



New DEA/Location Models with Interval Data

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Abstract

Recently the concept of facility efficiency, which defined by data envelopment analysis (DEA), introduced as a location modeling objective, that provides facilities location's effect on their performance in serving demands. By combining the DEA models with the location problem, two types of "efficiencies" are optimized: spatial efficiency which measured by finding the least cost location and allocation patterns for facilities, and the facility efficiency in serving demands which measured by DEA efficiency score. In this paper, location-allocation models with DEA in interval inputs and outputs environments are combined. A new pair of interval DEA/location models are constructed and run.

Keywords: Data envelopment analysis; Interval DEA model; UPLP; CPLP

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1. Introduction

Many types of location-allocation models have been presented to find optimal facilities location patterns with respect to several criteria like cost, demands coverage, time and others. Some of these models have been formulated in a multi objective programming framework eliciting trade-offs among these sometimes conflicting objectives (Klimberg and Ratick 2008). Most of these models follow just spatial efficiency and disregard to facilities efficiency, while final purpose of facilities installation and location is maximization of yield and efficiency and to achieve this purpose, the facilities efficiency in serving demands must be maximized. On the other hand, minimization of cost and time and maximization of demands coverage aren't enough to achieve optimal efficiency. Therefore, some models recently developed that use the concept of efficiency as defined by DEA as another location modeling objective to help providing insights into the performance of facilities at different potential sites. However, the previous models do not deal with imprecise data and assume that all input and output data are exactly known. In real world condition, however, this assumption may not always be true. Due to the existence of uncertainty, DEA sometimes faces the situation of imprecise data, especially when a set of decision-making units (DMUs) contains missing data, judgment data, forecasting data or ordinal preference information. Generally speaking, uncertain information or imprecise data can be expressed in interval or fuzzy

numbers. Therefore, how to evaluate the management or operation efficiency of a set of DMUs in interval and/or fuzzy environments is a worth-studying problem. This is the need of both the developments of DEA theory and methodology and its real applications (Wang et al. 2005). Therefore, in this paper, location-allocation models are combined with DEA in interval inputs and outputs environments to improve performance of these models.

Location-allocation problems have several types. We have used uncapacitated facility location problem (UPLP) model and the capacitated facility location problem (CPLP) model as the base location modeling framework for our model formulations. The uncapacitated facility location problems take a great variety of forms, depending on the nature of the objective function (mini sum, mini max, problems with covering constraints). The uncapacitated model assumes each facility has unlimited capacity, and as a result, if a facility supplies a demand node, it will satisfy all the demand, i.e., only one facility is necessary to serve a particular demand. The mathematical formulation of the UPLP is

$$\text{Min} \sum_k \sum_l c_{kl} \text{dem}_l t_{kl} + \sum_k F_k t_k \quad (1)$$

s.t:

$$\sum_k t_{kl} = 1 \quad \forall l$$

$$t_{kl} \leq t_k \quad \forall k, l$$

$$t_{kl}, t_k = 0, 1$$

Where

k : index of facility locations,

l : index of demand locations,

c_{kl} : cost of shipping one unit of demand from facility k to demand l ,

F_k : fixed cost of opening/using facility k ,

dem_l : the amount of demand at node,

$$t_k = \begin{cases} 1 & \text{if facility } k \text{ open} \\ 0 & \text{OW} \end{cases}$$

$$t_{kl} = \begin{cases} 1 & \text{if facility } k \text{ serves demand } l \\ 0 & \text{OW} \end{cases}$$

