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An Additive Model for Estimation Return to Scale in Regulated Environment with Quasi-Fixed Inputs in Data Envelopment Analysis (DEA)

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Abstract

The measurement of RTS amounts measures a relationship between inputs and outputs in a production structure. There are many different ways to calculate RTS in primal or dual space. But in more realistic cases, governments usually intervene on DMU's behavior as regulatory agency, this clearly represent a set of limitations and restrictions on behaviors of DMUs, So very few decisions in DMUs are made without intersecting some regulations. Therefore it is essential to be able to assess the impact of regulation on the behavior of the DMUs, and this would be ideally done by estimating returns to scale with and without the effect of the regulation.

In this paper we use additive model to provide an alternative approach for estimating returns to scale in regulated environments. The proposed model is developed to determining returns to scale in the presence of quasi-fixed inputs in Data Envelopment Analysis.

Keywords: Returns to scale, Regulation, Quasi-fixed inputs.

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1. Introduction

Measuring the efficiency of a decision making unit (DMU) has long been considered as a difficult task because one is dealing with complex economic and behavioral entities. This task becomes more difficult when it involves multiple inputs and multiple outputs. Data Envelopment Analysis (DEA) is a managerial powerful tool to evaluate the relative efficiency of each decision making unit. It was introduced by Charnes et al in 1978, with CCR model [1]. For a DMU, the production process is to consume the inputs to get the outputs, and the efficiency is to obtain more outputs with fewer inputs as much as possible. A number of different DEA models have now appeared in the literature for efficiency measurement.

It should be noted that in the production process all the inputs and outputs can be varied at the discretion of management or other users. These may be called “discretionary variables.” But “Non-discretionary variables,” not being subject to management control, may also need to be considered. The conceptual meaning of non-discretionary inputs contains a big class of variables our focus here is on inputs. For example the number of faculties of a university can be considered as non-discretionary inputs. Banker and Morey (1986) introduced non-discretionary inputs [2] and after that Charnes et al 1987 extended the additive model in order to accommodate non-discretionary variables [3].

One of the most important concepts in the theory of production is the scale of operations (RTS). It can provide beneficial information about the size of DMUs. RTS in DEA was introduced by Banker (1984) [4]. Since then, there have been many attempts to evaluate RTS within the DEA context. For example, Banker et al [5] provided an approach based on supporting hyperplane. Fare and Grosskopf [6] provided an alternative approach to estimate returns to scale which is based on optimal solutions of BCC, CCR, and CCR-BCC models. In a more realistic environment of the DMUs, not all inputs are fully discretionary and the environment in which they operate is regulated, Ouellette et al (2012) [7] showed how to introduce these refinements of the firm’s environment into the calculation of RTS. They consequently introduced regulations as an important part of the DMU´s environment. The focus of this paper is on estimating returns to scale for DEA models when DMUs face a complex environment that includes regulation and quasi-fixed inputs.

It is noteworthy that, since an inefficient DMU has more than one projection on the empirical function hence, different returns to scales can be obtained for projections of the inefficient DMU by using the proposed approach.

2. Preliminaries

In this section, BCC model for estimating returns to scale in DEA is described.

Production possibility set (PPS) is defined as PPS = $\{(X,Y) \mid Y\geq 0$ can be produced by $X \geq 0\}$ and here supposed that PPS = PPS_{BCC} in which:

T_v = PPS$_{BCC}$= $\{(X,Y) | X \geq \sum_{j=1}^{n} \lambda_j X_j,\ Y \leq \sum_{j=1}^{n} \lambda_j Y_j, \sum_{j=1}^{n} \lambda_j = 1, \lambda_j \geq 0,\ j = 1, \dots n\}$.

Let α (β) = max {α|(βx, αy,)ϵPPS} (†)

Banker defines $\mu^+$and μ^- as bellows:

$\mu^+ = \lim_{\beta \to 1^+} \frac{\alpha(\beta)-1}{\beta-1}$, $\mu^- = \lim_{\beta \to 1^-} \frac{\alpha(\beta)-1}{\beta-1}$

Now according to definition of μ^+, μ^-, the following theorem identify quality of RTS for DMU_O.

Theorem 1 Suppose that $DMU_O\ \epsilon\ \partial T_v$ then

$(i)\ \mu^+ > 1\ and\ \mu^- > 1\ if\ and\ only\ if$
$DMU_O\ has\ increasing\ RTS(IRS)$.

