model through directional distance formulation. The initial steps in the development are (i) the construction of the feasibility set and (ii) the estimation of the maximum feasible contraction of the outputs and/or expansion of the inputs of the DMU within feasibility set. The feasibility set T is defined such that

If $(x_j, y_j) \in T$ for $j = 1,2, \dots, n$

$(x,y) \in T$ $(u,v) \in R^+$, $(x', -y') \le (x,-y)$ $x' = x - u$ $y' = y+v$ $(x', y') \in T$

if $(x,y) \in T$ $(x', y') \in T$ $(x^*, y^*) = \lambda(x,y) + (1-\lambda)(x', y')$ ۰≤η≤۱ $\rightarrow (x^*, y^*) \in T$

if $(x,y) \in T$ $\rightarrow$ $(\lambda x, \lambda y) \in T$ $\forall \lambda >$۰

Therefore, the possibility of the following occurs:

$T_{VRS}^{BR} = \{(x, y): x \le \sum_{j=1}^{n} \lambda_j x_{ij} , y \ge \sum_{j=1}^{n} \lambda_j y_{rj}; \sum_{j=1}^{n} \lambda_j = 1; \lambda_j \ge 1\}$(E2)

So for estimate the bankruptcy measure of DMU_o therefore modify an improved efficiency measure through directional distance formulation of DEA–minimization model developed by Shetty and pakkala [23] for the feasibility set defined in (E2) with Malmquist index:

$$\text{Min } D(x_o^t, y_o^t) = \delta = \frac{1-(\sum_{i=1}^{m}\frac{\beta_{io}^-}{m})}{1+(\sum_{r=1}^{s}\frac{\beta_{ro}^+}{s})}$$

$$\begin{aligned}
\text{s.t } & \sum_{j=1}^{n} \lambda_j y_{rj}^t + \beta_{ro}^+ g_y \le y_{ro}^t \\
& \sum_{j=1}^{n} \lambda_j x_{ij}^t - \beta_{io}^- g_x \ge x_{io}^t \qquad (M_3) \\
& \sum_{j=1}^{n} \lambda_j = 1 \\
& \beta_{ro}^+, \beta_{io}^+, g_y, g_x, \ge 0 \\
& \lambda_j \ge 0 \quad , \quad j=1,2,\dots,n \quad , \quad i=1,2,\dots,m \quad , \quad r=1,2,\dots,s
\end{aligned}$$

$$\text{Min } D(x_o^{t+1}, y_o^{t+1}) = \delta = \frac{1-(\sum_{i=1}^{m}\frac{\beta_{io}^-}{m})}{1+(\sum_{r=1}^{s}\frac{\beta_{ro}^+}{s})}$$

$$\begin{aligned}
\text{s.t } & \sum_{j=1}^{n} \lambda_j y_{rj}^{t+1} + \beta_{ro}^+ g_y \le y_{ro}^t \\
& \sum_{j=1}^{n} \lambda_j x_{ij}^{t+1} - \beta_{io}^- g_x \ge x_{io}^t \qquad (M_3) \\
& \sum_{j=1}^{n} \lambda_j = 1 \\
& \beta_{ro}^+, \beta_{io}^+, g_y, g_x, \ge 0 \\
& \lambda_j \ge 0 \quad , \quad j=1,2,\dots,n \quad , \quad i=1,2,\dots,m \quad , \quad r=1,2,\dots,s
\end{aligned}$$

$$\text{Min D}\ (x_o^{t+1}, y_o^{t+1}) = \delta = \frac{1-(\sum_{i=1}^{m}\frac{\beta_{io}^{-}}{m})}{1+(\sum_{r=1}^{s}\frac{\beta_{ro}^{+}}{s})} \qquad (3a)$$

s.t $\sum_{j=1}^{n}\lambda_j y_{rj}^{t+1} + \beta_{ro}^{+} g_y \leq y_{ro}^{t+1}$

$\sum_{j=1}^{n}\lambda_j x_{ij}^{t+1} - \beta_{io}^{-} g_x \geq x_{io}^{t+1}$ (M_3)

