Efficiency Analysis Based on Separating Hyperplanes for Improving Discrimination among DMUs

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Abstract

Data envelopment analysis (DEA) is a non-parametric method for evaluating the relative technical efficiency for each member of a set of peer decision making units (DMUs) with multiple inputs and multiple outputs. The original DEA models use positive input and output variables that are measured on a ratio scale, but these models do not apply to the variables in which interval scale data can appear. However, with the widespread use of interval scale data, the emphasis has been directed towards the simultaneous consideration of the ratio and interval scale data in DEA models. This study, introduces a measure of inefficiency and identifies efficient units as is done in DEA models with VRS technology based on separating hyperplanes. The basic idea in the approach is to obtain a separating hyperplane of DMUs so that the hyperplane can separate the maximum number of DMUs whose performances are not better than a DMU under evaluation, from the rest of the DMUs. Performance measure is defined as a ratio of not-better units to all units. Also, this paper presents a relationship between the performance measures with those in DEA models with VRS technology.

Keywords: Data envelopment analysis; efficiency analysis; variable return to scale; separation hyperplanes; discrimination power.

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1. Introduction

DEA is a mathematical optimization technique which determines the efficiency of each DMU by maximizing the ratio of a weighted sum of its outputs to a weighted sum of its inputs while ensuring that the efficiencies of other units do not exceed one [1]. Beside of its popularity, DEA has some drawbacks such as the inputs and outputs are measured on a (positive) ratio scale and models gives poor discrimination on the performance of DMUs.

The original DEA models use (positive) ratio scale input and output variables but these models do not apply to variables in which interval scale data can appear. With the widespread use of interval scale variables, such as profit or the increase/decrease in bank accounts emphasis has led to the development of alternative models aiming at assessing efficiency in presence of ratio and interval scale data in DEA model [2]. However, interval scale inputs/outputs cannot be used widely in DEA models. The problem with interval scale variables arises from the fact that rations of measurement on such a scale are meaningless. Consequently, a DEA model can be used to handle interval scale variables in which the ratio of virtual effect inputs and/or outputs and observed inputs and/or outputs does not have any role in the calculations. Halme et al. introduced a combined DEA problem with VRS (BCC model) based on the directional distance function where both outputs were maximized and inputs were minimized [3]. As acknowledged by Halme et al as a drawback, since the number of the

variables augments as a consequence of the decomposition of the interval-scale variables, some of the inefficient units may become efficient. Furthermore, there are some cases in which the interval-scale variables used in DEA applications are not the result of the subtraction of the two ratio-scale variables [2]. Various efforts have been devoted to develop methods without a priori information to improve discrimination in DEA.

Sexton et al. first introduce the concept of cross-efficiency in DEA by using peer evaluation instead of a self-evaluation [4]. Andersen and Petersen present the procedure referred to SE-CCR model for ranking efficient units [5]. Their basic idea is to compare the unit under evaluation with all other units in the sample, i.e., the DMU itself is excluded. Doyle and Green further extend the work by Sexton et al. (1986) by introducing aggressive and benevolent cross-efficiency referred to CEM [6]. Tofallis addresses the discrimination problem by presenting the profiling method [7]. He uses the original DEA but taking one input at a time and only with related outputs. Seiford and Zhu develop a supper-efficiency DEA model referred to SE-BCC model [8]. Li and Reeves propose a multiple criteria approach to DEA referred to MCDEA [9]. Tone proposes the super-efficiency model using the slacks-based measure of efficiency [10]. However, none of research works in DEA literature has been done to improve discrimination a set of DMUs in the presence of interval scale data.

Many methods are considered that incorporate

a priori information to improve the discrimination of traditional DEA models. Direct weight restrictions was initially developed by Allen et al. [11], used by Beasley [12]. The Cone Ratio Model was developed by Charnes et al. [13], [14]. Wong and Beasley explore the use of such virtual input and output restrictions [15]. Based on the Russell measure, Zhu presented some models to introduce a preference structure in DEA models, using weights to do so [16]. The method of value efficiency analysis was developed by Halme et al. as a way of incorporating the decision maker's value judgments and preferences into the analysis, using two stages [17], [18].