$(ii)\ \mu^+ < 1\ and\ \mu^- < 1\ if\ and\ only\ if$
$DMU_O\ has\ decreasing\ RTS(DRS)$.

$(iii)\ \mu^+ < 1\ and\ \mu^- > 1\ if\ and\ only\ if$
$DMU_O\ has\ conctant\ RTS(CRS)$.

To use the BCC model to calculate the returns to scale, Suppose we have n DMUs in which (DMU_j: $j = 1, \dots, n$) use m inputs x_{ij} ($i = 1, \dots, m$) to produce s outputs y_{rj} ($r = 1, \dots, s$). Moreover, the BCC multiplier model for efficiency evaluated of DMU_O is as follows:

$$\begin{aligned} Max\quad & \sum_{r=1}^{s} u_r y_{ro} + u_o \\ s.t\quad & \sum_{i=1}^{m} v_i x_{io} = 1 \qquad (1) \\ & -\sum_{i=1}^{m} v_i x_{ij} + \sum_{r=1}^{s} u_r y_{rj} + u_o \leq 0 \\ & j = 1, \dots n \\ & u_r \geq 0 \text{ و } v_i \geq 0 \end{aligned}$$

Now suppose that (u^*, v^*, u_0^*) be an optimal solution for model (1). Banker and Thrall presented the following theorem for estimatingRTS of BCC-efficient DMUs[8]

Theorem 2. Suppose that (x_0, y_0) is a point on the BCC-efficient frontier. Then, the following conditions identify the situation for RTS at the point:

(i) Increasing RTS (IRS) prevail at (x_0, y_0) if and only if $u_0^* > 0$ for all optimal solutions of model (1).

(ii) Decreasing RTS (DRS) prevail at (x_0, y_0) if and only if $u_0^* < 0$ for all optimal solutions of model(1)

(iii) Constant RTS (CRS) prevail at (x_0, y_0) if and only if $u_0^* = 0$ for at least one optimal solution of model (1).

2.1 Khodabakhshi et al. model to estimate returns to scale.

Khodabakhshi et al provided a DEA approach to calculate the returns to scale based on additive model as follows:

Suppose that DMU$_O$ is a BCC-efficient DMU and consider the following additive model that has been presented by Charnes et al. [9] to evaluate the DMU$_o$:

$$\begin{aligned} Max\ & \sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+ \\ s.t.\ & \sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- = x_{io} \quad i=1,2,\dots,m \\ & \sum_{j=1}^{n} \lambda_j y_{rj} + s_r^+ = y_{ro} \quad r=1,2,\dots,s \\ & \sum_{j=1}^{n} \lambda_j = 1 \qquad (2) \\ & \lambda_j \geq 0 \qquad j=1,2,\dots,n \\ & s_i^-, s_r^+ \geq 0 \end{aligned}$$

Definition 2. DMU$_o$ is called efficient if and only if the obtained optimal value of objective function from model (3) is zero.

Theorem3. Suppose that DMUo with input–output combination (x_0, y_0) is efficient. Therefore, we have:

(i) There is ξ>1 so that $(\xi X_o, \xi Y_o) \epsilon\ PPS$ is $inefficient$ if and only if has DMU$_o$ has IRS

(ii) There is 0<ξ<1 so that $(\xi X_o, \xi Y_o) \epsilon$ *PPS is inefficient if and only if has* DMU_o *has DRS*

(iii) There is ξ>0 so that $(\xi X_o, \xi Y_o) \epsilon$ *PPS is efficient if and only if has* DMU_o *has CRS*

Now in order to estimate returns to scale of DMU_o, the following non-radial model was proposed by Khodabakhshi et al.[10]

$$
\begin{aligned}
& Max \sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+ \\
& s.t \ \sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- = \xi x_{io} \qquad i=1,2,\dots,m \\
& \sum_{j=1}^{n} \lambda_j y_{rj} - s_r^+ = \xi y_{ro} \qquad r=1,2,\dots,s \\
& \sum_{j=1}^{n} \lambda_j = 1 \qquad (3) \\
& \lambda_j \geq 0 \qquad j=1,2,\dots,n \\
& s_i^-, s_r^+ \geq 0
\end{aligned}
$$

Now according to model (3), the RTS of DMU_o are detected as follows:

Theorem 4. *Suppose that* DMU_o *with input-output combination* (X_o, Y_o) *is efficient. The following conditions estimate returns to scale of* DMU_o *being evaluated by model (4):*

(i) The optimal value of the objective function is non-zero and $\xi^* > 1$ *if and only if* DMU_o *has IRS*

(ii) The optimal value of the objective function is non-zero and $0<\xi^*<1$ *if and only if* DMU_o *has DRS*

(iii) The optimal value of the objective function is zero if and only if DMU_o *has CRS.*

2.2 Quasi-fixed Inputs in Regulated Environments

In this section, we introduce quasi-fixed inputs in the production process, as the firm cannot adjust the quantity used as it wishes at decision time and it does not have any control over them. In order to evaluate the efficiency of a target DMU, we use the following model:

$$
\begin{aligned}
& \text{Min } \theta \\
& s.t \ \sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{ro} \qquad r = 1 \dots s \\
& \sum_{j=1}^{n} \lambda_j x_{ij} \leq \theta x_{io} \qquad i = 1 \dots m \\
& \sum_{j=1}^{n} \lambda_j k_{qj} \leq k_{qo} \qquad q = 1 \dots Q \\
& \sum_{j=1}^{n} \lambda_j = 1 \qquad (4) \\
& \lambda_j \geq 0 \qquad j = 1 \dots n
\end{aligned}
$$

Definition 3. DMUo is fully efficient if and only if the following two conditions are both satisfied:

(a) θ = 1

(b) All slacks are zero

The additive model for efficiency measurement with quasi-fixed input is as follows:

$$
\begin{aligned}
& Max \sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+ \\
& s.t \ \sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- = x_{io} \qquad i=1,2,\dots,m \\
& \sum_{j=1}^{n} \lambda_j y_{rj} - s_r^+ = y_{ro} \qquad r=1,2,\dots,s \\
& \sum_{j=1}^{n} \lambda_j k_{qj} + s_q^- = k_{qo} \qquad q=1,2,\dots,Q \\
& \sum_{j=1}^{n} \lambda_j = 1 \qquad (5) \\
& s_i^-, s_r^+, s_q^- \geq 0 \\
& \lambda_j \geq 0 \qquad j=1,2,\dots,n
\end{aligned}
$$

It should be noted that the Q-vector of variables k, representing the state of quasi-fixed inputs and in the objective function of model (5), the slack of quasi-fixed variables (s_q^-) are not included.

Definition4. All slacks at zero in the objective are a necessary and sufficient condition for full efficiency with model (5).

It should be noted that the environment where firms are, generally changed by a number of constraints other than technological. One of those important factors is regulation. In other words very few decisions in a firm are made without intersecting some regulation.

Ouellette and Vigeant 2004 [11], and Ouellette and Vigeant 2001 [12], model the regulation through introducing new transformation function. Their proposed model was as follows:

$$\begin{aligned} &\text{Min } \theta \\ &\text{s.t} \quad \sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{ro} \qquad r = 1 \dots s \\ &\quad \sum_{j=1}^{n} \lambda_j x_{ij} \leq \theta x_{io} \qquad i = 1 \dots m \\ &\quad \sum_{j=1}^{n} \lambda_j k_{qj} \leq k_{qo} \qquad q = 1 \dots Q \\ &\quad \sum_{j=1}^{n} \lambda_j r_{lj} \geq r_{lo} \qquad l = 1 \dots L \\ &\quad \sum_{j=1}^{n} \lambda_j = 1 \qquad\qquad (6) \\ &\quad \lambda_j \geq 0 \qquad j = 1 \dots n \end{aligned}$$

Note that in model (6) the L-vector of variables r, represents the state of the regulation. The definition of production possibilities set in regulated environments is presented as follows:

$\text{PPS}_\text{R} = \{(x, y, k) \mid (x, y, k)\ is$ feasible under regulation defined by r }

The additive model for regulated environment is as follows:

$$\begin{aligned} &Max \sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+ + \sum_{l=1}^{L} s_l^+ \\ &s.t \quad \sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- = x_{io} \qquad i=1,2,\dots,m \\ &\quad \sum_{j=1}^{n} \lambda_j y_{rj} - s_r^+ = y_{ro} \qquad r=1,2,\dots,s \\ &\quad \sum_{j=1}^{n} \lambda_j r_{lj} - s_l^+ = r_{lp} \qquad l = 1,2,\dots,L \\ &\quad \sum_{j=1}^{n} \lambda_j k_{qj} + s_q^- = k_{qo} \qquad q=1,2,\dots,Q \\ &\quad \sum_{j=1}^{n} \lambda_j = 1 \qquad\qquad (7) \\ &\quad s_i^-, s_r^+, s_l^+, s_q^- \geq 0 \\ &\quad \lambda_j \geq 0 \qquad j=1,2,\dots,n \end{aligned}$$

Definition 5. All slacks at zero in the objective are a necessary and sufficient condition for full efficiency with model (7).

