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Calculation of the Efficiency of Two-Stage Network Structures with Additional Inputs to the Second Stage by SBM Approach: A Case Study on Credit Branches of an Iranian State Bank in Guilan Province

Saeed Jahangard Patavani [a*], Nasima Mahboubi [b]

(a) *M.Sc, Department of Applied Mathematics, Islamic Azad University, Lahijan Branch, Lahijan, Assessor Keshavarzi Bank, Guilan, Iran.*

(b) *M.Sc, Department of Applied Mathematics, Islamic Azad University, Lahijan Branch, Lahijan, Iran.*

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Abstract

Many studies have been conducted to determine the efficiency of two-stage network structures in the recent years. The two-stage network with additional inputs to the second stage, in which the second stage is independent of the first stage are one of these structures. Thus, there is a need for a model capable of calculating the efficiency of two-stage structures as well as efficiency of each stage which can then provide managers with recommendations to increase the efficiency of the entire system and its sub-processes. In this study, a non-cooperative game adapted from game theory and SBM approach is used to calculate the efficiency of a two-stage network structure to provide a unique analysis of the overall efficiency as the product of efficiency scores of the two stages. SBM approach is a non-radial DEA model capable of providing modification recommendations for inputs and outputs. The model then is implemented on 29 credit branches of an Iranian state bank in Guilan province and the results are analyzed.

Keywords: DEA, Two-Stage Network, Stackelberg Game, SBM Model, Bank.

[*] Corresponding Author: berjis.sj@gmail.com

1. Introduction

Data Envelopment Analysis (DEA) was first introduced and developed by Charnes et al. [1]. DEA is a non-parametric mathematical programming method for analyzing the relative efficiency of homogeneous decision making units (DMU). It has received much attention due to its many advantages compared to parametric methods. The literature has indicated that DEA can be applied in different situations such as bank performance, bankruptcy evaluation, R & D, agriculture economic, etc. [2].

In primary DEA models, DMUs are used as a black box and the DMU's internal structure is ignored [2]. Recent studies such as F¨are & Grosskopf [3], Tone & Tsutsui [4], Liang et al. [5], Ashrafi et al.[6] and Li et al.[2]. however, have aimed to resolve this issue. DEA network structure has also been studied And its applications. For example, Ghafoorian et al. [7] used the two-stage data envelopment analysis (DEA) model to investigate the effects of credit risks on bank efficiency. Keramati and Shaeri [8] also investigated credit risk management as well as efficiency management in 19 Iranian banks using network DEA. They used a three-stage DEA model which once assessed bank efficiency without considering the credit risk factor, and used the credit risk factor for a second time as an additional input to the third stage and assessed the decision unit efficiency employing the same model. Zha et al. [9] used a dynamic Slack-Based Measure (SBM) two-stage DEA model to study Chinese banks. Chilingerian and Sherman [10] developed another two-stage process for Healthcare applications. In recent studies on two-stage network systems, geometric/arithmetic means are used to decompose overall efficiency.

Kao and Hwang [11] developed a method in which, for instance, if a set of insurance companies include the two-stage operations “premium acquisition” and “profit generation”, then the overall efficiency is the efficiency multiplication of the two stages. Chen et al. [12] developed envelopment and multiplier models to decompose two-stage systems' efficiencies as well as to project an inefficient unit on the efficient frontier. Moreover, they proposed a variable intermediate measures SBM (VSBM) model to assess the efficiency of two-stage systems. They also proved through a dual VSBM model that the overall system inefficiency is obtained by summing the inefficiency of the two stages. Tone and Tsutsui [4] developed a network DEA model by SBM which assessed both efficiency types, i.e. the overall efficiency and the sub-process efficiency. the overall efficiency of a network in this model is a weighted harmonic mean of its divisional scores with the weights set exogenously.

Liang et al. [5], Li et al. [2], and Kao and Hwang [11] defined overall efficiency as the multiplication of the two stages, while Chen et al. [13] and Chen and Zhu [14] defined it as the averaged summation of the two stages. Tone and Tsutsui [15] proposed a dynamic DEA model within the framework of a slacks-based measure approach; however, decomposition of model overall efficiency was not studied.

Figure 1 was investigated by Liang et al. [5] and Kao & Hwang [11]. This Figure assumes that all the inputs of the second stage are the outputs of the first stage (intermediate measures).

Figure 2 which was proposed by Li et al. [2] shows the model presented in Figure 1 with inputs to the second stage in addition to the intermediate measures.