In this paper in order to increase the discriminatory power a set of DMUs in the presence of interval scale data, it is obtained a normal vector to a separating hyperplane of DMUs that the hyperplane can separate the maximum number of DMUs whose performances are not better than a DMU under evaluation, from the rest of the DMUs. Performance measure is defined as a ratio of not-better units to all units. Besides, in this paper we present a relationship between the performance measures with those in DEA models with VRS technology. This paper is organized as follows: In section 2 we introduce preliminaries of DEA. In section 3 we present an approach to efficiency analysis based on separating hyperplanes of DMUs and the relationship between the efficiency

measures with those in DEA models with variables return to scale technology. Conclusions are summarized in section 4.

2. Preliminaries

We assume that there are n peer observed DMUs to be evaluated. Each DMU produces $\boldsymbol{s}$ different outputs by consuming $\boldsymbol{m}$ different inputs. Specifically, $\boldsymbol{DMU_j}$ consumes amount $\boldsymbol{x_{ij}}$ of input $\boldsymbol{i}$ and produces amount $\boldsymbol{y_{rj}}$ of output $\boldsymbol{r}$. The input and output vectors of $\boldsymbol{DMU_j}$ are denoted by $\boldsymbol{x_j}$ and $\boldsymbol{y_j}$, respectively. We assume $\boldsymbol{x_j}$ and $\boldsymbol{y_j}$ are semi-positive, i.e., $\boldsymbol{x_j \geq 0, x_j \neq 0, \ y_j \geq 0, \ y_j \neq 0}$ for $\boldsymbol{j = 1, \dots, n}$, and further assume that each $\boldsymbol{DMU}$ has at least one positive input and one positive output value. We use $\boldsymbol{DMU_o} \ (\boldsymbol{o \epsilon \{1, 2, \dots, n\}})$ as the $\boldsymbol{DMU}$ under evaluation. Throughout this paper, vectors will be denoted by bold letters.

2.1. The BBC MODEL

The production set $\boldsymbol{P_v}$ of the BCC model is defined as a set of semi-positive $(\boldsymbol{x}, \boldsymbol{y})$ as follows:

$P_v = \{(x, y) : x \geq \sum_{j=1}^{n} \lambda_j x_j \ , \ y \leq \sum_{j=1}^{n} \lambda_j y_j,$

$\sum_{j=1}^{n} \lambda_j = 1, \ \lambda_j \geq 0; \ (j = 1 \dots, n)\},$

where $(\lambda_1, \dots, \lambda_n)$ is a semi-positive in $\mathbb{R}_n$. The input-oriented BCC model evaluates the efficiency of each DMU_o by solving the following linear program:

min θ

$S.t. \qquad \sum_{j=1}^{n} \lambda_j x_j \leq \theta x_o,$

$\qquad \sum_{j=1}^{n} \lambda_j y_j \geq y_o,$

$$\sum_{j=1}^{n} \lambda_j y_j \geq y_o,$$

$$\sum_{j=1}^{n} \lambda_j = 1,$$

$$\lambda_j \geq 0; \ j = 1, \dots, n \qquad (1)$$

where θ is a scalar. Also, the output-oriented BCC model evaluates the efficiency of each DMU_o by solving the following linear program:

$$\max \quad \varphi$$

$$s.t. \quad \sum_{j=1}^{n} \lambda_j x_j \leq x_o,$$

$$\sum_{j=1}^{n} \lambda_j y_j \geq \varphi y_o,$$

$$\sum_{j=1}^{n} \lambda_j = 1,$$

$$\lambda_j \geq 0; \ j = 1, \dots, n. \qquad (2)$$

where φ is a scalar.