$\sum_{j=1}^{n}\lambda_j = 1$

$\beta_{ro}^{+}, \beta_{io}^{+}, g_y, g_x, \geq 0$

$\lambda_j \geq 0$, j=1,2 ,…,n , i=1,2 ,…,m , r=1,2 ,…,s

$$\text{Min D}\ (x_o^{t}, y_o^{t}) = \boldsymbol{\delta} = \frac{1-(\sum_{i=1}^{m}\frac{\beta_{io}^{-}}{m})}{1+(\sum_{r=1}^{s}\frac{\beta_{ro}^{+}}{s})}$$

s.t $\sum_{j=1}^{n}\lambda_j y_{rj}^{t} + \beta_{ro}^{+} g_y \leq y_{ro}^{t+1}$

$\sum_{j=1}^{n}\lambda_j x_{ij}^{t} - \beta_{io}^{-} g_x \geq x_{io}^{t+1}$ (M_3)

$\sum_{j=1}^{n}\lambda_j = 1$

$\beta_{ro}^{+}, \beta_{io}^{+}, g_y, g_x, \geq 0$ (3f)

$\lambda_j \geq 0$, j=1,2 ,…,n , i=1,2 ,…,m , r=1,2 ,…,s

This model is non-radial so that at the time evaluating bankruptcy score indicates. $\beta_{ro}^{+} g_y$ and $\beta_{io}^{-} g_x$ are the distance between the bankruptcy frontier to the inputs and outputs, respectively of the evaluating DMU_o . Let, a DMU_o is performing well then its $\beta_{ro}^{+} g_y$, $\beta_{io}^{-} g_x$ are larger relative to other DMUs and it shows that the DMUo is away from the bankruptcy frontier .This model is resistance of expanding $\beta_{io}^{-} g_x$ and reducing $\beta_{ro}^{-} g_y$. $\sum_{i=1}^{m}\frac{\beta_{io}^{-}}{m}$ is the possible mean expansion of inputs and $1-(\sum_{i=1}^{m}\frac{\beta_{io}^{-}}{m})$ is the bankruptcy score of inputs. β_{ro}^{+} in denominator evaluates the possible reduction of outputs 'r'and $\sum_{r=1}^{s}\frac{\beta_{ro}^{+}}{s}$ is the possible mean reduction of outputs $1+(\sum_{r=1}^{s}\frac{\beta_{ro}^{+}}{s})$ is the output bankruptcy score. β_{io}^{-} is the relative possible expansion rate in the input 'i', β_{ro}^{+} is evaluates the possible reduction of outputs 'r'**.** Therefore, δ is the product of input and output bankruptcy scores. Bankruptcy occurs if $1=\delta^{*}$ so $\beta_{io}^{-}=0$, $\beta_{ro}^{+}=0$. D_o^t (x_o^{t+1}, y_o^{t+1}) is score bankruptcy at the time t+1and relative to t. When all the data are positive, usual choice of directional vectors (g_x , g_y) are observed input and output. When some data are negative, employing observed input and output would violate the constraint (3f). In order to overcome this limitation, we define directional vectors through anti-ideal DMU as defined in (E1)

$g_x = \max\{x_{ij}\} - x_{io}$ i=1, 2… m (E3)

$g_y = y_{ro} - \min\{y_{rj}\}$ $r = 1,2,\dots,s$ (E4)

The directional vectors defined in above are non-negative and satisfy the restriction in (3f). They are possible worsening of inputs and outputs because, expansion of inputs and or reductions of outputs are undesirable activities that lead to deterioration in the DMUs performances. This is contrary to the Silva Portela et al. (2004) range of possible improvement. Using Charnes and Cooper's (1962) transformation to fractional programming of (M_3), this model can be converted into following linear programming as in Tone. (2001). multiply a scalar variable't' greater than zero with both denominator and numerator of (3a). This does not cause any change in the value of δ. Further adjust't' so that the denominator becomes one and, move the denominator term to constraints.