This model assumes that each DMU_j (j=1,...,n) includes m inputs, x_{ij} (i=1,...,m) and D outputs (intermediate measures), z_{dj} (d=1,...,D) in the first stage. D outputs of the first stage become a part of inputs to the second stage. The other inputs of the second stage are x_{hj}^{2} (h=1,...,H). The set, y_{rj} (r =1,...,s) are outputs of the second stage.

Ashrafi et al. [6] exploited the Tone and Tsutsui's study [4] to use SBM model for evaluating the overall efficiency of the system shown in Figure 1 without additional inputs.

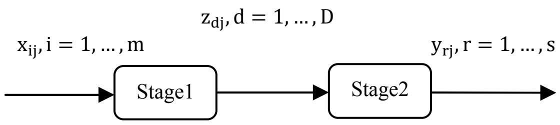

Figure 1: A two-stage process of DMU_j

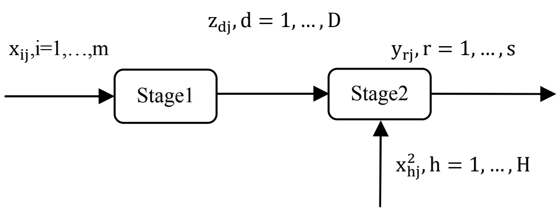

Figure 2: A two-stage process with additional inputs to the second stage for DMU_j

$$\rho_{overal}^{*} = min \frac{1 - 1/m \sum_{i=1}^{m} \frac{s_i^-}{x_{io}}}{1 + 1/s \sum_{r=1}^{s} \frac{s_r^+}{y_{ro}}}$$

s.t:

$$\begin{aligned} & x_{io} = \sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- \ , \ i = 1, \dots, m \\ & y_{ro} = \sum_{j=1}^{n} \mu_j y_{rj} - s_r^+ \ , \ r = 1, \dots, s \quad (1) \\ & \sum_{j=1}^{n} \lambda_j z_{dj} = \sum_{j=1}^{n} \mu_j z_{dj} \ , \ d = 1, \dots, D \\ & \lambda, \mu, s^-, s^+ \geq 0 \end{aligned}$$

In this model, the third constraint is used to connect two sub-processes, but it does not specify how the processes are related to each other. If stage 1 be more important than stage 2, how it is reflected in the model? In addition, the Model (1) does not provide a decomposition for the overall efficiency of the system. Is it possible to present a model that provides a decomposition of the overall efficiency of the system?

Liang et al. [5] used the game theory to present cooperative and non-cooperative (Stackelberg) multiplier models to resolve aforementioned issues. Li et al. [2] used Liang's model with additional inputs to the second stage to develop cooperative and non-cooperative (leaders and followers) multiplier models with the help of a heuristic approach. Their model is able to evaluate the overall efficiency and efficiencies of all stages. It should be noted that the optimal solutions of multiplier models are optimal weights. Thus, the zero weights will not help managers in analyzing the system. Therefore, there is a need for a model to clearly determine the efficiency score of each unit and sub-Processes and to reduce or increase inputs and outputs, respectively.

The aim of the present study is to calculate the overall efficiency of the network and its sub-processes (Fig. 2) assuming the additional inputs for the second stage using SBM model and non-cooperative game to provide a unique decomposition of the overall efficiency as the product of the efficiency scores of the two stages. the rest of the article is organized as follows. Section 2 introduces the production possibility set. Section 3 presents the model and projected DMU for an inefficient DMU is calculated. The relevant theorems are presented in Section 4. Section 5 presents the results obtained from implementation of the model on 29 credit branches of an Iranian state bank in Guilan province and concluding remarks.

2. Production possibility set

According to Tone & Tsutsui [4], the production possibility set of the model shown in Fig. 2 is defined as follows:

$$P=\{(x,y,z,x^2)\mid x \geq X\lambda,\ x^2 \geq X^2\mu,\ y \leq Y\mu,\ Z\lambda = Z\mu,\ z \leq Z\lambda\ (if\ Z\ is\ output\ vector)\ or\ z \geq Z\mu\ (if\ Z\ is\ input\ vector)\ ; \lambda \geq 0, \mu \geq 0\}$$

x^2, y and Z are defined in section 1 under the introduction to Fig. 1.The fifth property of the above production possibility set shows that the constraints z≤ Zλ and z ≥ Zμ cannot simultaneously exist in a mathematical model.

3. Model

We need to determine the leader and follower in the non-cooperative game. The first stage is assumed to be the leader. Therefore, this stage is more important. The second stage is known as the follower player which takes decisions according to the decisions of the leader.