Definition 1. (Input-oriented BCC-efficient)
The performance of DMU_o is the input-oriented BCC-efficient if and only if $\theta^* = 1$ [19].

The dual problem of model (2) is expressed as:

$$
\begin{aligned}
z^* = \max \quad & u^t y_o + u_o \\
& v^t x_o = 1, \\
s.t. \quad & u^t y_j + u_o - v^t x_j \leq 0; j = 1, \dots, n, \\
& u \geq 0, v \geq 0.
\end{aligned}
$$

$$(3)$$

Definition 2. (Output-oriented BCC-efficient)
The performance of DMU_o is the output-oriented BCC - efficient if and only if $z^* = 1$. Also, The dual problem of model (2) is expressed as:

$$
\begin{aligned}
t^* = \min \quad & v^t x_o + v_o \\
& u^t y_o = 1, \\
s.t \quad & u^t y_j - (v^t x_j + v_o) \geq 0; j = 1, \dots, n, \\
& u \geq 0, v \geq 0.
\end{aligned}
$$

$$(4)$$

Definition 3. (Input-oriented BCC-efficient)
The performance of DMU_o is the output-oriented BCC-efficient if and only if $t^* = 1$.

2.2. THE TWO-PHASE PROCESS FOR INPUT-ORIENTED BCC MODEL

The two-phase process for BCC model evaluates the efficiency of DMU_o by solving the following linear program:

$$\min \quad \theta - \varepsilon(1^t s^- + 1^t s^+)$$

$$s.t. \quad \sum_{j=1}^{n} \lambda_j x_j + s^- = \theta x_o,$$

$$\sum_{j=1}^{n} \lambda_j y_j - s^+ = y_o,$$

$$\sum_{j=1}^{n} \lambda_j = 1,$$

$$s^- \geq 0, s^+ \geq 0, \lambda_j \geq 0; \ j = 1, \dots, n \qquad (5)$$

where $\varepsilon > 0$ is the non-Archimedean element.

Definition 4. (BCC-efficient). The performance of DMU_o is BCC-efficient if only if an optimal solution $(\theta^*, \lambda^*, s^{-*}, s^{+*})$ of the two-phase model (3) satisfies: $\theta^* = 1$, $s^{-*} = 0$, and $s^{+*} = 0$. The dual multiplier form of program model (2) is expressed as:

$$
\begin{aligned}
max \quad & u^t y_o + u_o \\
& v^t x_o = 1, \\
s.t. \quad & u^t y_j + u_o - v^t x_j \leq 0; \ j = 1, \dots, n, \\
& u \geq 1\varepsilon, v \geq 1\varepsilon.
\end{aligned}
$$

$$(6)$$

By definition (2.2) and strong duality theorem, the performance of DMU_o is BCC-efficient if only if an optimal solution $(\boldsymbol{u}^*, \boldsymbol{v}^*, u_o^*)$ of model (4) satisfies

$$\boldsymbol{u}^{*t} \boldsymbol{y}_o + u_o^* = 1$$

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Definition 5. (Reference set) Reference set of DMU_o denoted by E_o is defined as:

$E_o =$
$$\left\{ DMU_j \mid \begin{array}{l} j\epsilon\{1,\dots,n\}\ \lambda_j^* > 0\ in\ some\ optimal\ solution \\ (\theta^*, \lambda^*, s^{-*}, s^{+*})\ of\ model\ (2.5) \end{array} \right\}$$

The DMUs in E_o are BCC-efficient [20].

Definition 6. (Extreme BCC-efficient) DMU_o is extreme BCC-efficient if only if $E_o = \{DMU_o\}$.

If DMU_o be extreme BCC-efficient, then DMU_o
is BCC-efficient [20].