$$\text{Min } \kappa = t - \left(\sum_{i=1}^{m} \frac{\beta_{io}^{-}}{m}\right)$$

$$\begin{aligned} \text{s.t} \quad & t + \left(\sum_{r=1}^{s} \frac{\beta_{ro}^{+}}{s}\right) = 1 \\ & \sum_{j=1}^{n} \lambda_j y_{rj}^{t} + \beta_{ro}^{+} g_y \le t y_{ro}^{t} \\ & \sum_{j=1}^{n} \eta_j x_{ij}^{t} - \beta_{io}^{-} g_x \ge t x_{io}^{t} \qquad (M_4) \\ & \sum_{j=1}^{n} \lambda_j = t \\ & \beta_{ro}^{+}, \beta_{io}^{+}, g_y, g_x, \ge 0 \\ & \lambda_j \ge 0, \quad j=1,2,\ldots,n, \quad i=1,2,\ldots,m, \quad r=1,2,\ldots,s \end{aligned}$$

Let the optimal solution for (M_4) be $(\beta_{ro}^{+*}\ \beta_{io}^{-*}, \lambda^*, t^*, \kappa^*)$, we have the optimal solution of (M_3) defined as Based on the optimal solution$(\delta^*=\kappa^*, A^* = \lambda^*/t^*, \beta_{io}^{-*} = \beta_{io}^{-*}/t^*, \beta_{ro}^{+*} = \beta_{ro}^{+*}/t^*)$ .The value of bankruptcy score lies between zero and one. Similarly, we can evaluate the case should be taken at other time

2.4. Interpretation of bankruptcy score

The model (M_3) has least feasible solution $\beta_{io}^{-} = 0$, $\beta_{io}^{+} = 0, \lambda_0 = 0, \lambda_0 = 1, (j \neq 0)$ and the optimal solution denoted with $\delta^*$is not greater than one. The constraints of (M_3) should be the activity ($g_y\ \beta_{ro}^{+}$ - y_{ro} , $\beta_{io}^{-} g_x +\ x_{io}$) belong to productivity possibility set while the objective seeks to maximize β_{ro}^{+} وβ_{io}^{-} that expands the input vector x_{io} non-radial to $\beta_{io}^{-} g_x +\ x_{io}$. In addition, reduces the output non-radial to $g_y \beta_{ro}^{+}$-y_{ro} Therefore,(β_{ro}^{+}- y_{ro},$\beta_{io}^{-} g_x x_{io}$) outperforms $(\sum_{j=1}^{n} \lambda_j x_{ij}, \sum_{j=1}^{n} \lambda_j y_{rj})$ .When optimal solution is less than one. The output constraints hold the equality i.e.

$$y_{ro} = \sum_{j=1}^{n} \lambda_j^* y_{rj} + \beta_{ro}^{+*} g_y \qquad \sum_{j=1}^{n} \lambda_j^* y_{rj} = y_{ro}^*$$

$$\frac{y_{ro} - y_{ro}^*}{y_{ro} - \min\{y_{rj}\}} = y_{ro} - \beta_{ro}^{+*} g_y = y_{ro}^*$$

y_{ro}^* is the threshold level for optimal reduction of outputs. This is that β_{ro}^{+*} is equal to ratio of optimal reduction of output. Similarly for the input constraints

$$x_{i\,o} = \sum_{j=1}^{n} \lambda_j^* x_{ij} - \beta_{io}^{-*} g_x \qquad \sum_{j=1}^{n} \lambda_j^* x_{ij} = x_{io}^*$$

$$\frac{x_{io}^* - x_{i\,o}}{\max\{x_{ij}\} - x_{i\,o}} = \quad x_{i\,o} + \beta_{io}^{-*} g_x \quad = x_{io}^*$$

2.5. An illustrative example

Five companies agricultural bankrupt using by Malmquist productivity index productivity score calculated. Data in this problem selected from a sample of Janova et al. [17].