The following model calculates the efficiency of the first stage (leader):

$$\rho_1^* = min \frac{1-\frac{1}{m}\sum_{i=1}^{m}\frac{s_i^-}{x_{io}}}{1+\frac{1}{D}\sum_{d=1}^{D}\frac{l_d^+}{z_{do}}}$$

s.t:

$$x_{io} = \sum_{j=1}^{n}\lambda_j x_{ij} + s_i^-\ ,\ i=1,\dots,m$$
$$z_{do} = \sum_{j=1}^{n}\lambda_j z_{dj} - l_d^+\ ,\ d=1,\dots,D$$
$$\lambda, s^-, l^+ \geq 0 \qquad (2)$$

The Model (2) is the primary SBM model for the single-stage DMUs where λ^*, s^{-*} and l^{+*} are the optimal solutions. Since the two sub-processes are linked together by intermediate measures and the follower's decisions are intended to be taken according to the decisions of the leader, therefore s^{-*} and l^{+*} should be considered in the calculation of the efficiency of the follower (see Doyl & Green [16]). Based on the Li et al. [2], the following model is proposed to calculate the efficiency of the second stage:

$$\rho_2^* = min \frac{1-\frac{1}{D+H}\left(\sum_{d=1}^{D}\frac{l_d^-}{z_{do}}+\sum_{h=1}^{H}\frac{p_h^-}{x_{ho}^2}\right)}{1+\frac{1}{s}\sum_{r=1}^{s}\frac{s_r^+}{y_{ro}}}$$

s.t:

$$x_{io} = \sum_{j=1}^{n}\lambda_j x_{ij} + s_i^-\ ,\ i=1,\dots,m$$

$z_{do} = \sum_{j=1}^{n} \mu_j z_{dj} + l_d^-\ ,\ d = 1, \dots, D$

$x_{\text{ho}}^2 = \sum_{j=1}^{n} \mu_j x_{\text{hj}}^2 + p_h^-\ ,\ h = 1, \dots, H$

$y_{ro} = \sum_{j=1}^{n} \mu_j y_{rj} - s_r^+\ ,\ r = 1, \dots, s$

$\sum_{j=1}^{n} \lambda_j z_{dj} = \sum_{j=1}^{n} \mu_j z_{dj}\ ,\ d = 1, \dots, D$

$$\frac{1 - \frac{1}{m}\sum_{i=1}^{m} \frac{s_i^-}{x_{io}}}{1 + \frac{1}{D}\sum_{d=1}^{D} \frac{l_d^+}{z_{do}}} = \rho_1^* \qquad (3)$$

$\lambda, \mu, s^+, s^-, l^-, l^+, p^- \geq 0$

The constraint number 6 guarantees that s^{-^*} and l^{+^*} related to the efficiency of the first stage are considered in the calculation of the efficiency of the second stage. the above models are under the constant returns-to-scale assumption(CRS). By adding the relations (4) to the Models (2) and (3), calculation can also be done under the variable returns-to-scale assumption (VRS).

$$\sum_{j=1}^{n} \lambda_j = 1 \ \text{ and } \ \sum_{j=1}^{n} \mu_j = 1 \qquad (4)$$

After calculating the efficiency of the second stage, the overall efficiency of the system is calculated as follows:

$$\rho^{non,1,*} = \rho_1^* * \rho_2^* \qquad (5)$$

Equation (5) gives an decomposition of the overall efficiency of the system. Moreover, clearly, $\rho^{non,1,*} = 1$ if and only if $\rho_1^{o} = 1$ and $\rho_2^{o} = 1$.

Definition 1 (efficient NSBM): A DMU_o is an efficient network SBM, if and only if $\rho^{non,1,*} = 1$.

This definition is equivalent to s^-=0, l^+=0, l^-=0, p^-=0 and s^+=0. In other words, there is no inputs excesses and outputs Shortfalls in the inputs and outputs of the model.

If the second stage is the leader, then:

$$\eta_2^* = min \frac{1 - \frac{1}{D+H}\left(\sum_{d=1}^{D} \frac{l_d^-}{z_{do}} + \sum_{h=1}^{H} \frac{p_h^-}{x_{\text{ho}}^2}\right)}{1 + \frac{1}{s}\sum_{r=1}^{s} \frac{s_r^+}{y_{ro}}}$$

s.t:

$z_{do} = \sum_{j=1}^{n} \mu_j z_{dj} + l_d^-\ ,\ d = 1, \dots, D$

$x_{\text{ho}}^2 = \sum_{j=1}^{n} \mu_j x_{\text{hj}}^2 + p_h^-\ ,\ h = 1, \dots, H$