Theorem 1. DMU_o is extreme BCC-efficient iff

$$\min \quad \theta - \varepsilon \left(\sum_{j \neq o} \lambda_j \right)$$

$$s.t. \quad \sum_{j=1}^{n} \lambda_j x_j + s^- = \theta x_p,$$

$$\sum_{j=1}^{n} \lambda_j y_j - s^+ = y_o,$$

$$\sum_{j=1}^{n} \lambda_j = 1,$$

$$s^- \geq 0, s^+ \geq 0, \lambda_j \geq 0;\ j = 1,\dots,n. \quad (7)$$

has an optimal objective function value of one.

Proof. Let DMU_o not be extreme BCC-efficient. Then, there exists an optimal solution $(\theta^*, \lambda^*, s^{-*}, s^{+*})$ to model (2) such that a $\lambda_j^* > 0\ (j \neq o)$, and also noting that $\theta^* \leq 1$, thus $\theta^* - \varepsilon \sum_{j \neq o} \lambda_j^* < 1$. Therefore, since $(\theta^*, \lambda^*, s^{-*}, s^{+*})$ is a feasible solution to model (7), the optimal objective function value of model (5) is less one.

Let the optimal objective function value of the model (7) be less one, and let $(\tilde{\theta}, \tilde{\lambda}, \widetilde{s^-}, \widetilde{s^+})$ be an optimal solution of the model, then either

$\tilde{\theta} < 1$ or ($\tilde{\theta} = 1$ and $\sum_{j \neq o} \widetilde{\lambda_j} > 0$). If $\tilde{\theta} < 1$, since $(\tilde{\theta}, \tilde{\lambda}, \widetilde{s^-}, \widetilde{s^+})$ is a feasible solution of the model (5), then DMU_o isn't BCC-efficient, thus DMU_o isn't extreme BCC-efficient. If $\tilde{\theta} = 1$ and $\sum_{j \neq o} \widetilde{\lambda_j} > 0$, then either $(\tilde{s}^-, \tilde{s}^+) \neq (0,0)$ or $(\widetilde{s^-}, \widetilde{s^+}) = (0,0)$ If $(\widetilde{s^-}, \widetilde{s^+}) \neq (0,0)$, since $(\tilde{\theta}, \tilde{\lambda}, \widetilde{s^-}, \widetilde{s^+})$ is a feasible solution of model (5), then DMU_o isn't BCC-efficient, thus DMU_o isn't extrem BCC-efficient. If $(\tilde{s}^-, \tilde{s}^+) = (0,0)$, then either $(\tilde{\theta}, \tilde{\lambda}, \widetilde{s^-}, \widetilde{s^+})$ is an optimal solution of mode (5) or isn't. If $(\tilde{\theta}, \tilde{\lambda}, \widetilde{s^-}, \widetilde{s^+})$ be an optimal solution of model (5), then, since $\sum_{j \neq o} \widetilde{\lambda_j} > 0$, thus DMU_o isn't extrem BCC-efficient. If $(\tilde{\theta}, \tilde{\lambda}, \widetilde{s^-}, \widetilde{s^+})$ not be an optimal solution of model (5) then there exists an optimal solution $(\theta^*, \lambda^*, s^{-*}, s^{+*})$ of the model such that $\theta^* = 1$ with $(s^{-*}, s^{+*}) \neq (0,0)$ which in this case also, DMU_o isn't extreme BCC-efficient.

3. Efficiency analyses based on separating hyperplanes of DMUs

Theorem 2. DMU_o is input oriented BCC – efficient iff

$$\min \quad \sum_{j=1}^{n} t_j$$

$$s.t. \quad \begin{array}{l} v^t x_o - (u^t y_o + u_o) = 0,\ \ v^t x_o \geq \varepsilon, \\ v^t x_j - (u^t y_j + u_o) + Mt_j \geq 0;\ j = 1,\dots n, \end{array}$$

$$u \geq 0,\ \ v \geq 0,\ \ \ \ t_j \epsilon\{0,1\}, \quad (8)$$

where M is a sufficiently large number, has an optimal objective function value of zero ∎.