In the CPLP, a number of capacitated facilities are to be located among possible sites in order to satisfy demands of customers by minimizing total costs of transportation and fixed charges of establishing facilities. The CPLP has been effectively implemented to solve real-world applications such as plants location, power stations location, warehouses location, to just name a few. The CPLP is generalization of the simple plant location problem. In the CPLP each facility has limited capacity and so more than one facility may supply a demand node. The formulation for the CPLP is:

$$\text{Min} \sum_k \sum_l c_{kl} b_{kl} + \sum_k F_k t_k \quad (2)$$

s.t:

$$\sum_k t_{kl} \geq 1 \quad \forall l$$

$$t_{kl} \leq t_k \quad \forall k, l$$

$$\sum_k b_{kl} = dem_l \quad \forall l$$

$$b_{kl} \leq \text{Min}[dem_l, cap_k] t_k \quad \forall k, l$$

$$t_{kl}, t_k = 0, 1$$

$$b_{kl} \geq 0$$

Where

cap_k : capacity of facility k

b_{kl} : amount of units which shipped from facility k to demand location l . The other parameters and variables are the same with model (1).

In this paper, we combine UPLP and CPLP with DEA in interval inputs and outputs to find optimal and efficient facility location/allocation patterns.

2. Literature Review

Location analysis is a specialized branch of combinatorial optimization that has grown from early foundations to maturity, with most growth occurring since the 1960s. A wide range of problems has emerged, which may be characterized in general as finding optimal locations for facilities.

There are many researches which have been done in facility location's context. Here, we review some of these researches which are in the context of UPLP or CPLP. Warehouse location problem which has appeared under a variety of different names, including uncapacitated/simple and warehouse/plant/facility/site location, studied with Kuehn and Hamburger (1963). They developed basic greedy heuristic algorithm (drop, add, swap) for solving locating warehouses problem. Efroymson and Ray (1966) and Khumawala (1972) presented an efficient branch and bound algorithm for the warehouses location problem. Erlenkotter (1978) developed and tested a method for the uncapacitated facility

location problem that was based on a linear programming dual formation. A simple ascent and adjustment procedure frequently produces optimal dual solutions, which in turn often correspond directly to optimal integer primal solutions. If not, a branch-and-bound procedure completes the solution process. Erlenkotter (1978)'s method used Lagrangian relaxation with solutions to the dual problem achieves significantly quicker results in finding integer solutions, and can itself be said to be a milestone in algorithmic terms. Guignard and Spielberg (1979) give a direct dual method, consisting of several phases (each of which appears essential for some data), to resolve a strong relaxed form of the mixed plant location problem (mixed in the sense of allowing capacitated as well as uncapacitated plants) with additional constraints over the integer variables (user specified, or derived from the data themselves). Bilde and krarup (1977) presented a sharp lower bounds and efficient algorithms for the simple plant location problem. Cornuejols et al. (1991) presented an excellent theoretical analysis of all possible Lagrangian relaxations and the linear programming relaxation for the CPLP. DEA is an extension of Farrell (1957)'s idea of linking the computation of technical efficiency with production frontiers. The first DEA model was developed by Charnes et al. (1978). The CCR model is a fractional programming model, which measures the relative technical

efficiency of a firm by calculating the ratio of weighted sum of its outputs to the weighted sum of its inputs. The fractional program is run for each firm to determine the set of input-output weights, which maximizes the efficiency of that firm subject to the condition that no firm can have a relative efficiency score greater than unity for that set of weights. Thus, the DEA model calculates a unique set of factor weights for each firm. The set of weights has the following characteristics: It maximizes the efficiency of the firm for which it is calculated and it is feasible for all firms. The model (3) is linear programming of CCR model

$$\begin{aligned} & \text{Max} \sum_r u_r y_{rk} \\ & \sum_r u_r y_{rj} - \sum_{i=1}^s v_i x_{ij} \quad j = 1, 2, \dots, n \\ & \sum_i v_i y_{ik} = 1 \\ & u_r, v_i \geq 0 \quad r = 1, \dots, m, \quad i = 1, \dots, s \end{aligned} \quad (3)$$

Here u_{ij}, v_{rj} are inputs and outputs of DMU_j and $u_r, v_i \geq 0$ are the variable weights to be determined by the solution of the problem.