$y_{ro} = \sum_{j=1}^{n} \mu_j y_{rj} - s_r^+\ ,\ r = 1, \dots, s \qquad (6)$

$\dots . \mu, s^+, l^-, p^- \geq 0$

$$\eta_1^* = min \frac{1 - \frac{1}{m}\sum_{i=1}^{m} \frac{s_i^-}{x_{io}}}{1 + \frac{1}{D}\sum_{d=1}^{D} \frac{l_d^+}{z_{do}}}$$

s.t:

$x_{io} = \sum_{j=1}^{n} \lambda_j x_{ij} + s_i^-\ ,\ i = 1, \dots, m$

$z_{do} = \sum_{j=1}^{n} \lambda_j z_{dj} - l_d^+\ ,\ d = 1, \dots, D$

$x_{\text{ho}}^2 = \sum_{j=1}^{n} \mu_j x_{\text{hj}}^2 + p_h^-\ ,\ h = 1, \dots, H$

$y_{ro} = \sum_{j=1}^{n} \mu_j y_{rj} - s_r^+\ ,\ r = 1, \dots, s$

$\sum_{j=1}^{n} \lambda_j z_{dj} = \sum_{j=1}^{n} \mu_j z_{dj}\ , d = 1, \dots, D \quad (7)$

$$\frac{1 - \frac{1}{D+H}\left(\sum_{d=1}^{D} \frac{l_d^-}{z_{do}} + \sum_{h=1}^{H} \frac{p_h^-}{x_{\text{ho}}^2}\right)}{1 + \frac{1}{s}\sum_{r=1}^{s} \frac{s_r^+}{y_{ro}}} = \eta_2^*$$

$\lambda, \mu, s^+, s^-, l^-, l^+, p^- \geq 0$

And the overall efficiency is calculated as follows:

$$\eta^{\text{non},2,*} = \eta_1^* * \eta_2^* \qquad (8)$$

3.1. SBM projection

It is assumed that the optimal solutions of Models (3) and (7) are available for DMU_o like $(\lambda^*, s^{-^*}, \mu^*, s^{+^*}, p^{-^*})$. Then, the DMU_o projection is obtained from the following relations:

$x_{io}{}^* \leftarrow x_{io} - s_i^{-^*}$

$y_{ro}{}^* \leftarrow y_{ro} + s_r^{+^*}$

$x_{ho}^2{}^* \leftarrow x_{ho}^2 - p_h^{-^*}$

$$z_{do}{}^{*} \leftarrow (\sum_{j=1}^{n} \lambda^{*}{}_{j} z_{dj} = \sum_{j=1}^{n} \mu^{*}{}_{j} z_{dj}) \qquad (9)$$

4. Theorems

Theorem 1: The virtual DMU obtained from the Model (9) is globally efficient.

Proof: Assume that the virtual DMU obtained from the Model (9) is not globally efficient. It is also assumed that the optimum solution of the virtual DMU is obtained from the Model (3) as follows:

$$(\acute{\lambda}, \acute{s}^{-}, \acute{\mu}, \acute{s}^{+}, \acute{p}^{-})$$

Therefore:

$$x_{io}{}^{*} = \sum_{j=1}^{n} \acute{\lambda}_j x_{ij} + \acute{s}_i^{-}.$$
$$x_{ho}^{2}{}^{*} = \sum_{j=1}^{n} \acute{\mu}_j x_{hj}^{2} + \acute{p}_h^{-}. \qquad (10)$$
$$y_{ro}{}^{*} = \sum_{j=1}^{n} \acute{\mu}_j y_{rj} - \acute{s}_r^{+}.$$

Substituting relation (9) in relation (10):

$$x_{io} = \sum_{j=1}^{n} \acute{\lambda}_j x_{ij} + \acute{s}_i^{-} + s_i^{-*}$$
$$x_{ho}^{2} = \sum_{j=1}^{n} \acute{\mu}_j x_{hj}^{2} + \acute{p}_h^{-} + p_h^{-*} \qquad (11)$$
$$y_{ro} = \sum_{j=1}^{n} \acute{\mu}_j y_{rj} - \acute{s}_r^{+} - s_r^{+*}$$

Given the objective function of the Model (3):

$$\acute{\rho}_2 = min \frac{1 - \frac{1}{D+H}(\sum_{d=1}^{D} \frac{l_d^{-}}{z_{do}} + \sum_{h=1}^{H} \frac{p_h^{-} + p_h^{-*}}{x_{ho}^{2}})}{1 + \frac{1}{s}\sum_{r=1}^{s} \frac{s_r^{+} + s_r^{+*}}{y_{ro}}} \qquad (12)$$

According to proof by contradiction, since the solution of the Model (9) is not efficient, then:

$$\acute{p}_h^{-} > 0 \quad and \quad \acute{s}_r^{+} > 0$$

Therefore: $\acute{\rho}_2 \leq \rho_2^{*}$

This is inconsistent with the optimality of $\rho_2{}^{*}$. Thus, the absurd hypothesis is canceled and the virtual DMU obtained from the Model (9) is globally efficient.