Proof. Let the optimal objective function value of model (8) is zero. Then, letting $(\bar{u}, \bar{v}, \bar{u}_o, \bar{t})$ be an optimal solution of model

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(8), we have:

$\bar{v}^t x_o = \bar{u}^t y_o + \bar{u}_o, \ \bar{u} \geq 0, \ \bar{v} \geq 0, \ \bar{v}^t x_o \geq \varepsilon,$

$\bar{v}^t x_j - (\bar{u}^t y_j + \bar{u}_o) \geq 0; \ j = 1, .., n.$

So that by taking $\hat{u} = \tilde{u}/k, \ \hat{v} = \tilde{v}/k$ with $k = \tilde{u}^t y_o$, we have

$\hat{v}^t x_o = (\hat{u}^t y_o + \hat{u}_o),$

$\hat{v}^t x_j - (\hat{u}^t y_j + \hat{u}_o) \geq 0; \ (i = 1, ..., n),$

$(\hat{u}, \hat{v}) \geq (0,0) \ \hat{v}^t x_o = 1,$

Thus, $(\hat{u}, \hat{v}, \hat{u}_o)$ is an optimal solution for model (3). Therefore, DMU_o is input oriented BCC-efficient.

Conversely, let DMU_o is input oriented BCC - efficient, and let $(\bar{u}, \bar{v}, \bar{u}_o)$ is an optimal solution of model (2), then

$\tilde{u} \geq 0, \tilde{v} \geq 0, \bar{v}^t x_o = 1,$

$\tilde{v}^t x_j - (\tilde{u}^t y_j + \tilde{u}_o) \geq 0; \ j = 1, .., n.$

Thus $(\tilde{u}, \tilde{v}, \tilde{u}_o, \tilde{t})$, where $\tilde{t} = 0 \epsilon \mathbb{R}^n$, is a feasible solution for model (8). Therefore, the optimal objective function value of model (8) is zero ∎

Theorem 3. DMU_o is both input oriented BCC - efficient and output oriented BCC-efficient if

$\min \quad \sum_{j=1}^{n} t_j$

$s.t. \quad \begin{array}{l} v^t x_o - (u^t y_o + u_o) = 0 \\ v^t x_o \geq \varepsilon, \quad u^t y_o \geq \varepsilon, \\ v^t x_j - (u^t y_j + u_o) + Mt_j \geq 0; \ j = 1, ..., n \\ u \geq 0, v \geq 0, t_j \epsilon \{0,1\}. \end{array}$

(9)

has an optimal objective function value of zero.

Proof. Let the optimal objective function value model (9) is zero, thus letting $(\bar{u}, \bar{v}, \bar{u}_o, \bar{t})$ be an optimal solution for the

model, we have $\bar{u} \geq 0, \ \bar{v} \geq 0, \ \bar{u}^t y_o \geq \varepsilon,$

$\bar{v}^t x_o \geq \varepsilon, \qquad \bar{v}^t x_j - (\bar{u}^t y_j + \bar{u}_o) \geq 0;$

$(j = 1, ..., n), \quad (\bar{v}^t x_o) = (\bar{u}^t y_o + \bar{u}_o).$

So that by taking $\tilde{u} = (\bar{u}/k), \tilde{v} = (\bar{v}/k)$ with $k = \bar{u}^t y_o$ we have

$\tilde{u} \geq 0, \quad \tilde{v} \geq 0, \quad \tilde{v}^t x_o = 1, \tilde{v}^t x_o = \tilde{u}^t y_o + \tilde{u}_o, \tilde{v}^t x_j - (\tilde{u}^t y_j + \tilde{u}_o) \geq 0; \ j = 1, ..., n.$

Thus $(\bar{u}, \bar{v}, \bar{u}_o)$ is a feasible solution of model (4), and since $\tilde{v}^t x_o = 1$, it follows that the optimal objective function value of model (9) is equal to one. Hence, DMU_o is input oriented BCC - efficient. Similarly, we can show that DMU_o is output oriented BCC-efficient.