Tabel 1

		DMU1	DMU2	DMU3	DMU4	DMU5
Year 2007	In1	1.113	1.117	1.112	1.113	1.115
	In2	0.510	0.509	0.514	0.509	0.513
	In3	0.513	0.513	0.514	0.511	0.514
	O1	0.481	0.482	0.483	0.479	0.483
	O2	0.076	0.077	0.078	0.077	0.078
	O3	-0.073	-0.072	-0.071	-0.071	-0.075
	O4	-0.104	-0.103	-0.102	-0.102	-0.106
	O5	-0.153	-0.152	-0.151	-0.151	-0.155
	O6	-0.098	-0.097	-0.096	-0.096	-0.100
	O7	-0.072	-0.071	-0.070	-0.074	-0.070

Tabel 2

		DMU1	DMU2	DMU3	DMU4	DMU5
Year 2008	In1	1.112	1.116	1.111	1.112	1.114
	In2	0.509	0.508	0.513	0.508	0.512
	In3	0.512	0.512	0.513	0.510	0.513
	O1	0.481	0.482	0.483	0.479	0.483
	O2	0.076	0.077	0.078	0.077	0.078
	O3	-0.073	-0.072	-0.071	-0.071	-0.075
	O4	-0.104	-0.103	-0.102	-0.102	-0.106
	O5	-0.153	-0.152	-0.151	-0.151	-0.155
	O6	-0.098	-0.097	-0.096	-0.096	-0.100
	O7	-0.072	-0.071	-0.070	-0.074	-0.070

Tabel 3

		DMU1	DMU2	DMU3	DMU4	DMU5
Year 2009	In1	1.114	1.115	1.113	1.114	1.116
	In2	0.511	0.510	0.514	0.510	0.513
	In3	0.513	0.514	0.515	0.511	0.515
	O1	0.481	0.482	0.483	0.479	0.483
	O2	0.076	0.077	0.078	0.077	0.078
	O3	-0.073	-0.072	-0.071	-0.071	-0.075
	O4	-0.104	-0.103	-0.102	-0.102	-0.106
	O5	-0.153	-0.152	-0.151	-0.151	-0.155
	O6	-0.098	-0.097	-0.096	-0.096	-0.100
	O7	-0.072	-0.071	-0.070	-0.074	-0.070

Tabel 4

		DMU1	DMU2	DMU3	DMU4	DMU5
Year 2010	In1	1.115	1.116	1.114	1.115	1.113
	In2	0.512	0.511	0.515	0.511	0.517
	In3	0.514	0.515	0.516	0.512	0.517
	O1	0.482	0.483	0.484	0.480	0.486
	O2	0.077	0.078	0.079	0.078	0.081
	O3	-0.072	-0.073	-0.070	-0.070	-0.068
	O4	-0.103	-0.102	-0.101	-0.101	-0.099
	O5	-0.152	-0.151	-0.150	-0.150	-0.150
	O6	-0.097	-0.096	-0.095	-0.095	-0.093
	O7	-0.071	-0.070	-0.096	-0.073	-0.067