In the next section, we will present our proposed approach for estimating RTS of efficient DMUs in the presence of quasi-fixed inputs in regulated environments.

3. New insights in to estimating returns to scale in the presence of quasi-fixed inputs when the firm is regulated.

Consider n DMUs, $\{DMU_j \mid j = 1, \dots, n\}$ with input-output combination (x_j, k_j, y_j) in regulated environment. Note that k_j is quasi-fixed inputs of DMU_j.

The dual (multiplier) form associated with model (6) is as follows:

$$\begin{aligned} &Max \quad -\sum_{q=1}^{Q} v_q^k k_{qo} + \sum_{l=1}^{L} u_l^r r_{lo} + \sum_{r=1}^{s} u_r y_{rj} + u_o \\ &s.t \quad -\sum_{i=1}^{m} v_i x_{ij} - \sum_{q=1}^{Q} v_q^k k_{qj} + \sum_{l=1}^{L} u_l^r r_{lj} + \sum_{r=1}^{s} u_r y_{rj} + u_o \leq 0 \quad j=1,\dots,n \\ &\sum_{i=1}^{m} v_i x_{io} = 1 \qquad\qquad (8) \\ &\quad u_r, v_i, \ v_q^k, u_l^r \geq 0 \end{aligned}$$

By considering variable RTS assumption, we have the following production possibility set (PPS):

$\text{PPS}_\text{R} = \{(x, k, y) \mid (x, k, y)\ is$ feasible under regulation defined by r }

And PPS-BCC define as follow in regulated environment

$$PPS = \{(x, k, y, r) \mid \sum_{j=1}^{n} \lambda_j x_j \leq x, \sum_{j=1}^{n} \lambda_j y_j \geq y,$$

$$\sum_{j=1}^{n}\lambda_j k_j \leq k, \sum_{j=1}^{n}\lambda_j = 1, \sum_{j=1}^{n}\lambda_j r_j \geq r, \lambda_j \geq 0;\ j = 1 \dots n \}$$

Theorem 5. *Suppose that DMU_o be efficient DMU by using model (8). Then, we have:*

(i) DMU_o has IRS iff ($u_o^* > -U^{r^*}R_o$) for all optimal solutions of model (11).

(ii) DMU_o has DRS iff ($u_o^* < -U^{r^*}R_o$) for all optimal solutions of model (11).

(iii)DM U_o has CRS iff $(u_o^* = U^{r^*}R_o)$ for at least one optimal solution of model (11).

Proof. Case (i): first assume that DMU_o has IRS, then according to theorem1, $\mu^+ > 1\ and\ \mu^- > 1$. Since $\mu^+ > 1$ then $\alpha(\beta) > \beta$. Moreover, DMU_o is efficient, therefore: $V^*X_o + V^{k^*}K_o - U^*Y - U^{r^*}R_o - u_o^* = 0$

According to (†), we imply that:

$$-V^*(\beta X_o) - V^{k^*}(\beta K_o) + U^*(\alpha(\beta)Y) + U^{r^*}R_o + u_o^* = 0$$

Since $\alpha(\beta) > \beta$, thus, we have:

$$-\beta(V^{x^*}(X_o)) - \beta(V^{k^*}(K_o) + \beta\left(U^*\left(Y_o^b\right)\right) + U^{r^*}R_o + u_o^* < 0$$

$$-\beta(V^*(X_o) - V^{k^*}(K_o) + U^*\left(Y_o^b\right) + U^{r^*}R_o + u_o^*) + (1-\beta)(U^{r^*}R_o + u_o^*) < 0$$

So, we have $(1-\beta)(U^{r^*}R_o + u_o^*) < 0$. Since β>1 then

$$\left(U^{r^*}R_o + u_o^*\right) > 0 \Rightarrow u_o^* > -U^{r^*}R_o$$

Similarly for $\mu^- > 1$, we obtain $u_o^* > -U^{r^*}R_o$.