Theorem 4. DMU_o is BCC – efficient iff

$\min \quad \sum_{j=1}^{n} t_j$

$s.t. \quad \begin{array}{l} v^t x_o - (u^t y_o + u_o) = 0 \\ v^t x_j - (u^t y_j + u_o) + Mt_j \geq 0; \ j = 1, ..., n, \\ u \geq 1\varepsilon, v \geq 1\varepsilon, t_j \epsilon \{0,1\}. \end{array}$

…………………………………… … …..(10)

has an optimal objective function value of zero.

Proof. Let the optimal objective function value model (10) is zero, then letting $(\bar{u}, \bar{v}, \bar{u}_o, \bar{v}_o, \bar{t})$ be an optimal solution for the model, we have

$\bar{u} \geq 1\varepsilon, \ \bar{v} \geq 1\varepsilon, \ \bar{v}^t x_o = \bar{u}^t y_o + \bar{u}_o, \ \bar{v}^t x_j - (\bar{u}^t y_j + \bar{u}_o) \geq 0; \ j = 1, .., n.$

Thus $(\bar{u}, \bar{v}, \bar{u}_o)$ is a feasible solution for the following model

$\min \quad v^t x_o - (u^t y_o + u_o)$

$s.t. \quad \begin{array}{l} v^t x_j - (u^t y_j + u_o) \geq 0; \ j = 1, ..., n, \\ u \geq 1\varepsilon, \ v \geq 1\varepsilon, \ u_o \ free, \end{array}$

(11)

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which for the feasible solution the objective function value of the model is equal to zero. Therefore, the optimal objective function value model (10) is zero. Thus, by strong duality theorem, the optimal objective function value of the following model:

$$\max \quad \sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+$$

$$s.t. \quad \sum_{j=1}^{n} \lambda_j x_{ij} + s⬚ = x_{io}; \quad i = 1, \ldots, m,$$

$$\sum_{j=1}^{n} \lambda_j y_{ij} - s⬚ = y_{ro}; \quad r = 1, \ldots, s,$$

$$\lambda_j \geq 0; \; \forall j, s_i^- \geq 0; \; \forall i, \quad s_r^+ \geq 0,; \forall r \quad (12)$$

which is dual form of model (11), equal to zero. Consequently DMU_o is BCC − efficient. It is easy to show that if DMU_o is BCC − efficient, then the optimal objective function value model (11) is zero.

Theorem 5. DMU_o is extreme BCC − efficient iff

$$\min \quad \sum_{j \neq o} t_j$$

$$s.t. \quad \begin{array}{l} v^t x_o - (u^t y_o + u_o) = 0, v^t x_o \geq \varepsilon, \\ v^t x_j - (u^t y_j + u_o) + M t_j \geq \varepsilon, \quad j \neq o, \\ u \geq 0, \; v \geq 0, \; t_j \epsilon\{0,1\}, j \neq o, \end{array} \quad (13)$$

has an optimal objective function value of zero.

Proof. Let the optimal objective function value model (13) is zero, then letting $(\bar{u}, \bar{v}, \bar{u}_o, \bar{t})$ be an optimal solution for the model, where $\bar{t} = (\bar{t}_1, \ldots, \bar{t}_{o-1}, \bar{t}_{o+1}, \ldots, \bar{t}_n) = 0 \epsilon \mathbb{R}^{n-1}$, we have

$$\bar{v}^t x_o = \bar{u}^t y_o + \bar{u}_o,$$

$$\bar{v}^t x_j - (\bar{u}^t y_j + \bar{u}_o) \geq 1; \; j \epsilon \{1, \ldots, n\} - \{o\},$$

$$\bar{v}^t x_o \geq 1, \; \bar{u} \geq 0, \; \bar{v} \geq 0, \; \bar{u}_o \geq 0, \; \bar{v}_o \geq 0.$$

