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Research Article

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Estimating Most Productive Scale Size with Double Frontiers in Data Envelopment Analysis using Negative Data

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Abstract

In this paper, it is assumed that the "Decision Making Units"(DMU_s) are consist of positive and negative input and output. Firstly, the optimistic and pessimistic models have been suggested by using negative data and then units with most productive scale size are measured in optimistic and pessimistic models. These productive values are compared with double frontiers and Hurwicz's Criterion to obtain *DMU* with *MPSS*.

***Keywords**:* Data envelopment analysis; Most Productive Scale Size; Optimistic efficiency; Pessimistic efficiency; double frontiers; Negative data .

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1. Introduction

One of the most common DEA models is the CCR model, which was initially proposed by Charnes, Cooper and Rhodes [1], in 1978 to measure the efficiency of a set of DMU_s .This model is the extension of Farrel measure used for multiple inputs and outputs and it deals with the calculation of radial efficiency in PPS under Constant returns to scale (CRS) and it has two characteristics of input orientation (envelopment form), and output orientation (multiplier form), that all the input and output values are non negative, whereas in many applications, the negative inputs and outputs could be appear as loss when the net profit is an output variable. Later on, various approaches presented which have the way for using negative data in this model and other models like the Semi-Oriented measurement. The DEA was presented based on the need for scientific method to analyze economic unit's performance. Therefore, returns to scale (RTS) as an economical concept could be evaluated under DEA models. Indeed, returns to scale is related to the economical interpretation of the efficiencies of DEA. Returns to scale is the effect of means of production over production and has three type of "increasing", "decreasing" and "constant".

In special case, if a DMU has a Constant returns to scale (CRS) – when any multi of inputs, Produce the same multi of outputs, than the DMU in this state, has the highest $MPSS$ which represent a very important in DEA and connected with the RTS. The concept of the $MPSS$ was introduced into DEA by Banker (1984). Later, Cooper et al. (1996) provided a fractional objective function model for determining the $MPSS$. Jahanshahloo and Khodabakhshi (2003) proposed an input-output orientation model for estimating the $MPSS$ with a linear objective function. Banker et al. (2004) reviewed of the development of $MPSS$ as one part of the literature review of RTS. Khodabakhshi (2009) discussed the estimation of the $MPSS$ when the stochastic data are obtained (see [7]). However, all the papers about the $MPSS$ in DEA are based on the optimistic point of view. Since the performances of decision making units (DMU_s) can also be measured from the pessimistic point of view (see [5]). Since, the results of $MPSS$ application in different evaluation system might give different results, hence by applying Double Frontiers and Hurwicz's Criterion, the performance of each unit is assessed in both optimistic and pessimistic point of view (see [3], [4]).

The purpose of this paper is to study the $MPSS$ with double frontiers data envelopment analysis by using negative data.

2. Data Envelopment Analysis using negative data

Suppose we have $j=1,...,n$, DMU_s as, $\left(X_j,Y_j\right)$, where $X_j=\left(x_{1j},x_{2j},...,x_{mj}\right)$ is a vector of observed inputs and $Y_j=\left(y_{1j},y_{2j},...,y_{sj}\right)$ is a vector of observed outputs for DMU_j . Each DMU_j used for efficiency comparisons is assumed to have used the same inputs and produced the same outputs (see [2]). Suppose, without disturbing the generality of the problem:

$I = \{i / x_{ij} \geq 0; \forall i = 1,...,n\}$

k= {i/ x_{ij} DMU_s and negative for others}

can takes positive values for some when

$I \cap K = \varnothing$ And $I \cup K = \{1, 2,..., m\}$

And suppose:

$R = \{r / y_{rj} \geq 0; \forall j = 1,...,n\}$

T={r/ y_{rj} can takes positive values for some DMU_s and negative for others} when