There have been some applications of DEA to evaluating the efficiency of spatial location patterns. Shroff et al. (1998) developed a model which incorporates efficiency measurement methodologies utilizing DEA to estimate the relative sitting efficiency of 26 potential sites. Their study identified several Pareto-optimal sites as potential locations for the proposed long-term care facility.

Klimberg and Ratick (2008) mentioned that Fisher and Rushton (1979), Desai and Storbeck (1990), Desai et al. (1995), and Athanassopoulos and Storbeck (1995), are a series of related papers which applied DEA to measure the relative spatial efficiency of location decisions. As part of their DEA models, they used two measures of access as input variables, the total travel distance and the extent of non-coverage populations not within a specified distance of a facility (Klimberg and Ratick 2008). Thomas et al. (2002) have broadened the classic anti-center models to include generalized-distance measures. It includes in the inherent proximity measures other cost/benefit metrics. Unlike the classic data envelopment analysis (DEA), the combined location/DEA model proposed in their research assumes disposability of input/output's only. It represents a more flexible formulation. The locations of multiple sites are analyzed using a binary integer program, while evaluation is performed by the full strength of a DEA model. Through a case study, they show how location and DEA models can be used together to more realistically characterize a sitting decision.

In previous studies, these two objectives i.e., spatial efficiency and facility efficiency have not been simultaneously applied to find and evaluate solutions to location problems. However, Klimberg and Ratick (2008) combined the DEA problem with the location problem and spatial efficiency with the facility

efficiency were simultaneously optimized. Their model does not deal with imprecise data and assumes that all inputs and outputs data are exactly known. In real world situations, however, this assumption may not always be true. Due to the existence of uncertainty, we sometimes face the situation of imprecise data, especially when a set of decision-making units (DMUs) contains missing data, judgment data, forecasting data or ordinal preference information. Generally speaking, uncertain information or imprecise data can be expressed in interval or fuzzy numbers. Therefore, how to evaluate the management or operation efficiency of a set of DMUs in interval and/or fuzzy environments is a worth-studying problem. This is the need of both the developments of DEA theory and methodology and its real applications (Wang et al. (2005)). So in the next section we develop and present formulations combining the uncapacitated and capacitated facility location problem with the DEA problem in interval inputs and outputs environments.

3. Model Development

The model which presented by Klimberg and Ratick (2008), deals with crisp data. They assume that inputs and outputs are measured by exact values. In some applications, however, inputs and outputs of DMUs are ever-changeful. In the real world situations, there always exists uncertainty in human thinking and judgment. As some authors point out, the precise data are not always possible

(Wang et al. (2005)). However, in real world condition many complicated factors are involved that makes difficult to measure inputs and outputs precisely. This makes a case where we need to measure the efficiency of DMUs with inexact values or interval data.

3.1. CCR Model With Interval Data

The approach proposed in this paper is based on Kabnurkar (2001)'s research. He presupposed that the decision-maker can define the risk free and impossible bounds for each interval input and output. Risk-free bounds are the conservative values that are most realistically attainable in real world condition, whereas impossible bounds are associated with those values, which represent management scenarios that are the least realistic. The risk-free and impossible bounds are used for determining the membership functions for the input and output data. All membership functions are assumed to vary linearly between the bounds. In addition, all membership functions have a value equal to zero at the impossible bounds and a value equal to one at the risk-free bounds (Kabnurkar 2001).