Therefore, $(\bar{u}, \bar{v}, \bar{u}_o)$ is a feasible solution for the following model:

$$\min \quad v^t x_o - (u^t y_o + u_o)$$
$$\bar{v}^t x_o \geq 1,$$

$$s.t. \quad v^t x_o - (u^t y_o + u_o) \geq 0,$$
$$v^t x_j - (u^t y_j + u_o) \geq 1; \; j \epsilon \{1, \ldots, n\} - \{o\},$$

$$u \geq 0, \quad v \geq 0, u_o \; free, \quad (14)$$

Therefore, by strong duality theorem, the optimal objective function value of the following model:

$$max \quad \varphi + \sum_{j \neq o} \lambda_j$$

$$s.t \quad \sum_{j=1}^{n} \lambda_j x_j + \varphi x_o \leq x_o,$$

$$\sum_{j=1}^{n} \lambda_j y_j \geq y_o,$$

$$\sum_{j=1}^{n} \lambda_j = 1,$$

$$\lambda_j \geq 0; \; j = 1, \ldots, n,$$

which is dual form of model (14), equal to zero. Hence, by theorem (4), DMU_o is extreme BCC-efficient.

It is easy to show that if DMU_o is extrem BCC − efficient, then the optimal objective function value model (14) is zero ∎

4. Illustrative example

To illustrate, we consider an example consisting of twelve DMUs each consuming two outputs to produce a single output (see Table 1) each one of which can be measured on interval scale variables.

Table1: Data set

	Input	Output 1	Output 2
U. 1	1	1	2.5
U. 2	1	2.5	1
U. 3	1	2.5	4
U. 4	1	4	5.5
U. 5	1	5	3.5
U. 6	1	2	6
U. 7	1	6	4
U. 8	1	1.5	6
U. 9	1	8	2
U. 10	1	8	1
U. 11	1	5.5	5
U. 12	1	7	3

By using the data set from Table 1, we solve models of (3), (6), (8), (9), (10) and (13) for each DMU. The results are reported in Table 2. Where, for example, Eff (3) represents the efficiency obtained by model (3).

Model (3) indicates DMU_o is radial BCC-efficient, thus according to the table 2, all of 12 units are radial efficient. Model (6) indicates DMU_o is BCC-efficient, therefore according to the table 2 units of 1, 2, 3, 10 and 10 are not CCR-efficient, while units of 4, 5, 6, 7, 9, 11, and 12 are BCC-efficient. Model (8) indicates normal vector of a hyperplane in the PPS with VRS technology so that the DMU under evaluation is on the hyperplane and it can separate the maximum number of DMUs whose performances are not better than the DMU under evaluation, from the rest of DMUs. For example, maximum number of DMUs whose performances are not better than

the U.1 is 12. Thus, the efficiency score of U.1 is 12/12 = 12. Also, model (8) diagnoses radial efficient units correctly. Model (9) indicate normal vector with nonzero components of a hyperplane in the PPS with VRS technology so that the hyperplane can separate the maximum number of DMUs whose performances are not better than the DMU under evaluation, from the rest of DMUs. In addition, virtual input and virtual output of the DMU under evaluation be positive. For example, the maximum number of DMUs whose performances are not better than U.3 is 12-5=7. Then, the objective function value of model (9) in the evaluation of U.3 is equal to 5 thus the efficiency score of Unit 3 is 7/12 =0.5833333. Also, model (10) diagnoses BBC-efficient units correctly, although the efficiency scores for units that are not BCC-efficient will differ from efficiency scores obtained from model (6). Finally, model (13) diagnoses extreme BBC-efficient units correctly, although the efficiency scores for units that are not BCC-efficient will differ from efficiency scores obtained from model (7). For example, the objective function value of model (13) in the evaluation of U.8 is equal to one, thus the efficiency score of U.8 is 11/12 = 0.9166667. More importantly, Model (13) both indicates extreme BCC-efficiency of unites and provides efficiency scores for the units that are not extreme BCC-efficient.

