



Revise Approach to Measuring Congestion Based on the Comparison of Inputs

AbbasAli. Noura^a, Elnaz. Hosseini^{b*}

(a) *Faculty Of Mathematics, Sistan and baluchestan University, Danshgah street, Zahedan, IRAN*

(b) *Faculty Of Mathematics, Sistan and baluchestan University, Danshgah street, Zahedan, IRAN*

Received 28 September 2014, Revised 08 November 2014, Accepted 07 February 2015

Abstract

the method for measuring the congestion of Noura et al. [A.A. Noura, F. Hosseinzadeh Lotfi, G.R. Jahanshahloo, S. Fanati Rashidi, R.P. Barnett, A new method for measuring congestion in data envelopment analysis, *Socio-Economic Planning Sciences*, 44 (2010) 240-246], there is no problem for congestion detection in the case of one input and one output but in higher space is not able to detect congestion of some units. we offer modification of the method of measuring the congestion of Noura et al. The proposed method ability congestion units go up and this method detect all congestion units.

Keywords: Data envelopment analysis; Congestion; Efficiency; Decision Making Unit.

1. Introduction

The focus of this paper is on the problem of congestion, which refers to a situation where the use of a particular input has increased by so much that output actually falls. Congestion can be viewed as an extreme form of technical inefficiency and, as such, can be regarded as a potentially serious practical problem. Fare and Svensson Proportional to the congestion by varying rule were defined and developed. [6]. Fare and Grosskopf to determine the functional role of input in the proposed

* Corresponding author: eh_hoseni@yahoo.com.

congestion.[5,4]. Then Brocket et al [2] and cooper et al [3] Have developed a new method based on DEA to obtain input congestion. Other methods proposed by other scientists to evaluate congestion but In this paper, we pay to a method that is presented by Noura et al. the method for measuring the congestion of Noura et al and his colleagues studied And we find that this approach ,there is no problem for congestion detection In the case of one input and one output but in higher space is not able to detect congestion of some units. we offer modification of the method of measuring the congestion of Noura et al. The proposed method ability congestion units go up and this method detect all congestion units. Sections of this paper is as follows: In section 2 we expressed modification of the method of measuring the congestion of Noura et al. In section 3 By providing examples We describe the proposed method And compare the results with other methods of measuring congestion. And the end results are expressed.

2. Revise approach

Noura et al. [5] have proposed a method for measuring the congestion at which they use. Comparing the measured densities of the inputs. We found that this approach, there is no problem for congestion detection In the case of one input and one output but in higher space is not able to detect congestion of some units.[8]

We propose the following method for determining the congestion.

Suppose we have n DMUs with m inputs and s outputs, and that the vectors $x_j = (x_{1j}, \dots, x_{mj})^T$ and $y_j = (y_{1j}, \dots, y_{sj})^T$ denote the input and output values of DMU_j (j = 1, ..., n) respectively. First, we solve the output-oriented BCC (Banker, Charnes, Cooper) model (1), which assumes variable returns to scale (VRS), in order to obtain the efficiency of each DMU.

$$\begin{aligned} \phi_o^* = \text{Max } \phi_o + \varepsilon \left(\sum_{r=1}^s s_{ro}^+ + \sum_{i=1}^m s_{io}^- \right) \\ \text{s.t.} \\ \sum_{j=1}^n x_{ij} \lambda_j + s_{io}^- = x_{io}, \quad i = 1, \dots, m \\ \sum_{j=1}^n y_{rj} \lambda_j + s_{ro}^+ = \phi_o y_{ro}, \quad r = 1, \dots, s \\ \sum_{j=1}^n \lambda_j = 1 \\ (\lambda_j, s_{ro}^+, s_{io}^-) \geq 0, \quad j = 1, \dots, n, \quad r = 1, \dots, s, \quad i = 1, \dots, m \end{aligned} \tag{1}$$

In (1), $\varepsilon > 0$ is a Non-Archimedean element smaller than any positive real number. we solve Model (1), above, for each DMU_j (j = 1, ..., n) and obtain the optimal solution: $(\phi_o^*, \lambda_o^*, s_{ro}^{+*}, s_{io}^{-*})$.

Denoting the ϕ_o^* corresponding to DMU_j by ϕ_j^* we define set E as follows:

$$E = \{j | \phi_j^* = 1, \sum_{r=1}^s s_{rj}^* = 0\}$$

As the congestion measurement method based on the comparison of inputs, Highest value in each entry for each component are calculated.

We show Such input with x_i^* ($i = 1, \dots, m$).

Step one: A) if $\phi_o^* > 1$ (or $\sum_{r=1}^s s_{ro}^* > 0$) and $x_{io} > x_i^*$, DMU_o is congestion. and the congestion of the input (i) against is $s_i^{\hat{c}} = x_{io} - x_i^*$.