Conversely, assume that $u_o^* > -U^{r^*}R_o$ for all optimal solutions of model (8).

Now consider Z_ε as below:

$$Z_\varepsilon = ((1+\varepsilon)X_O, (1+\varepsilon)K_o, (1+\varepsilon)Y)$$

Where ε is a small positive number. Therefore,

$$-V^*\big((1+\varepsilon)X_O\big) - V^{K^*}\big((1+\varepsilon)K_O\big) + U^*((1+\varepsilon)Y) + U^{R^*}R_O + u_o^*$$
$$= (1+\varepsilon)\big(-V^{X^*}(X_O) - V^{K^*}K_O + U^*Y + U^{R^*}R_O + u_o^*\big) - \varepsilon\big(U^{R^*}R_O + u_o^*\big).$$

So we include that $-\varepsilon\big(-U^{R^*}R_O + u_o^*\big) < 0$. Thus Z_ε does not lie on the efficient frontier. Hence DMU_O has IRS.

Other case can be proved similarly.

It should be noted that, the definition of the RTS when the regulation component is binding differs from the case that they do not binding, in the other words the regulatory variables impact the behavior of all dual variables and in turn will lead to returns to scale that differ from those measured when the regulation is not accounted for.

Theorem 6. *Suppose that DMU_o is efficient DMU by using model (11). Then, we have:*

(i) There is $\xi > 1$ so that $(\xi X_o, \xi K_o, \xi Y_O, R) \in$ PPS is inefficient if and only if DMU_o has IRS.

(ii) There is $0 < \xi < 1$ so that $(\xi X_o, \xi K_o, \xi Y_O, R) \in PPS$ is inefficient if and only if DMU_o has DRS

(iii) There is $\xi > 0$ so that $(\xi X_o, \xi K_o, \xi Y_O, R) \in PPS$ is efficient if and only if DMU_o has CRS.

Proof: *Case (1):* Assume that $\left(V^*, V^{k^*}, U^*, U^{r^*}, u_o^*\right)$ be an obtained optimal solution for mode (8). Since DMU_o is efficient so,

$V^*X_o + V^{k^*}K_o + U^*Y_O + U^{r^*}R_o + u_o^* = 0.$

Also, $(\xi X_o, \xi K_o, \xi Y_O, R) \in PPS$ *is inefficient*, thus we have:

$$-V^*(\xi X_O) - V^{k^*}(\xi K_O) + U^*(\xi Y_O) + U^{r^*}R_O + u_o^* < 0.$$

$$\Rightarrow \xi\left(-V^*(X_O) - V^{k^*}K_O + U^*Y_O + U^{r^*}R_O + u_o^*\right) + (1-\xi)\left(U^{r^*}R_O + u_o^*\right) < 0$$

Therefor we conclude that $(1-\xi)\left(U^{*^R}R_O + u_o^*\right) < 0$. Since $\xi > 1$

Then $U^{r^*}R_O + u_o^* > 0 \Rightarrow u_o^* > -U^{r^*}R_O$. thus according to theorem 5, DMU_o has IRS.

Conversely suppose that DMU_ohas IRS, then according to theorem 5, ($u_o^* > -U^{r^*}R_o$).

Contrary assume that for each $\xi > 1$, $(\xi X_o, \xi K_o, \xi Y_O, R) \in PPS$ is efficient.

Therefore, each convex combination of $(\xi X_o, \xi K_o, \xi Y_O, R) \in PPS$ and (X_o, K_o, Y_O, R) lies on the efficient frontier. Thus there is supporting hyperplane $-\bar{V}X - \bar{V}^kK + \bar{U}Y + \bar{U}^rR + \bar{u}_o = 0$ of PPS which passes from $(\xi X_o, \xi K_o, \xi Y_O, R)$ and (X_o, K_o, Y_O, R), so if $\alpha = \bar{V}X_o$ then the following optimal solution of model (7) in assessing DMU_O which is active on $(\xi X_o, \xi K_o, \xi Y_O, R)$ and (X_o, K_o, Y_O, R)

$$\left(V^*, V^{k^*}, U^*, U^{r^*}, u_o^*\right) = (\alpha^{-1}\bar{V}, \alpha^{-1}\bar{V}^k, \alpha^{-1}\bar{U}, \alpha^{-1}\bar{U}^r, \alpha^{-1}\bar{u}_o)$$