5. Conclusion

We provided models to improve discrimination

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Table2: Results

	Eff (3)	Eff (6)	Eff (8)	Eff (9)	Eff (10)	Eff (13)
U. 1	1.0000000	0.5500000	1.0000000	0.3333333	0.3333333	0.3333333
U. 2	1.0000000	0.4999850	1.0000000	0.4166667	0.3333333	0.3333333
U. 3	1.0000000	0.7666667	1.0000000	0.5833333	0.5000000	0.5000000
U. 4	1.0000000	1.0000000	1.0000000	1.0000000	1.0000000	1.0000000
U. 5	1.0000000	1.0000000	1.0000000	1.0000000	1.0000000	0.9166667
U. 6	1.0000000	1.0000000	1.0000000	1.0000000	1.0000000	1.0000000
U. 7	1.0000000	1.0000000	1.0000000	1.0000000	1.0000000	0.8333333
U. 8	1.0000000	0.9999975	1.0000000	1.0000000	0.9166667	0.9166667
U. 9	1.0000000	1.0000000	1.0000000	1.0000000	1.0000000	1.0000000
U. 10	1.0000000	0.9999950	1.0000000	1.0000000	0.9166667	0.9166667
U. 11	1.0000000	1.0000000	1.0000000	1.0000000	1.0000000	1.0000000
U. 12	1.0000000	1.0000000	1.0000000	1.0000000	1.0000000	0.9166667

among DMUs, in the presence of interval scale data, based on separation hyperplanes in PPS with VRS technology. For this purpose we obtained a vector normal to a separating hyperplane of DMUs so that the hyperplane can separate the maximum number of DMUs whose performances are not better than a DMU under evaluation, from the rest of the DMUs. In addition, we presented the relationship between the performance measures with those in the DEA with VRS technology. Furthermore, these performance measures can be applied as a criterion for efficiency analysis and improving discrimination of a set of peer DMUs.

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Efficiency Analysis Based on Separating Hyperplanes for Improving Discrimination among DMUs

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تحلیل کارایی بر اساس ابرصفحه های جداکننده برای بهبود تشخیص میان واحدهای تصمیم گیرنده

چکیده: تحلیل پوششی داده ها یک روش ناپارامتریک برای ارزیابی کارایی تکنیکی نسبی مجموعه ای از واحدهای تصمیم گیرنده متجانس با ورودیها و خروجیهای چند گانه است. علیرغم محبوبیت این روش، تحلیل پوششی داده ها دارای برخی اشکالات نظیر وزن های ورودی-خروجی غیر واقعی وعدم تشخیص میان واحد های کارا است. این مقاله، یک اندازه کارایی و ناکارایی برای واحدها براساس ابرصفحه های جداکننده معرفی می کند، همانگونه که در مدلهای تحلیل پوششی داده ها با تکنولوژی بازده به مقیاس متغیر معرفی می شود. ایده اساسی در این روش به دست اوردن یک ابرصفحه ی جداکننده واحدها است، به گونه ای که ان ابرصفحه بتواند حداکثر واحدهایی که عملکردشان از واحد تحت ارزیابی بهتر نیست را جدا کند. همچنین، این مقاله یک رابطه میان اندازه های عملکرد به دست امده در این روش با آنچه در تحلیل پوششی داده ها با تکنولوژی بازده به مقیاس متغیر به دست می اید را به دست میدهد.

کلمات کلیدی: تحلیل پوششی داده ها، تحلیل کارایی، بازده به مقیاس متغیر، ابرصفحه های جداکننده، قدرت تشخیص