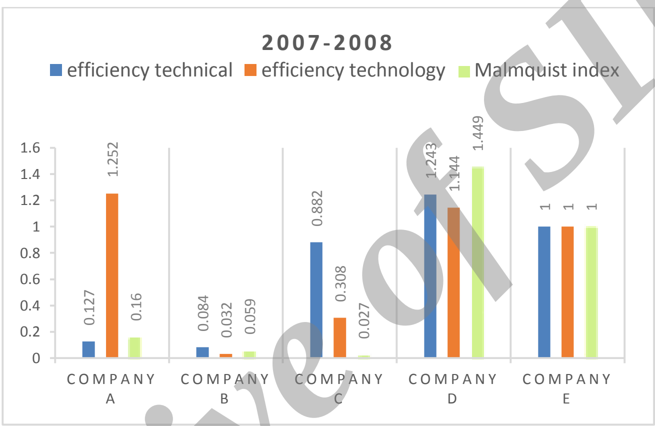

Figure 1

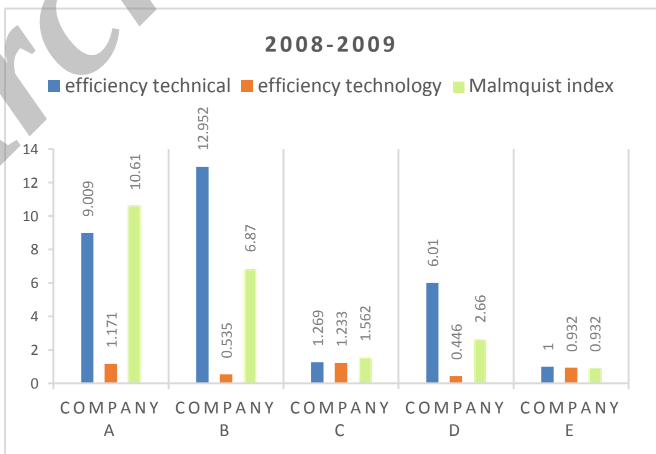

Figure 2

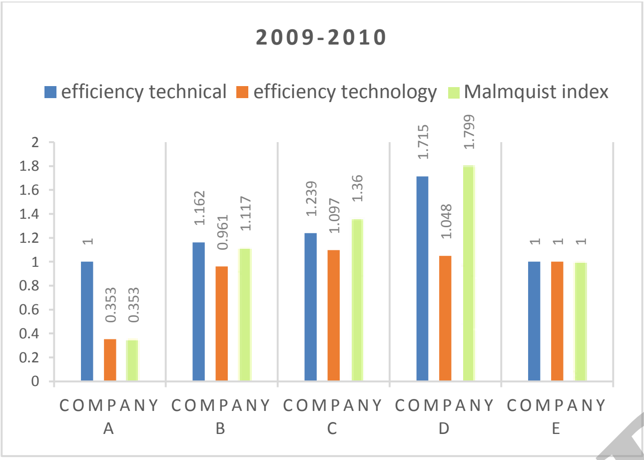

Figure 3

2.6. Results and Analysis

We apply the model (M_3) for the selected firms. The model shows that when productivity score in each period is greater than one, thus the firm to be bankrupt. The first calculating efficiency score between two various time. This efficiency score is always between zero and one. In contrast, changes technology shows moving efficiency frontier. If technical efficiency and technology efficiency more than one so malmquist index has decrease. In year 2009-2008 both the technical efficiency and technology efficiency in company A is more than one, so the company is to be bankrupt. The company A has productivity score greater than one so the company does not have improvement. In year 2009-2008, the company E has the best productivity score. When the technical efficiency and the efficiency technology are less than one, a productivity index will be progressed. Company B year 2008-2007 the technical efficiencies is 0/084, therefore the company B have many distance of to be bankrupt. In 2010-2009 the company E both efficiency technical and efficiency technology is one and shows that the productivity score is one.

3. Input and output variables

Ten financial ratios are selected out of which three are input variables and seven are output .These financial ratios are selected based on previous literature on bankruptcy assessment like Serrano-Cinca [22] ,Canbas et al. [6] ,Alfaro et al. [3] and premachandra et al. [20] are select. Variables the ratios used are Total Debt to Total Assets (TDTA), Current Liabilities to Total Assets (CLTA) and Total Liabilities to Total Assets (TLTA) as the three input variables. Larger values for TDTA, CLTA and TLTA would result in firms coming on the bankruptcy frontier. Return to Total Assets (RTTA), Profit before Interest and Taxes to Total Assets (PBIT), Cash Flow to Total Assets (CFTA), Current Assets to Total Assets (CATA), Working Capital to Total Assets (WCTA), Market Value of Equity to Book Value of Common Equity (MVCE), and earnings before Interest and Taxes to Total Assets (EBITTA) are the seven output variables. A high value of output variables would tend to result in a value nearer to zero for the objective function of the bankruptcy model. Hence, firms with high values for these output variables may not appear on the bankruptcy frontier. The detailed descriptions of input and output variables to assess the bankruptcy are given below.

Total Debt to Total Assets (TDTA): This is handspike scale and most important is application in risk index and if this scales being a large, thus company would be bankrupt.

Current Liabilities to Total Assets (CLTA): if a firm have high CLTA ratio therefore a firm difficulty for to meet short-term debt obligations and the company being in threshold bankrupt.

Total Liabilities to Total Assets (TLTA): A firm with had a high TLTA ratio would being to difficulty to meet short term and long-term debt obligations. In this time the company would have difficulty in running its day-to-day operations.
Return to Total Assets (RTTA): This ratio is the indicator of the profitability of the firm. This means how productively the total assets of the firm are profit in production it returns.