Hence we have:

$$-V^*X_o - V^{k^*}K_o + UY_O + U^{r^*}R + u_o^* = 0$$

$$-V^*(\xi X_o) - V^{k^*}(\xi K_o) + U^*\left(\xi Y_o^g\right) + U^{r^*}R + u_o^* = 0 \Rightarrow$$

$$\xi(-V^*X_o - V^{k^*}K_o + U^*Y_O + U^{r^*}R + u_o^*) + (1-\beta)(U^{r^*}R_O + u_o^*) = 0 \Rightarrow$$

$(1-\beta)(U^{r^*}R_O + u_o^*) = 0.$ since $\xi > 1$ then $U^{*^R}R_O + u_o^* = 0.$

Then according to theorem (7) DMU_o has CRS. So the contrary suppose us false and proof is complete.

Other cases con be proved similarly.

Now the following additive model for efficiency measurement of DMU_o in the presence of undesirable outputs in regulated environment were introduced:

$$\begin{aligned}
& Max \sum_{i=1}^m s_i^- + \sum_{r=1}^s s_r^+ + \sum_{l=1}^L s_l^+ \\
& s.t \ \sum_{j=1}^n \lambda_j x_{ij} + s_i^- = x_{io} \qquad i=1,2,\dots,m \\
& \sum_{j=1}^n \lambda_j y_{rj} - s_r^+ = y_{ro} \qquad r=1,2,\dots,s \\
& \sum_{j=1}^n \lambda_j r_{lj} - s_l^+ = r_{lp} \qquad l=1,2,\dots,L \\
& \sum_{j=1}^n \lambda_j k_{qj} + s_q^- = k_{qo} \qquad q=1,2,\dots,Q \\
& \sum_{j=1}^n \lambda_j = 1 \qquad\qquad (9) \\
& s_i^-, s_r^+, s_l^+, s_q^- \geq 0 \\
& \lambda_j \geq 0 \qquad j=1,2,\dots,n
\end{aligned}$$

Definition7. DMU_o is called efficient under model (9) if and only if the optimal value of its objective function is zero.

Now in other to estimate the RTS of DMU_o we present the following non-radial DEA model

$$\begin{aligned}
& Max\ \omega_o = \sum_{i=1}^m s_i^- + \sum_{r=1}^{s_1} s_r^+ + \sum_{l=1}^L s_l^+ \\
& s.t \ \sum_{j=1}^n \lambda_j x_{ij} + s_i^- = \xi x_{io} \qquad i=1,2,\dots,m \\
& \sum_{j=1}^n \lambda_j y_{ro} - s_r^+ = \xi y_{ro} \qquad r=1,2,\dots,s_1 \\
& \sum_{j=1}^n \lambda_j r_{lj} - s_l^+ = r_{lp} \qquad l=1,2,\dots,L \\
& \sum_{j=1}^n \lambda_j k_{qj} + s_q^- = \xi k_{qo} \qquad q=1,2,\dots,Q \\
& \sum_{j=1}^n \lambda_j = 1 \qquad\qquad (10) \\
& s_i^-, s_r^+, s_l^+, s_q^- \geq 0 \\
& \lambda_j \geq 0 \qquad j=1,2,\dots,n
\end{aligned}$$

Suppose $(\omega_o^*, \xi^*, \lambda^*, S^{x^*}, S^{y^*}, S^{r^*}S^{k^*})$ be an optimal solution of model (10) now the theorem (7) identify R.T.S of DMU_o.

Theorem 7 Suppose that DMU_o be efficient by using model (11). The following The following conditions estimate returns to scale of evaluated DMU by model (23):

(i) The optimal value of the objective function is non-zero and $\xi^* > 1$ if and only if DMU has IRS.

(ii) The optimal value of the objective function is non-zero and $0 < \xi^* < 1$ if and only if DMU has DRS.

(iii) The optimal value of the objective function is zero if and only if DMU has CRS.

Proof : *Case (i):* Assume that the optimal value of the objective function of model (10) is non-zero and $\xi^* > 1$. Thus $(\xi^* X_o, \xi^* K_o, \xi^* Y_o, R) \in PPS$ is inefficient under model (10).

So, associated with Theorem 6, DMU_o has IRS.