If superscripts 0 and 1 represent impossible and risk-free bounds for input data, then the membership function associated with the i^{th} fuzzy input (x_{ij}) for the j^{th} DMU is given by:

$$\mu_x(x_{ij}) = \frac{x_{ij}^0 - x_{ij}}{x_{ij}^0 - x_{ij}^1} \tag{4}$$

Furthermore, if superscripts 0 and 1 represent impossible and risk-free bounds for output

data, then the membership function associated with the r^{th} output (y_{rj}) for the j^{th} DMU is given by:

$$\mu_y(y_{rj}) = \frac{y_{rj} - y_{rj}^1}{y_{rj}^0 - y_{rj}^1} \tag{5}$$

Both (x_{ij}) and (y_{rj}) can be expressed in terms of the risk-free and impossible bounds and the membership functions as follows:

$$x_{ij} = x_{ij}^0 - (x_{ij}^0 - x_{ij}^1)\mu_x \tag{6}$$

$$y_{rj} = (y_{rj}^0 - y_{rj}^1)\mu_y + y_{rj}^1 \tag{7}$$

Using these definitions, the model (3) can be modified as follows: Find u and v such that,

$$\begin{aligned} &Max \sum_r u_r [(y_{rk}^0 - y_{rk}^1)\mu_y + y_{rk}^1] \\ &s.t: \end{aligned} \tag{8}$$

$$\begin{aligned} &\sum_i v_i [x_{ik}^0 - (x_{ik}^0 - x_{ik}^1)\mu_x] = 1 \\ &\sum_r u_r [(y_{rj}^0 - y_{rj}^1)\mu_y + y_{rj}^1] - \sum_i v_i [x_{ij}^0 - (x_{ij}^0 - x_{ij}^1)\mu_x] \leq 0 \\ &v_i \geq \epsilon, u_r \geq \epsilon, i = 1, \dots, m, r = 1, \dots, s \end{aligned}$$

As was mentioned in Carlsson and Korhonen (1986), the decision for the above model is achieved when,

$$\mu_x = \mu_y = \mu = \min\{\mu_x, \mu_y\} \tag{9}$$

So, the model (8) can be written as

$$\begin{aligned} &Max \sum_r u_r [(y_{rk}^0 - y_{rk}^1)\mu + y_{rk}^1] \\ &s.t: \end{aligned} \tag{10}$$

$$\begin{aligned} &\sum_i v_i [x_{ik}^0 - (x_{ik}^0 - x_{ik}^1)\mu] = 1 \\ &\sum_r u_r [(y_{rj}^0 - y_{rj}^1)\mu + y_{rj}^1] - \sum_i v_i [x_{ij}^0 - (x_{ij}^0 - x_{ij}^1)\mu] \leq 0 \\ &v_i \geq \epsilon, u_r \geq \epsilon, i = 1, \dots, m, r = 1, \dots, s \end{aligned}$$

Finally, the membership function μ , which is the parameter here, is varied at pre specified

intervals to observe the variations of the efficiency profile. Here, $\mu = 0$ would yield overly optimistic values of technical efficiency and $\mu = 1$ would yield ultra conservative values.

3.2. Combined UPLP/interval DEA model

In this section, we combine UPLP model and CCR model with DEA model by interval data i.e., model (10), which have been presented in previews section. The model (10) is non-linear because of existing μv_i and μu_r , therefore, to make it easier to solve, we convert it into a linear programming model as follows:

$$\mu u_r = q_r \Rightarrow 0 \leq q_r < u_r \tag{11}$$

$$\mu v_i = p_i \Rightarrow 0 \leq p_i < v_i \tag{12}$$

With replacement of p_i and q_r in model (10) we get model (13).

$$\begin{aligned} &Max \sum_r q_r [(y_{rk}^0 - y_{rk}^1) + u_r y_{rk}^1] \\ &s.t: \end{aligned} \tag{13}$$

$$\begin{aligned} &\sum_i [v_i x_{ik}^0 - p_i (x_{ik}^0 - x_{ik}^1)] = 1 \\ &\sum_r q_r [(y_{rj}^0 - y_{rj}^1) + u_r y_{rj}^1] - \sum_i [v_i x_{ij}^0 - p_i (x_{ij}^0 - x_{ij}^1)] \leq 0 \\ &v_i \geq \varepsilon, u_r \geq \varepsilon, \quad 0 \leq q_r < u_r, \quad 0 \leq p_i < v_i, \\ &i = 1, \dots, m, \quad r = 1, \dots, s \end{aligned}$$