$R \cap T = \varnothing$ And $R \cup T = \{1, 2,..., r\}$

Let us define $x_{kj} = x_{kj}^1 - x_{kj}^2$ for $k \in K$ when there in $x_{kj}^1 \geq 0$ and $x_{kj}^2 \geq 0$ for all $j = 1,...,n$ and

$$x_{kj}^1 = \begin{cases} x_{kj} & if \quad x_{kj} \geq 0 \\ 0 & if \quad x_{kj} < 0 \end{cases} \quad \&$$

$$x_{kj}^2 = \begin{cases} 0 & if \quad x_{kj} \geq 0 \\ -x_{kj} & if \quad x_{kj} < 0 \end{cases}$$

And $y_{tj} = y_{tj}^1 - y_{tj}^2$ for $t \in T$ when there in $y_{tj}^1 \geq 0$ and $y_{tj}^2 \geq 0$ for all $j = 1,...,n$ and

$$y_{tj}^1 = \begin{cases} y_{tj} & if \quad y_{tj} \geq 0 \\ 0 & if \quad y_{tj} < 0 \end{cases} \quad \&$$

$$y_{tj}^2 = \begin{cases} 0 & if \quad y_{tj} \geq 0 \\ -y_{tj} & if \quad y_{tj} < 0 \end{cases}$$

Now the CCR model under evaluations as DMU_o with semi positive and negative inputs and outputs which has been defined by Emrouznejad et al., [2] is presented as follows:

$$\begin{aligned} & \theta_o^* = Min\, \theta \\ s.t. \quad & \sum_{j=1}^{n} \lambda_j x_{ij} \leq \theta x_{io} \qquad ; \forall i \in I \\ & \sum_{j=1}^{n} \lambda_j x_{ij}^1 \leq \theta x_{ko}^1 \qquad ; \forall k \in K \\ & \sum_{j=1}^{n} \lambda_j x_{ij}^2 \geq \theta x_{ko}^2 \qquad ; \forall k \in K \\ & \sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{ro} \qquad ; \forall r \in R \\ & \sum_{j=1}^{n} \lambda_j y_{rj}^1 \geq y_{to}^1 \qquad ; \forall t \in T \\ & \sum_{j=1}^{n} \lambda_j y_{rj}^2 \leq y_{ro}^2 \qquad ; \forall t \in T \\ & \lambda_j \geq 0 \;; \quad \forall j = 1,...,n \end{aligned} \tag{1}$$

3. Estimating Most Productive Scale Size with Double Frontiers in Data Envelopment Analysis using negative data

3.1. We name model (1) as CCR optimistic model under evaluation as DMU_o with semi positive and negative inputs and outputs. And in pessimistic CCR model under evaluation as DMU_o with semi positive and negative inputs and outputs as follows:

$$\begin{aligned} & \phi_o^* = Max\ \phi \\ s.t. \quad & \sum_{j=1}^{n} \lambda_j x_{ij} \geq \phi x_{io} \qquad ; \forall i \in I \\ & \sum_{j=1}^{n} \lambda_j x_{ij}^1 \geq \phi x_{ko}^1 \qquad ; \forall k \in K \\ & \sum_{j=1}^{n} \lambda_j x_{ij}^2 \leq \phi x_{ko}^2 \qquad ; \forall k \in K \\ & \sum_{j=1}^{n} \lambda_j y_{rj} \leq y_{ro} \qquad ; \forall r \in R \\ & \sum_{j=1}^{n} \lambda_j y_{rj}^1 \leq y_{to}^1 \qquad ; \forall t \in T \\ & \sum_{j=1}^{n} \lambda_j y_{rj}^2 \geq y_{ro}^2 \qquad ; \forall t \in T \\ & \lambda_j \geq 0 \;; \quad \forall j = 1,...,n \end{aligned} \tag{2}$$

Definition 1: If θ_o^* in the model (1), is equal to one, then DMU_o is $MPSS$ in the measurement of optimistic model.

Definition 2: If ϕ_j^* is the optimal solution to model (2) under evaluation DMU_j ($j=1,...,n$) and $\phi_h^* = Max\{\phi_j^* / \forall j\}$, then DMU_h is called $MPSS$ in the pessimistic measurement.

Definition 3: The double frontiers approach, consider two efficiency frontier for decision making units, that one of them is efficiency frontier corresponding the best or optimistic efficiency and the other, is inefficiency frontier that is defined as an input frontier. This frontier represents the worst efficiency or pessimistic efficiency.