Profit before Interest and Taxes to Total Assets (PBITTA): This ratio, like the (RTTA), is the indicator of the profitability of the firm. This means how productively the total assets of the firm are important for generating the earnings before interest and taxes. Than others with decrease in these ratios. Cash Flow to Total Assets

(CFTA): This ratio, like the RTTA and PBITTA, is the indicator of the profitability of the firm. This means how productively the total assets of the firm are important in generating the earnings in the form of cash flows

Current Assets to Total Assets (CATA): This ratio indicates the proportion of the current assets to total assets. This means how productively the total assets of the firm are important production the liquid assets. If in the company of current assets is high also shows that the firm has been able to use the total assets for production the current assets in the form of inventory, debtors, cash and bank.

Working Capital to Total Assets (WCTA): This ratio is the indicator of the ability of the firm for production the working capital. This ratio considers the difference between the current assets and current liabilities in the numerator this means how productively the total assets of the firm are employed in generating the net working capital.

Market Value of Equity to Book Value of Common Equity (MVBV): We use a market-based growth for measures an output variable. The target is to maximize the shareholders' value if in the company this value being a low thus the company would be bankrupt.

Earnings before Interest and Taxes to Total Assets (EBITTA): This ratio is the indicator of the profitability of the firm. This means how productively the total assets of the firm are employed in generating the earnings before interest and taxes.

4. Conclusions

The objective of this study is to develop a precise and a comprehensive with malmquist productivity index for bankruptcy prediction that facilitates build an early warning system. This paper uses a directional distance function formulation of DEA using by malmquist index to assess the bankruptcy. The directional distance function measures the efficiencies of DMUs, which measures the best performances of DMUs and determine an efficiency frontier, we develop a model to measure worst relative efficiency within the interval of 'zero to one' and locate worst performance of DMUs and determine an inefficiency frontier and we are doing this action for various period times thus we calculated malmquist productivity index. The productivity scores of upper than one, the company to be bankrupt and productivity are decrease and if productivity being less than one thus productivity increase. If productivity is equal to one thus productivity has no change. Our model incorporates possible decrease of outputs and increase of inputs in the direction of anti-ideal DMU, one, which takes maximum inputs, and minimum outputs, which leads to bankruptcy of the firms. The worst performing firms are not necessarily declared as bankrupt. The scores indicate that these firms are prone financial distress and may experience problems of meeting the obligations to creditors and others. This study could be used for measure total productivity between various times in the firms and other institutions.

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A Modified Directional Distance Formulation of DEA with Malmquist Index to Assess Bankruptcy

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فرمول فاصله جهت داراصلاح شده تحلیل پوششی داده ها با کمک شاخص مالمکوییست برای ارزیابی ورشکستگی

چکیده

ورشکستگی از نقطه نظرزمان و تاریخ بسیاررایج می باشد و در نتیجه می تواند مانعی برای چشم انداز آینده باشد. با استفاده از تحلیل پوششی داده ها و شاخص مالمکوییست می توان ارزیابی دقیقی از واحدهای تصمیم گیرنده مانند بیمارستان ها، دانشگاه ها، شرکت های تجاری و غیره به دست آورد. در این مقاله با استفاده از شاخص بهره وری مالمکوییست و فرمول فاصله جهت دارتحلیل پوششی داده ها به ارزیابی ورشکستگی می پردازیم. این روش یکی از کلی ترین مدل های فاصله جهت دار غیر شعاعی است. متد فوق به بررسی اندازه گیری بدترین کارایی نسبی در بازه صفر تا یک در زمان ها و دوره های مختلف می پردازد. مدل در واقع بدترین واحد های تصمیم گیرنده را با تعیین مرز ناکارآمد به طور همزمان با کاهش خروجی و افزایش ورودی تعیین می سازد. با استفاده از شاخص مالمکوییست بهره وری درسال های مختلف مورد اندازه گیری قرار می گیرد.این مطالعه به اندازه گیری ورشکستگی به طور دقیق و جامع می پردازد که می تواند معرفی کننده یک سیستم هشدار دهنده برای ارزیابی ورشکستگی باشد.

کلمات کلیدی: تحلیل پوششی داده ها، تابع فاصله جهت دار ، ورشکستگی، بهره وری، مالمکوییست