The model (14) maximizes the sum of the efficiencies for all DMUs. Where d_k is the level of inefficiency of DMU k . (Klimberg and Ratick 2008)

$$\begin{aligned} &Max \sum_k (1 - d_k) \\ &s.t: \end{aligned} \tag{14}$$

$$\sum_i v_i x_{ik} = 1 \quad \forall k$$

$$\sum_r u_r y_{rk} + d_k = 1 \quad \forall k$$

$$\sum_r u_r y_{rj} - \sum_i v_i x_{ij} \leq 0 \quad \forall j, k$$

$$d_k, v_i, u_r \geq 0 \quad \forall i, j, k, \quad i = 1, \dots, m, \quad r = 1, \dots, s$$

By Combination the model (13) and the model (14), we will get the model (15).

$$\begin{aligned} &Max \sum_k (1 - d_k) \\ &s.t: \end{aligned} \tag{15}$$

$$\sum_i [v_i x_{ik}^0 - p_i (x_{ik}^0 - x_{ik}^1)] = 1$$

$$\sum_r [q_r (y_{rk}^0 - y_{rk}^1) + u_r y_{rk}^1] + d_k = 1$$

$$\sum_r q_r [(y_{rj}^0 - y_{rj}^1) + u_r y_{rj}^1] - \sum_i [v_i x_{ij}^0 - p_i (x_{ij}^0 - x_{ij}^1)] \leq 0$$

$$v_i \geq \varepsilon, u_r \geq \varepsilon, \quad 0 \leq q_r < u_r, \quad 0 \leq p_i < v_i, \\ i = 1, \dots, m, \quad r = 1, \dots, s$$

The combination of model (15) with model UPLP (model (1)) results in the final model, i.e, model (16).

$$\begin{aligned} &Max \sum_k \sum_l (1 - d_{kl}) \\ &Min \sum_k \sum_l c_{kl} dem_l t_{kl} + \sum_k F_k t_k \\ &s.t: \end{aligned} \tag{16}$$

$$\sum_k t_{kl} = 1 \quad \forall l$$

$$t_{kl} \leq t_k \quad \forall k, l$$

$$\sum_i [v_{ikl} x_{ikl}^0 - p_{ikl} (x_{ikl}^0 - x_{ikl}^1)] = t_{kl} \quad \forall i, k, l$$

$$\sum_r [q_{rkl} (y_{rkl}^0 - y_{rkl}^1) + u_{rkl} y_{rkl}^1] + d_{kl} = t_{kl} \quad \forall r, k, l$$

$$\sum_r [q_{rsh} (y_{rsh}^0 - y_{rsh}^1) + u_{rkl} y_{rsh}^1]$$

$$- \sum_l [v_{ikl} x_{ish}^0 - p_{ish} (x_{ish}^0 - x_{ish}^1)] \leq 0$$

$$t_{kl}, t_k = 0, 1$$

$$\begin{aligned} \varepsilon t_{kl} &\leq q_{rkl} \leq u_{rkl} \\ \varepsilon t_{kl} &\leq p_{ikl} \leq v_{ikl}, \\ i &= 1, \dots, m, \quad r = 1, \dots, s \end{aligned}$$

Where

x_{ikl}^0 : impossible bound for i^{th} input of DMU kl

x_{ikl}^1 : risk free bound for i^{th} input of DMU kl

y_{rkl}^0 : risk free bound for r^{th} output of DMU kl

y_{rkl}^1 : impossible bound for r^{th} output of DMU kl

F_k : fixed cost of opening/using facility k

dem_l : the amount of demand at node l

$$t_k = \begin{cases} 1 & \text{if facility } k \text{ open} \\ 0 & \text{OW} \end{cases}$$

$$t_{kl} = \begin{cases} 1 & \text{if facility } k \text{ serves demand } l \\ 0 & \text{OW} \end{cases}$$

c_{kl} : cost of shipping one unit of demand from facility k to demand l ,

d_{kl} : the level of inefficiency of DMU kl .