The common measurement of DEA, only consider relative efficiencies one single group of DMU, while the pessimistic efficiencies were neglected in those approaches. In fact, if we suppose relative optimistic and pessimistic efficiencies simultaneously, then all decision making units could be classified without any more need in calculations and manager's knowledge of himself in giving priority. On the other side, the optimistic and pessimistic from various view point ends to two types of Rankings for units. Therefore, we need to have a general measurement performance to achieve a comprehensive Ranking. The application of a geometric mean can be considered as one of these methods which were introduced by Wang et al. (2007). The obtained efficiency defines a set of efficient units which these units have a better relative efficiency while the obtained inefficiency defines a set of units which have relative more weak performance. Hence, usually the best decision making units could be selected among the set of efficient units.

3.2. Hurwicz's decision rule

The Hurwicz's rule is a procedure applied within the decision making process under uncertainty ($DMUU$). This uncertainty is a consequence of the fact that it is not able to anticipate the future effectively. One may just forecast various phenomena and events, but in many cases it is extremely difficult to estimate the exact value of particular parameters (temperature, company profit, size of the mature crops, demand for a product, product prices, production costs etc.). If these data were known, it would be easy to indicate the best alternative (decision), e.g. the best investment strategy. But when many future factors are not deterministic at the time of the decision, the decision maker (DM) has to choose the appropriate alternative on the basis of some scenarios (states of nature, events) predicted by experts, him or herself (Hurwicz 1951) (see [6]).

Hurwicz's criterion method is an optimistic and pessimistic method that this procedure usually leads to reasonable answers. *Hurwicz* (1951; 1952) argued that the decision maker should rank alternatives according to the weighted average of the security and the optimism levels ($I_j; j = 1,...,n$).

Now Hurwicz's criterion h_j express as follows:

$$h_j = \lambda w_j + (1-\lambda) m_j, \forall j$$

Where h_j is the Hurwicz's criterion and λ is the coefficient of pessimism which fulfills the following condition: $\lambda \in [0, 1]$, the parameter λ is close to 0 for extreme optimists. Also w_j and m_j are the worst and the best values results could be appeared, respectively.

The most well-known Hurwicz's criterion, suggested by Hurwicz (1951), selects the minimum and the maximum payoff to each given act x, and then associates to each act to attain the following index of any two acts:

$$\lambda \max(x) + (1-\lambda) \min(x)$$

The one with the maximum index would be preferred.

In the Hurwicz's criterion, the parameter λ, which reflects the degree of the decision maker's optimism, is determined by the decision maker. Since different decision makers have different criteria, it is difficult to determine the appropriate value of λ. By varying the value of λ, the Hurwicz's criterion becomes various decision rules, e.g., when λ=0, it comes out the pessimistic criterion; when λ=1, the criterion becomes the optimistic criterion. In fact there is many Hurwicz's criterion. (Hurwicz, 1951, 1952).

In this paper in order to take both advantages of the two measurements, we employ the Hurwicz's criterion for determining the final efficiency of all DMU_s.

3.3. A double frontiers measurement

Since the optimistic efficiency measurement and the pessimistic efficiency measurement are two different decision making criteria. As different measurements reflect different information on different frontiers, any measurement which considered only one of them is biased. This may lead us to think that the two measurements should be considered together for identifying the best DMU which represents the $MPSS$. Based on this idea, the following part is to construct a double frontiers approach for examining the $MPSS$.

Wang and Chin (2007), Wang and Lan (2011) used geometric method to combine both information on optimistic frontier and pessimistic frontier. Then Wang and Chin (2009) proposed a new method to obtain a double frontiers approach.

Using the Hurwicz's criterion to integrate the optimistic and the pessimistic efficiency measurements, it has obtained standardized synthesis efficiency as follows:

$$\xi_j = (1-\lambda)\frac{\theta_j^{pes}}{Max\theta_j^{pes}} + \lambda\theta_j^{opt} \quad ; \quad \lambda \in [0,1]$$

Where θ_j^{pes} stands for pessimistic efficiency obtaining from model pessimistic CCR, θ_j^{opt} stands for optimistic efficiency obtaining from model optimistic CCR and ζ_j stands for the standardized synthesis efficiency of the double frontiers approach of the DMU_j.