B) if $\phi_o^* > 1$ (or $\sum_{r=1}^s s_{ro}^* > 0$) and $x_{io} = x_i^*$ = $x_{io} - s_{io}^-$, DMU_o on input i is not congestion.

C) if $\phi_o^* > 1$ (or $\sum_{r=1}^s s_{ro}^* > 0$) and $x_{io} \leq x_i^* \neq x_{io} - s_{io}^-$, go to the second step.

Step two: find:

$$x_{ij} \leq x_{it} < x_i^*, x_{io} - s_{io}^- \leq x_{it} \quad \forall j, t \in E \quad (2)$$

If there was not in equations (2) applies, DMU_o on input i is not congestion.put:

$$x_{io} - s_{io}^- = x_{it}$$

Put $x_{it} = x_i^*$ and go to the first step.

Total $s_i^{\hat{c}}$ is the congestion value of DMU_o.

The proposed method for every DMU_o ($o \notin E$)

One $DMU_o^* = (x_{1o}^*, \dots, x_{mo}^*, \phi^* y_{1o} + s_{1o}^+, \dots, \phi^* y_{so} + s_{so}^+)$ Determined.

Theorem 1. If $DMU_o^* = (x_{1o}^*, \dots, x_{mo}^*, \phi^* y_{1o} + s_{1o}^+, \dots, \phi^* y_{so} + s_{so}^+)$ Then $DMU_o^* \in PPS_{TV}$.

Proof. The method of determining x_{io}^* And that s_{io}^- The maximum amount of assistance, we have:

$$x_{io}^* \geq x_{io} - s_{io}^-$$

Further, since $(x_o - s_o^-, \phi^* y_o + s_o^+) \in PPS_{TV}$ and, given the production

possibility principle [1], Output can be generated by input greater than $x_o - s_o^-$, so

$$DMU_o^* =$$

$$(x_{1o}^*, \dots, x_{mo}^*, \phi^* y_{1o} + s_{1o}^+, \dots, \phi^* y_{so} + s_{so}^+) \in PPS_{TV}.$$

Consider the following model:

$$\beta_o^* = \text{Max } \beta_o$$

s.t.

$$\sum_{j=1}^n x_{ij} \lambda_j = x_{io}, \quad i = 1, \dots, m$$

$$\sum_{j=1}^n y_{rj} \lambda_j - \beta_o y_{ro} \geq 0 \quad r = 1, \dots, s$$

$$\sum_{j=1}^n \lambda_j = 1 \quad (2)$$

$$\lambda_j \geq 0, \quad j = 1, \dots, n$$

$$(\lambda_j, s_{ro}^+, s_{io}^-) \geq 0, \quad j = 1, \dots, n, \quad r = 1, \dots, s, \quad i = 1, \dots, m.$$

Theorem 2: if $x_{io} > x_{io}^*$, then $\phi_o^* > \beta_o^*$ and FGL method based on the strong and weak access DMU_o is congestion.

Proof. Note that the first inequality constraint in model (1) with equality in the model (2) is replaced. Therefore, the auxiliary input is not possible. So we have: $\phi_o^* \geq \beta_o^*$ Suppose $x_{io} > x_{io}^*$ as $x_{io}^* \geq x_{io} - s_{io}^-$ so $x_{io} > x_{io} - s_{io}^-$ so $\phi_o^* > \beta_o^*$ and FGL method based on the strong and weak access DMU_o is congestion.

3. Numerical Examples

Example 1

Six decisions With two inputs and one output is shown in Figure 1. The output units A,B,C,D,G Is equal to one And the output value of R is equal to 10.

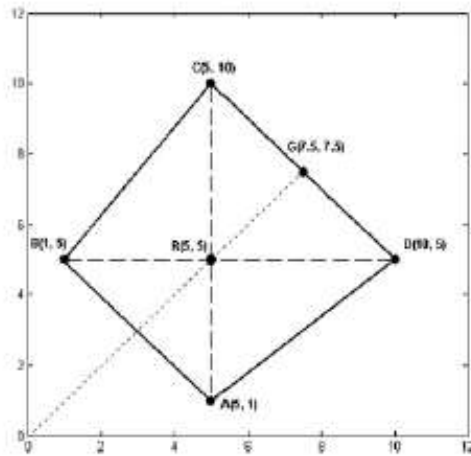


Figure 1

The measurement method is based on comparing the input congestion (Noura et al.) $E=\{A,B,R\}$ And congestion is calculated as follows: $\emptyset_C^*, \emptyset_D^*, \emptyset_G^* > 1$

$$\left\{ \begin{array}{l} \mathbf{x}_C = (5, 10), \mathbf{x}_C^* = (5, 5) \\ s_1^c = 0, s_2^c = 5 \rightarrow s_c^c = 5 \\ \mathbf{x}_D = (10, 5), \mathbf{x}_D^* = (5, 5) \\ s_1^c = 5, s_2^c = 0 \rightarrow s_D^c = 5 \\ \mathbf{x}_G = (7.5, 7.5), \mathbf{x}_G^* = (5, 5) \\ s_1^c = 2.5, s_2^c = 2.5 \rightarrow s_G^c = 5 \end{array} \right.$$

So by Noura et al, the DMU G, D, C are subject to condensation.