Theorem 2: If ρ_1^{*} and ρ_2^{*} are respectively the efficiencies of the first and second stages when the first stage is the leader and η_1^{*} and η_2^{*} are the corresponding efficiencies when the second stage is leader, then:

$$\rho_1^{*} \leq \eta_1^{*} \quad , \quad \rho_2^{*} \geq \eta_2^{*}$$

Proof: Assume that the solution of the Model (7) is as follows:

$$(\lambda^{non,2,*}, \mu^{non,2,*}, s^{+non,2,*}, s^{-non,2,*}, l^{-non,2,*}, l^{+non,2,*}, p^{-non,2,*})$$

φ is defined as follows:

$$\varphi = (\lambda^{non,2,*}, s^{-non,2,*}, l^{+non,2,*})$$

φ is a feasible solution for the Model (2). Given the objective function of the Model (2):

$$\rho_1^{*} \leq min \frac{1 - \frac{1}{m}\sum_{i=1}^{m} \frac{s^{-non,2,*}}{x_{io}}}{1 + \frac{1}{D}\sum_{d=1}^{D} \frac{l^{+non,2,*}}{z_{do}}} \qquad (13)$$

The term on right hand of the relation (13) is the optimal solution of the Model (7). Thus, $\rho_1^{*} \leq \eta_1^{*}$.

Similarly, it can be proved that $\rho_2^{*} \geq \eta_2^{*}$.

According to the first theorem in the Li et al. model [2], $e_1^{*} \geq \pi_1^{*}$ (where e_1^{*} and $\pi_1{}^{*}$ are the efficiencies of the first stage when the first and second stages are leaders respectively). However, the opposite occurred in the above theorem because of the nature of SBM model seeking the maximum value of inputs excesses and outputs shortfalls. Therefore, when the first stage is chosen as leader, the SBM model suggests maximum slack variables to modify inputs and outputs. Accordingly, the efficiency of the leader stage will be minimal.

Using the SBM model, we were able to determine the overall efficiency of a two-stage network with additional inputs to the second stage as well as the efficiency of the sub-processes, and ultimately derive a unique decomposition for the overall efficiency. We also defined projections of inefficient DMUs on the efficient frontier so that CEOs could distinguish efficient units from inefficient ones, and lead the inefficient units towards efficiency. An application of the model is presented as follows.

5. Application

The models and findings of this paper are applied on 29 credit branches of an Iranian state bank in Guilan province. Figure 3 shows the processes of the branches to attract and distribute resources which consist of two sub-processes of resources outfit and credits.

In the first stage (resources outfit), bank branches try to attract deposits and give the absorbed deposits to the second stage (credits) to be used for payments and revenue. In addition to new payment loans in the second stage, it also has the task of collecting paid loans. Since a part of the new loans are supplied by reception of the previously paid loans, the lack of reception of loans would be costly for the branch. As a result, the claims from customers who received loans and did not pay more than three months past their payment due can be considered as an input for the second stage. Measures, which sure would be costly, should be taken to collect loans.

The bank service posts include cashier, Head of fund, resources outfit, Head of finance, senior user, collection, credits, expert, assessor, archivist, vice president and president. Given the role of each post, the branch staffs are partitioned into two sets. staff at the first 5 roles are a member of resource outfit set and the personnel in the next 5 roles are placed in the credits group. Given the supervisory nature of the roles of vice president and president, these were placed in the resource outfit and credits groups respectively based on the recommendations of the bank experts.

Given the above description, the inputs and outputs of each stage are explained in Figure 3.