v_{ikl} : input weight for i^{th} input of DMU kl

u_{rkl} : output weight for r^{th} output of DMU kl

3.3. Numerical Example I

An example with four facilities and two demand nodes was created to test model (16). The input and output values, fixed cost of opening/using facilities, the amount of demand at node l and cost of shipping one unit of demand from facilities to demand nodes are listed in Table 1 and Table 2. The model (16) was run with the data discussed above by assigning weights to objective functions and yielded the following results which are shown in Table 3 and Table 4. Here, w_1 is the weight on the DEA objective function and w_2 is the

weight on the cost objective function.

3.4. Combined CPLP/interval DEA model

Like the section 3.2, we combine the CPLP model (model (2)) with the model (15) and get the simultaneous CPLP/interval DEA model i.e, model (17).

$$Max \sum_k \sum_l (1 - d_{kl}) \tag{17}$$

$$Min \sum_k \sum_l c_{kl} b_{kl} + \sum_k F_k t_k$$

$$\sum_k t_{kl} \geq 1 \quad \forall l$$

$$t_{kl} \leq t_k \quad \forall k, l$$

$$\sum_l b_{kl} = dem_l \quad \forall k$$

$$b_{kl} \leq [dem_l, y_{kl}^0] t_k \quad \forall k, l$$

$$\sum_i [v_{ikl} x_{ikl}^0 - p_{ikl} (x_{ikl}^0 - x_{ikl}^1)] = t_{kl} \quad \forall i, k, l$$

$$\sum_r [q_{rkl} (y_{rkl}^0 - y_{rkl}^1) + u_{rkl} y_{rkl}^1] + d_{kl} = t_{kl} \quad \forall r, k, l$$

$$\sum_r [q_{rsh} (y_{rsh}^0 - y_{rsh}^1) + u_{rkl} y_{rsh}^1]$$

$$- \sum_l [v_{ikl} x_{ish}^0 - p_{ish} (x_{ish}^0 - x_{ish}^1)] \leq 0$$

$$t_{kl}, t_k = 0, 1$$

$$\varepsilon t_{kl} \leq q_{rkl} \leq u_{rkl}$$

$$\varepsilon t_{kl} \leq p_{ikl} \leq v_{ikl},$$

$$b_{kl} \geq t_{kl}$$

Here, b_{kl} is amount of shipping units from facility k to demand node l . The other parameters and variable are the same with the model (16).

3.5. Numerical Example II

The model (17) was run with the data discussed in Numerical Example I and the results are shown in Table 5 and Table 6.

Table 1. Fixed cost of opening facilities, input/output values and cost of shipping one unit of demand from facilities to demand nodes

| facilities | fixed cost of opening facilities | demand node | input values | output values | c_{kl} |
|------------|----------------------------------|-------------|--------------|---------------|----------|
| 1 | 220\$ | 1 | [11,17] | [31,38] | 5\$ |
| 1 | | 2 | [12,15] | [32,37] | 9\$ |
| 2 | 270\$ | 1 | [11,15] | [26,30] | 6\$ |
| 2 | | 2 | [12,16] | [27,31] | 7\$ |
| 3 | 240\$ | 1 | [5,13] | [25,30] | 5\$ |
| 3 | | 2 | [11,17] | [27,31] | 7\$ |
| 4 | 300\$ | 1 | [6,12] | [24,32] | 9\$ |
| 4 | | 2 | [11,18] | [31,38] | 7\$ |

Table 2. Demand requirement of each demand node

| demand node | 1 | 2 |
|------------------|---|----|
| amount of demand | 8 | 20 |