In this example we solve the revise approach. (Proposed in Section 2.)

With this method $E=\{A,B,R\}$. we have:

$$\emptyset_C^*, \emptyset_D^*, \emptyset_G^* > 1,$$

$$\left\{ \begin{array}{l} \mathbf{x}_C = (5,10), \mathbf{x}_C^* = (5,5) \\ s_1^c = 0, s_2^c = 5 \rightarrow s_c^c = 5 \\ \mathbf{x}_D = (10,5), \mathbf{x}_D^* = (5,5) \\ s_1^c = 5, s_2^c = 0 \rightarrow s_D^c = 5 \\ \mathbf{x}_G = (7.5,7.5), \mathbf{x}_G^* = (5,5) \\ s_1^c = 2.5, s_2^c = 2.5 \rightarrow s_G^c = 5 \end{array} \right.$$

The new method DMUs C,D,G are subject to condensation. As can be seen, for example, the results of both methods are equal.

Example 2

Consider the data in Table 1.

Table 1

DMU	Input	Input	Output	Output
	1	2	1	2
A	1	1	1	1
B	2	2	2	2
C	2	3	2	1
D	3	3	1	1

According to conducted Noura and colleagues $E=\{A,B,C\}$ and $\mathbf{x}^* = (2,3) \geq \mathbf{x}_j \forall j(j \in E)$

We have: $\emptyset_D^* > 1, \mathbf{x}_D = (3,3) \rightarrow s_D^c = 1.$

According to conducted Noura and colleagues Unit D is only subjected to compression.

The new method , $E=\{A,B\}$ and we have:

$$\emptyset_C^*, \emptyset_D^* > 1,$$

$$\left\{ \begin{array}{l} \mathbf{x}_C = (2,3), \mathbf{x}_C^* = (2,2) \\ s_1^c = 0, s_2^c = 1 \rightarrow s_c^c = 1 \\ \mathbf{x}_D = (3,3), \mathbf{x}_D^* = (2,2) \\ s_1^c = 1, s_2^c = 1 \rightarrow s_D^c = 2 \end{array} \right.$$

So with revise approach DMUs C,D are densities.

The results of the methods Tone and Sahoo (10) Sueyoshi (9) Noura et al. and revise approach for Example 2 is shown in Table 2.

Table 2

DMU	Noura et al	Tone and Sahoo	Sueyoshi	revise approach
A	Not congestion	Not congestion	Not congestion	Not congestion
B	Not congestion	Not congestion	Not congestion	Not congestion
C	Not congestion	Not congestion	Wide congestion	Not congestion
D	Not congestion	Not congestion	Wide congestion	Not congestion

As can be seen in the methods Tone and Sahoo, Sueyoshi and revise approach units C,D are densities. Method Noura et al. only Unit D has detect congestion and unit C does not detect the congestion.

Example 3

Consider the data in Table 3.

Table 3

DMU	Input 1	Input 2	Output 1	Output 2	Output 3	Output 4
A	2	2	2	2	2	2
B	2	2	2	3	2	2
C	2	2	2	2	3	2
D	2	2	2	2	2	3
H	1	1	2	2.5	3	2
F	1	3	2	2	2	4
G	2	1	2	2.5	2.25	3

based on the method of Noura et al $E=\{A,B,C,D,H,F,G\}$ and

$$x^* = (2,3) \geq x_j \quad \forall j(j \in E)$$

With this method, none of the units have condensation.

These are examples of revise approach to

solve ,with this method $E=\{B,H,F,G\}$. we

have:

$$\sum_{r=1}^2 s_{rA}^+, \sum_{r=1}^2 s_{rc}^+, \sum_{r=1}^2 s_{rD}^+ > 0,$$

$$\begin{cases} x_A = (2,2), x_A^* = (1,1) s_1^c = 1 \\ \quad , s_2^c = 1 \rightarrow s_A^c = 2 \\ x_C = (2,2), x_C^* = (1,1) s_1^c = 1 \\ \quad , s_2^c = 1 \rightarrow s_C^c = 2 \\ x_D = (2,2), x_D^* = (2,1) s_1^c = 0 \\ \quad , s_2^c = 1 \rightarrow s_C^c = 1 \end{cases}$$

so with the proposed method DMUs A,C,D are densities.