The inputs to the resource outfit stage include:

Cs: Operational, administrative, improvement and maintenance of buildings and computer

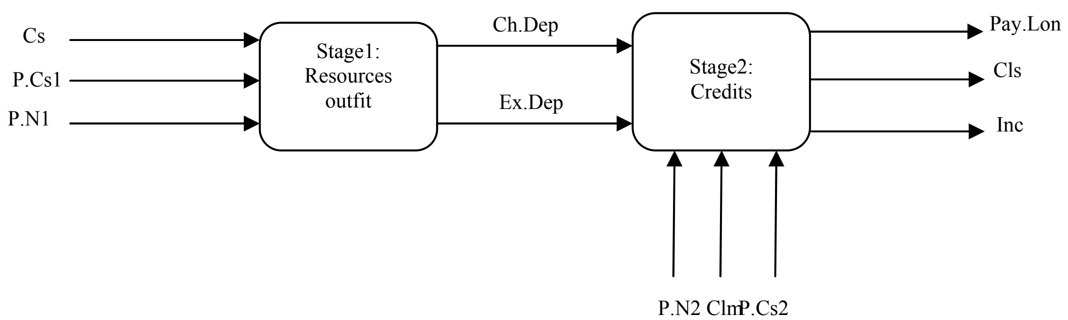

Figure 3: The two-stage model of credit branches with additional inputs to credits stage

systems costs.

P.Cs1: Personnel costs of the resources outfit division

P.N1: Number of personnel in the resource outfit division

The outputs of this stage include:

Ch.Dep: Public low-cost deposits such as checking accounts

Ex.Dep: Public long term deposits that the branch has to spend money for attracting them.

These outputs are inputs to the second stage.

The additional inputs to the second stage are independent of the first stage and include:

P.Cs2: Personnel cost of the credits stage

P.N2: Number of personnel in the credits stage

Clm: Branch claims consisting of past due loans and overdue and doubtful receivables

The outputs of the credits stage include:

Pay.Lon: payment loans from the resources collected by the branch.

Clc: Collection including the reception of all paid loans.

Inc: Total income branch.

Table 1 shows data from 29 credit branches of an Iranian state bank in Guilan province in 2013 collected from the management of the bank branches.

According to Table 2, Theorem 2 is true for the results.

Four branches are efficient when the first stage is leader and two branches are efficient when the second stage is leader. In most cases, $\rho^{non,1,*} \geq \eta^{non,2,*}$ reflecting the importance of the resources outfit stage in the bank branches.

6. Conclusion

In this study, based on the works of Tone & Tsutsui [4], Ashrafi et al. [6] and Li et al. [2], the overall efficiency of the two-stage system with additional inputs to the second stage was calculated using SBM model so that the merits of the model could be exploited. Using the game theory and Stackelberg game between the stages, leader and follower stages were chosen. The overall efficiency of the system was decomposed so that the efficiency score was obtained by multiplying the efficiencies of the two stages. The model was applied on 29 credit branches of an Iranian state bank in Guilan province and the results were studied. More sophisticated models for two-stage structures with independent and common inputs and outputs can also be used. This model can be of high significance to those organizations which, in addition to identifying their weaknesses and strengths, need solutions to cover the weaknesses. This is because SBMs are non-radial models that can provide managers and decision-makers of an organization with output shortage and input excess. Since in the present study we proved that the projection of an inefficient DMU is an overall efficiency, relying on a scientific approach, managers can lead the inefficient units to the efficiency frontier. Another advantage to this method is the use of non-