Table 3. Results for Numerical Example I

| t_k, t_{kl} | weights on objective functions | | | |
|--------------------------------|--------------------------------|----------------------------|----------------------------|------------------------|
| | $w_1 = 0$ $w_2 = 1$ | $w_1 = 0.4$ $w_2 = 0.6$ | $w_1 = 0.8$ $w_2 = 0.2$ | $w_1 = 1$ $w_2 = 0$ |
| t_1 | 0 | 0 | 0 | 1 |
| t_2 | 0 | 0 | 0 | 1 |
| t_3 | 1 | 1 | 1 | 1 |
| t_4 | 0 | 0 | 0 | 1 |
| t_{11} | 0 | 0 | 0 | 0 |
| t_{12} | 0 | 0 | 0 | 0 |
| t_{21} | 0 | 0 | 0 | 0 |
| t_{22} | 0 | 0 | 0 | 0 |
| t_{31} | 1 | 1 | 1 | 1 |
| t_{32} | 1 | 1 | 1 | 1 |
| t_{41} | 0 | 0 | 0 | 1 |
| t_{42} | 0 | 0 | 0 | 0 |
| Total fixed costs | 240\$ | 240\$ | 240\$ | 1030\$ |
| Total transport costs | 180\$ | 180\$ | 180\$ | 188\$ |
| Total costs | 420\$ | 420\$ | 420\$ | 1218\$ |
| Total sum of efficiency scores | 6.5351 | 6.6783 | 6.6783 | 7.8069 |

Table 4. Efficiency scores for DMUs in Numerical Example I

| weights on objective functions DMUs | $w_1 = 0$ $w_2 = 1$ | $w_1 = 0.4$ $w_2 = 0.6$ | $w_1 = 0.8$ $w_2 = 0.2$ | $w_1 = 1$ $w_2 = 0$ |
|--|------------------------|----------------------------|----------------------------|------------------------|
| (1,1) | 1 | 1 | 1 | 1 |
| (1,2) | 1 | 1 | 1 | 1 |
| (2,1) | 1 | 1 | 1 | 1 |
| (2,2) | 1 | 1 | 1 | 1 |
| (3,1) | 0.025 | 0.3404 | 0.3404 | 1 |
| (3,2) | 0.5101 | 0.3379 | 0.3379 | 0.8069 |
| (4,1) | 1 | 1 | 1 | 1 |
| (4,2) | 1 | 1 | 1 | 1 |

Table 5. Results of Numerical Example II

| t_k, t_{kl} | weights on objective functions | | | |
|--------------------------------|--------------------------------|----------------------------|----------------------------|------------------------|
| | $w_1 = 0$ $w_2 = 1$ | $w_1 = 0.4$ $w_2 = 0.6$ | $w_1 = 0.8$ $w_2 = 0.2$ | $w_1 = 1$ $w_2 = 0$ |
| t_1 | 1 | 1 | 1 | 1 |
| t_2 | 1 | 1 | 1 | 1 |
| t_3 | 1 | 1 | 1 | 1 |
| t_4 | 0 | 0 | 0 | 1 |
| t_{11} | 1 | 1 | 1 | 1 |
| t_{12} | 0 | 0 | 0 | 0 |
| t_{21} | 0 | 0 | 0 | 0 |
| t_{22} | 1 | 0 | 0 | 0 |
| t_{31} | 1 | 0 | 0 | 1 |
| t_{32} | 1 | 1 | 1 | 0 |
| t_{41} | 0 | 0 | 0 | 1 |
| t_{42} | 0 | 0 | 0 | 1 |
| Total fixed costs | 730\$ | 730\$ | 730\$ | 1030\$ |
| Total transport costs | 188\$ | 188\$ | 188\$ | 195\$ |
| Total costs | 608\$ | 608\$ | 608\$ | 1225\$ |
| Total sum of efficiency scores | 5.9643 | 7.5082 | 7.5082 | 8 |