Conversely, let DMU_o has IRS. So according to Theorem 7, there is $\xi^* > 1$ such that $(\xi^* X_o, \xi^* K_o, \xi^* Y_o, R) \in PPS$is inefficient, this implies that the value of its objective function is non-zero. Now, we must prove that, $\xi^* > 1$.

Contrary: suppose that $\xi^* \leq 1$. If $\xi^* < 1$. than according to Theorem 6, DMU_o has DRS and also, if $\xi^* = 1$ then DMU is inefficient. Thus, there are two contradictions. Hence, the contrary suppose is false and the proof is complete.

Other cases can be proved, similarly.

4. Application

In this section, to illustrate the proposed model for estimating RTS in regulated environment a numerical example is presented. In table 1, data and numerical results for three DMUs with single inputs and single output in regulated environment are presented. Note that regulation variable is shown by R.

In table 2 we calculate the RTS type of DMUs without regulatory constraint**.**

Table 1. Data of inputs and outputs with the obtained results from model (7).

	I_1	I_2	O_1	O_2	K	R	
DMU1	33940	7	19	10	7	0.85	Inefficient
DMU2	25450	5	38	14	9	0.96	Efficient
DMU3	31200	6	48	11	4	0.87	Efficient
DMU4	31580	5	73	18	17	0.94	Efficient
DMU5	35600	5	40	28	6	0.94	Efficient
DMU6	39160	4	33	38	23	0.90	Efficient
DMU7	42800	7	62	20	12	0.90	inefficient
DMU8	42480	7	78	27	13	0.95	Efficient
DMU9	45980	7	70	28	4	0.81	Efficient
DMU10	51000	8	59	15	3	0.86	Efficient
DMU11	51215	6	48	11	4	0.93	Efficient
DMU12	56000	7	56	26	10	0.6	Inefficient
DMU13	56700	7	59	33	7	0.6	Efficient
DMU14	58140	4	78	34	21	0.8	Efficient
DMU15	60100	7	19	10	11	0.94	Inefficient

Table2. Table 2 represents the obtained results from the proposed approach for R.T.S measurement.

DMUs	ξ^*	ω_o^*	Results of proposed model
D_2	1	0	CRS
D_3	1	0	CRS
D_4	1	0	CRS
D_5	1	0	CRS
D_6	1	0	CRS
D_8	1	0	CRS
D_9	1	0	CRS
D_{10}	1	0	CRS
D_{11}	1	0	CRS
D_{13}	0.8466	93	DRS
D_{14}	1	0	CRS

5. Conclusion

In this research, we first introduce a new input oriented model for determining efficient DMUs in the presence of Quasi-fixed inputs in regulated environment, then a new non-radial model is presented to estimate RTS of these DMUs in DEA.

Note that, since an inefficient DMU has more than one projection on the empirical function so, different returns to scales can be obtained for projections of the inefficient DMU by using the proposed RTS approach.

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An Additive Model for Estimation Return to Scale in Regulated Environment with Quasi-Fixed Inputs in Data Envelopment Analysis (DEA)

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مدلی جمعی جهت تخمین بازده به مقیاس در محیط تحت کنترل با حضور ورودی های شبه ثابت در تحلیل پوششی داده ها

چکیده: محاسبه بازده به مقیاس به رابطه بین ورودی و خروجی در ساختار تولید می پردازد. روشهای مختلفی جهت محاسبه بازده به مقیاس در فضای پرایمال و فضای دوآل وجود دارد. اما در عمل دولت ها معمولا به عنوان آژانس های نظارتی بر رفتار واحد تصمیم گیرنده نظارت می کنند.

این نظارت مجموعه ای از محدودیت ها را بر رفتار واحد تصمیم گیرنده اعمال می کند، لذا آگاهی از نحوه تاثیر و اثرگذاری قیود نظارتی بر واحد تصمیم گیرنده اساسی بنظر می رسد. این مهم با تعیین بازده به مقیاس در حضور محدودیت نظارتی و همچنین در غیاب آن انجام می گردد.

در این مقاله ما از مدلی جمعی جهت تعیین بازده به مقیاس واحدهای تصمیم گیرنده ای که در محیط های تحت کنترل قرار دارند و همچنین دارای ورودی های شبه ثابت هستند، استفاده کرده ایم.

کلمات کلیدی: بازده به مقیاس، ورودی شبه ثابت، نظارت