Table 1: Inputs and outputs of the 29 branches of a bank in Guilan province

DMU	input to the first stage			intermediate measures		additional inputs to the second			outputs of the second stage		
	Cs	P.Cs1	P.N1	Ch.Dep	Ex.Dep	Clm	P.Cs2	P.N2	Pay.Lon	Cls	Inc
B1	8,438	2,255	6	28,040	62,789	25,482	3,020	9	68,724	131,181	28,899
B2	12,764	1,993	6	23,925	82,092	34,589	1,787	4	56,532	122,890	25,993
B3	6,633	1,884	7	36,238	53,880	21,965	2,322	6	122,522	157,293	28,290
B4	12,287	2,464	8	30,167	75,479	46,915	1,610	4	129,082	218,904	35,890
B5	6,838	1,256	5	39,139	48,036	23,075	1,555	4	82,330	103,824	18,696
B6	12,281	3,380	11	44,705	86,940	104,302	2,375	6	128,143	259,097	57,879
B7	17,458	1,658	5	26,895	127,148	5,577	2,074	6	62,834	92,644	14,578
B8	7,599	1,170	3	25,530	48,967	6,577	1,308	4	23,310	41,820	8,792
B9	4,014	877	3	22,272	32,784	4,713	948	3	29,453	37,488	6,257
B10	6,657	2,089	6	28,177	46,842	24,723	1,644	5	129,126	166,483	26,059
B11	10,330	757	3	18,486	72,875	2,485	1,097	2	15,841	24,362	4,231
B12	9,978	2,058	6	32,064	73,512	21,048	2,303	7	33,785	69,708	10,912
B13	41,679	6,542	20	93,426	268,240	122,096	6,344	13	227,528	335,892	80,417
B14	8,176	1,878	6	34,489	59,372	41,178	1,854	6	87,309	153,895	27,836
B15	5,705	1,873	5	22,737	41,385	30,520	1,964	6	114,506	178,013	40,582
B16	2,954	1,229	4	17,978	20,456	8,486	1,133	4	42,810	66,498	12,658
B17	11,878	2,849	9	30,012	89,605	31,610	2,774	7	71,138	121,518	24,543
B18	9,511	2,214	6	47,684	70,084	33,820	1,970	5	250,001	293,432	48,047
B19	8,272	1,924	6	20,335	58,199	37,497	2,449	7	79,773	127,438	23,707
B20	3,849	951	3	18,257	23,359	16,691	1,111	3	46,308	62,777	16,146
B21	5,374	1,520	4	30,223	32,942	56,649	2,328	6	75,068	145,580	31,894
B22	7,880	2,428	6	50,967	58,399	86,146	3,615	9	286,538	437,148	78,919
B23	8,956	1,946	5	32,605	61,980	62,510	2,556	6	108,688	189,769	31,752
B24	6,640	1,171	3	29,971	45,846	28,534	1,569	5	40,766	67,406	18,321
B25	5,944	1,625	4	26,930	44,389	7,644	1,200	3	27,133	45,219	8,530
B26	8,971	3,180	9	48,675	53,930	36,099	4,929	13	86,165	148,485	29,227
B27	6,707	1,287	4	33,813	47,341	21,779	1,466	4	83,190	85,600	17,382
B28	6,686	2,754	8	32,664	47,726	55,329	3,366	9	157,850	217,954	45,623
B29	3,814	1,260	4	15,592	23,032	17,429	1,680	4	79,762	111,888	26,782

Table 2: The results of non-cooperative (Stackelberg) model

	Stage 1 is Leader			Stage 2 is Leader		
DMU	ρ_1^*	ρ_2^*	$\rho^{non,1,*}$	η_1^*	η_2^*	$\eta^{non,2,*}$
B1	0.680	0.429	0.292	0.680	0.429	0.292
B2	0.596	0.440	0.262	0.622	0.440	0.273
B3	0.817	0.625	0.511	0.817	0.625	0.511
B4	0.579	1.000	0.579	0.612	1.000	0.612
B5	1.000	1.000	1.000	1.000	0.474	0.474
B6	0.646	1.000	0.646	0.664	1.000	0.664
B7	1.000	1.000	1.000	1.000	1.000	1.000
B8	0.907	0.413	0.375	0.907	0.290	0.263
B9	1.000	0.632	0.632	1.000	0.380	0.380
B10	0.673	0.731	0.492	0.673	0.731	0.492
B11	1.000	1.000	1.000	1.000	0.407	0.407
B12	0.756	0.210	0.159	0.756	0.210	0.159
B13	0.665	0.407	0.271	0.686	0.407	0.279
B14	0.794	0.488	0.387	0.794	0.488	0.387
B15	0.660	1.000	0.660	0.674	1.000	0.674
B16	0.696	0.646	0.450	0.696	0.646	0.450
B17	0.579	0.370	0.214	0.579	0.370	0.214
B18	0.919	1.000	0.919	0.919	1.000	0.919
B19	0.561	0.486	0.272	0.561	0.486	0.272
B20	0.737	0.510	0.376	0.737	0.510	0.376
B21	0.803	0.519	0.417	0.803	0.519	0.417
B22	1.000	1.000	1.000	1.000	1.000	1.000
B23	0.782	0.530	0.415	0.782	0.530	0.415
B24	1.000	0.307	0.307	1.000	0.300	0.300
B25	0.820	0.393	0.323	0.825	0.312	0.257
B26	0.661	0.368	0.243	0.661	0.368	0.243
B27	0.974	0.459	0.447	0.974	0.459	0.447
B28	0.630	0.693	0.437	0.630	0.693	0.437
B29	0.583	1.000	0.583	0.583	1.000	0.583

cooperative (Stackelberg) model of behavior and its outcomes; since in practice, network processes consider leaders and followers in financial and nonfinancial organizations as well as state and private organizations.

We also proved that if the first stage is the leader, then its efficiency score is lower than when the second stage is considered as the leader. This is concluded from SBM model which indicates all units' inefficiencies. Therefore, careful attention must be paid to the selection of the leader stage so that it is in line with the nature of the studied network.The method presented in this paper can be used for multi-stage network structures.