4. Numerical example

In this example, we make a comparison with 10 DMU_s ,where there are one positive input (cost), one non-positive input (effluent), one positive output (saleable output) and two non-positive outputs (methane and co_2) that the data set of "the notional effluent processing system" extracted from Sharp et al (2006).

Table 1: Notional effluent processing system

DMU	I_1	I_2	O_1	O_2	O_3
DMU_1	1.03	0.05	0.56	-0.09	0.44
DMU_2	1.75	-0.17	0.74	0.24	-0.31
DMU_3	1.44	-0.56	1/37	0.35	-0.21
DMU_4	10.8	0.22	5.61	-0.98	3.79
DMU_5	1.3	-0.07	0.49	-1.08	0.34
DMU_6	1.98	0.1	1.61	-0.44	0.35
DMU_7	0.97	0.17	0.82	0.08	-0.43
DMU_8	9.82	-2.32	5.61	1.42	-1.94
DMU_9	1.49	2.32	0.52	0.52	-0.37
DMU_{10}	5.95	0.15	2.14	-0.52	0.18

Table 2: Three different measurements and the MPSS

DMU	CCR-R Optimistic (Efficiency / MPSS)	CCR-RB Pessimistic (Efficiency / MPSS)	Double frontiers (Efficiency / MPSS)
DMU_1	1 MPSS	1	0.762987
DMU_2	0.56242	1	0.5141072
DMU_3	1 MPSS	1.90849 MPSS	1 MPSS
DMU_4	1 MPSS	1.44425	0.878375
DMU_5	1 MPSS	1	0.761987
DMU_6	1 MPSS	1	0.761987
DMU_7	0.888555	1	0.706262
DMU_8	0.60047	1.13895	0.598633
DMU_9	1 MPS	1	0.761987
DMU_{10}	0.63278	1	0.578377

The results achieved in table No.2, shows that from optimistic view point, the "DMU_3" has a efficiency value equal to one, and at the same time from pessimistic viewpoint it is a MPSS (because it has the maximum efficiency value among other decision making units) and in double frontiers approach, it has the highest efficiency among the other DMU_s.

5. Conclusions and Future extension

In this paper, it has been estimated the $MPSS$ by using the pessimistic and optimistic CCR model and a double frontiers approach with negative inputs and outputs.

The decision making units represented the $MPSS$ that obtains the efficiency equal to one under optimistic model. Also the decision making unit, do obtain the maximum optimal value of objective function model among other units, are known as $MPSS$. Since the performance of the $MPSS$ measured from different view-points may be different. Thus, it has been estimated a double frontiers measurement with the Hurwicz's criterion to gauge the overall performance of each DMU. This double frontiers efficiency measurement integrates both optimistic and pessimistic efficiencies of each DMU and is therefore more comprehensive than either of them. This model is applicable for other models like "Fuzzy" and "integer".

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Estimating Most Productive Scale Size with Double Frontiers in Data Envelopment Analysis using Negative Data

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تعیین واحدهایی با بالاترین سطح بهره وری با بکار گیری مرزهای دو لایه در حضور داده های منفی در DEA

چکیده: در این مقاله فرض شده است که واحدهای تصمیم گیرنده شامل ورودی و خروجی مثبت و منفی می باشند. ابتدا مدل های خوش بینانه و بدبینانه را در حضور داده های منفی پیشنهاد دادیم و سپس واحدهایی با بالاترین سطح بهره وری در مدل خوش بینانه و بدبینانه مورد ارزیابی قرار گرفت و این مقادیر بهره وری را با روش مرزهای دولایه و معیار هورویکس برای بدست آوردن واحد تصمیم گیرنده با بالاترین سطح بهره وری مقایسه کرده ایم.

کلمات کلیدی: بالاترین سطح بهره وری، کارایی خوش بینانه ، کارایی بدبینانه، مرزهای دولایه، داده های منفی.