Table 6. Efficiency scores for DMUs in Numerical Example II

| weights on objective functions DMUs | $w_1 = 0$ $w_2 = 1$ | $w_1 = 0.4$ $w_2 = 0.6$ | $w_1 = 0.8$ $w_2 = 0.2$ | $w_1 = 1$ $w_2 = 0$ |
|--|------------------------|----------------------------|----------------------------|------------------------|
| (1,1) | 0.987 | 0.987 | 0.987 | 1 |
| (1,2) | 1 | 1 | 1 | 1 |
| (2,1) | 1 | 1 | 1 | 1 |
| (2,2) | 0.3593 | 1 | 1 | 1 |
| (3,1) | 0.3202 | 1 | 1 | 1 |
| (3,2) | 0.2978 | 0.5212 | 0.5212 | 1 |
| (4,1) | 1 | 1 | 1 | 1 |
| (4,2) | 1 | 1 | 1 | 1 |

4. Conclusion

In this paper, we combined location/allocation models with DEA in interval inputs and outputs environments to improve performance of these models. Solving for the DEA efficiency measure, simultaneously with other location modeling objectives, provides a promising rich approach to multi objective location problems. The ability to use location models to test trade-offs between spatial efficiency and facility efficiency provides a promising new rich approach for multi objective location analysis. We presented a new pair of location/DEA models for dealing with interval data. The presented models used the interval CCR model and combined it with UPLP and CPLP models to optimize two efficiencies, spatial efficiency and facilities efficiency. Due to the existence of uncertainty in real world conditions, our models dealt with interval inputs and outputs. The models were run with the data discussed in Numerical Example I and the results obtained. Since interval efficiencies measure the performances of DMUs more comprehensively than the traditional DEA efficiency, they are expected to have widely potential applications in the future.

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New DEA/Location Models with Interval Data

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ارائه مدل جدید مکان یابی - تخصیص، با استفاده از تحلیل پوششی داده های بازه ای

چکیده: به تازگی مفهوم کارایی تجهیزات، به گونه ای که در تحلیل پوششی داده ها مطرح می شود، به عنوان یکی دیگر از اهداف مکان یابی مطرح شده است تا تاثیر گذاری مکان تجهیزات بر کارایی آن ها در سرویس دهی به تقاضاها نیز در مساله لحاظ شود. با ادغام تحلیل پوششی داده ها در مساله مکان یابی، دو نوع کارایی و در نتیجه کارایی کل بهینه می شود. یکی کارایی مکانی که با یافتن الگوی مکانی با حداقل هزینه توسط تابع هدف های سابق برآورده می شود، و دیگری کارایی تجهیزات در سرویس دهی به تقاضاها که با امتیازات به دست آمده از تکنیک تحلیل پوششی داده ها برای الگوی منتخب برآورده می شود. با توجه به این که تصمیم گیری در مورد مکان یابی عمدتاً از تصمیم گیری های بلند مدت و استراتژیک است، با گنجاندن تحلیل پوششی داده ها در حالت های مختلف (غیر قطعی و دخیل کردن نظر کارشناسان در وزن ها) با مدل های مکان یابی، یک الگوی مکانی خوب، الگویی که هم تعامل مکانی بین تجهیزات و هم کارایی و عملکرد تجهیزات در آن مکان ها را بهینه کند، به دست می آید. در این مقاله سعی بر آن است تا مدل های مکان یابی - تخصیص مراکز با ظرفیت سرویس نامحدود و همچنین مکانیابی مراکز با ظرفیت سرویس محدود را با مدل های تحلیل پوششی داده ها در حالت عدم قطعیت (بازه ای) تلفیق کرده تا مدل های کارآمد تری بدست آیند.

کلمات کلیدی: تحلیل پوششی داده ها، مدل مکان یابی - تخصیص با ظرفیت سرویس محدود و نامحدود، داده های بازه ای.