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References:

[1] Charnes A, Cooper WW, Rhodes E. Measuring the efficiency of decision making units. European Journal of operational Research. 1978; 2(6):429-44.

[2] Li Y, Chen Y, liang L, Xie J. DEA models for extended two-stage network, structures. Omega. 2012; 5(40): 611-18.

[3] F¨are R, Grosskopf S. Network DEA. Socio-Economic Planning Sciences. 2000; 34, 35–49.

[4] Tone K, Tsutsui M. Network DEA: a slacks-based measure approach. European Journal of Operational Research. 2009; 197(1): 243–52.

[5] Liang L, Cook WD, Zhu J. DEA models for two-stage processes :game approach and efficiency decomposition. Naval Research Logistics. 2008; 55(7): 643-53.

[6] Ashrafi A, Jaafar AB, Lee LS, Abu Bakar MR. A Slacks-Based measure of sufficiency in two-stage data envelopment analysis. int.Journal of math. analysis. 2011; 5(29): 1435-44.

[7] Ghafoorian H, Anuar M, Norhan N, Abubakar M, Mahammadi F. Efficiency Considering Credit Risk in Banking Industry, using two-stage DEA. Journal of social and Development Sciences. 2013; 4(8):356-60.

[8] Keramati MA, Shaeri M. Assessment of credit Risk management and Managerial Efficiency of Banks Using Data Envelopment Analysis (DEA) Network. Biological Forum- An International Journal. 2014; 2(6): 320-28.

[9] Zha Y, Liang N, Wu M, Bian Y. Efficiency evaluation of banks in China: A dynamic two-stage slacks-based measure approach. Omega. 2016; 60: 60-72.

[10] Chilingerian J, Sherman HD. Health care applications in: From hospitals to physician, from productive efficiency to quality frontiers, Handbook on data envelopment analysis. Cooper ww, Seiford LM, Zhu j, Editors. Boston: Springer; 2004.

[11] Kao C, Hwang SN. Efficiency decomposition

in two-stage data envelopment analysis: an application to non-life insurance companies in Taiwan. European Journal of operational Research. 2008; 185(1): 418-29.

[12] Chen Y, Li Y, Liang L, Salo A, Wu H. Frontier projection and efficiency decomposition in two-stage processes with slacks-based measures. European Journal of Operational Research. 2016; 250(2): 543-54.

[13] Chen Y, Cook WD, Li N, Zhu J. Additive efficiency decomposition in two-stage DEA. European Journal of Operational Research. 2009; 196(3): 1170–6.

[14] Chen Y, Zhu J. Measuring information technology's indirect impact on firm performance. Information Technology & Management Journal. 2004; 5(1-2): 9–22.

[15] Tone K, Tsutsui M. Dynamic DEA with network structure: A slacks-based measure approach. Omega. 2014; 42(1): 124–31.

[16] Doyl J, Green R. Efficiency and cross-efficiency in DEA: derivation, meanings and uses. Journal of the operational Research Society. 1994; 45(5): 567-78.

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Calculation of the Efficiency of Two-Stage Network Structures with Additional Inputs to the Second Stage by SBM Approach: A Case Study on Credit Branches of an Iranian State Bank in Guilan Province

Saeed Jahangard Patavani, Nasima Mahboubi

محاسبه کارایی ساختار های شبکه دومرحله ای با ورودی های اضافی به مرحله دوم توسط روش SBM: مطالعه موردی روی شعب اعتباری یکی از بانک های دولتی ایران در استان گیلان

چکیده: در سال های اخیر مطالعات فراوانی جهت محاسبه کارایی ساختارهای شبکه دومرحله ای انجام گرفته است. شبکه دومرحله ای با ورودی های اضافی به مرحله دوم یکی از این همین ساختارهاست که در آن مرحله دوم دارای ورودی های اضافی و مستقل از مرحله اول است. این مقاله توسط بازی غیرتعاونی برگرفته از تئوری بازی ها و روش SBM که یک مدل غیر شعاعی DEA با توانایی پیشنهاد اصلاح ورودی ها و خروجی ها می باشد، کارایی این نوع از ساختار شبکه ای را محاسبه و یک تجزیه یکتا از کارایی کل به صورت ضرب امتیاز کارایی های دو مرحله ارائه می کند.سپس این مدل رار روی ۲۹ شعبه اعتباری یکی از بانک های دولتی استان گیلان اجرا و نتایج تجزیه و تحلیل می شود.

کلمات کلیدی: تحلیل پوششی داده ها، شبکه دومرحله ای، بازی غیر تعاونی، مدلSBM ، بانک