The results of the methods Cooper et al., Tone and saho, Sueyoshi, Noura et al. and revise approach for Example 3 is shown in Table 4.

Table 4

DMU	Cooper et al	Tone and saho	Sueyoshi	Noura et al	revise approach
A	Not congestion	Not congestion	Wide congestion	Not congestion	congestion
B	Not congestion	Not congestion	Not congestion	Not congestion	Not congestion
C	Not congestion	congestion	Wide congestion	Not congestion	congestion
D	Not congestion	congestion	Wide congestion	Not congestion	congestion
H	Not congestion	Not congestion	Not congestion	Not congestion	Not congestion
F	Not congestion	Not congestion	Not congestion	Not congestion	Not congestion
G	Not congestion	Not congestion	Not congestion	Not congestion	Not congestion

With Noura et al. and Cooper et al. method No one does compression. Tone and saho method has multiple optimal solutions.

Congestion determination in tone and sahuo method depend on which option to be choosen. with Sueyoshi method and revise approach units A,C,D are subject to condensation. Congestion determination in Sueyoshi method depend on real number of σ .

With revise approach units A,C,D are subject to condensation. Therefore, the proposed method can detect all congestion units.

5. Conclusions

In this paper, the method for measuring the congestion of Noura and his colleagues studied And we find that this approach ,there is no problem for congestion detection In the case of one input and one output but in higher space is not able to detect congestion of some units. Using the congestion of units that are modified to be able to identify. Revise approach don't have other methods difficulties and is a perfect method to determine congestion.

References:

- [1] R.D. Banker, A. Charnes, W.W. Cooper, Some models for estimating technical and scale inefficiencies in Data Envelopment Analysis, *Management Science*, 30 (1984) 1078–1092.
- [2] P.L. Brockett, W.W. Cooper, H.C. Shin, Y. Wang, Inefficiency and congestion in Chinese production before and after the 1978 economic reforms, *Socio-Economic Planning Sciences*, 32 (1998) 1-20.
- [3] W.W. Cooper, H. Deng, Z.M. Huang, S.X. Li, one-model approach to congestion in data

envelopment analysis, *Socio-Economic Planning Sciences*, 36 (2002) 231-238.

[4] R. Färe, S. Grosskopf, Measuring congestion in production, *Journal of Economics*, 43 (1983) 257-271.

[5] R. Färe, S. Grosskopf, When can slacks be used to identify congestion, *Socio-Economic Planning Sciences*, 35 (2001) 1-10.

[6] R. Färe, L. Svensson, Congestion of production factors, *Econometrica*, 48 (1980) 1745-1753.

[7] A.A. Noura, F. Hosseinzadeh Lotfi, G.R. Jahanshahloo, S. Fanati Rashidi, R.P. Barnett, A new method for measuring congestion in data envelopment analysis, *Socio-Economic Planning Sciences*, 44 (2010) 240-246.

[8] A.A. Noura, E. Hosseini, Measuring congestion in data envelopment analysis with common weights, *International Journal of Data Envelopment Analysis*, 3 (2013) 125-133.

[9] T. Sueyoshi, K. Sekitani, DEA congestion and returns to scale under an occurrence of multiple optimal projections, *European Journal of Operational Research*, 194 (2009) 592–607.

[10] K. Tone, B.K. Sahoo, Degree of scale economies and congestion: A unified DEA approach, *European Journal of Operational Research*, 158 (2004) 755–772.

Revise Approach to Measuring Congestion Based on the Comparison of Inputs

AbbasAli. Noura, Elnaz. Hosseini

اصلاحی بر روش اندازه گیری تراکم براساس مقایسه ورودی ها

چکیده: روش اندازه گیری تراکم نورا و همکارانش براساس مقایسه ورودی ها در حالت یک ورودی و یک خروجی مشکلی برای تشخیص تراکم ندارد اما در فضای بالاتر قادر به تشخیص برخی واحدهای دارای تراکم نیست. در این مقاله اصلاحی بر روش اندازه گیری تراکم براساس مقایسه ورودی ها پیشنهاد می کنیم و با اثبات قضایا و حل مثال های عددی درستی روش پیشنهادی را نشان می دهیم. با روش پیشنهادی قدرت تشخیص واحدهای دارای تراکم بالا می رود و این روش همه واحدهای دارای تراکم را تشخیص می هد.

کلمات کلیدی: تحلیل پوششی داده ها، تراکم، کارایی، واحد تصمیم گیرنده.