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Ranking DMUs by New Metric D_{TM} with Fuzzy Data in DEA

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Abstract
Technique of Data Envelopment Analysis, involved methods which conducted for desirable objective management of Decision Making unit, that is same increasing of efficiency level. An important topic in DEA interpretation is the ranking of DMUs. There exist many methods for it in crisp DEA. In this paper we use of the new metric D_{TMF} that proposed by T.Allahviranloo and M.Adabitabar for fuzzy ranking of all DMUs.

Keywords: Data Envelopment Analysis, Ranking, Fuzzy number.

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1. INTRODUCTION

Data envelopment Analysis (DEA), first proposed by Charnes, Cooper [1], is a non-parametric approach to evaluate the performance or efficiency of various organizations in public and private sectors with multiple inputs and multiple outputs. DEA is a mathematical programming approach that uses the production frontiers to evaluated relative efficiency. If decision making unit (DMU) lies on production frontiers, DMU is efficient, otherwise DMU is inefficient. The ranking concept used for comparison among the efficient DMUs. Several methods introduced for ranking of units (in case of crisp DEA with real data). [7,5]

Many authors have distinguished the need to present some kind of data uncertainty in the linear programming models of DEA (see e.g. [9]).The fuzzy system approach has many features, which are particulary suitable for the theory and practice of DEA models. Some fuzzy versions of DEA models are proposed in Sengupta [3], Cooper et al. [12], Kao and Lio [2], Guo and Tanaka [8], and Saati et al. [6].

The existed fuzzy approches for evaluting DMUs in fuzzy DEA are usually categorized in four groups: the fuzzy ranking approach, the defuzzification approach, the tolerance approach and the based approach (Letworasirikul, Shu-cherng, Joines and Nuttle If exact values are suggested these are only statistical inference from past data and their stabili, 2003).

The rest of this paper is organized as follows: In section 2, background fuzzy is presented, section 3 introduced the new metric D_{TM} into two sub section interval metric D_{TMI} and fuzzy metric D_{TMF}. The method for ranking DMUs with fuzzy data is in section 4, then presents the conclusion in the last section.

2. Background fuzzy

A generalized left right fuzzy number (GLRFN) of Dubois and Prade (1980), and Duckstein (2002) is a fuzzy set $\tilde{A}=(a_1,a_2,a_3,a_4)$ such that the membership function satisfies the following:

$$\mu_{\tilde{A}}(x)=\begin{cases} L\left(\dfrac{a_2-x}{a_2-a_1}\right), & a_1 \le x \le a_2 \\ 1, & a_2 \le x \le a_3 \\ R\left(\dfrac{x-a_3}{a_4-a_3}\right), & a_3 \le x \le a_4 \\ 0, & \text{otherwise} \end{cases} \tag{1}$$

Where L and R are strictly decreasing functions defined on [0,1] and satisfiyng the conditions:

$$\begin{aligned} &L(x)=R(x)=1 \ \text{ if } x \le 0, \\ &L(x)=R(x)=0 \ \text{ if } x \ge 1 \end{aligned} \tag{2}$$

For $a_2=a_3$, we have the classical definition of left right fuzzy number (LRFN) of Dubois and Prade (1980),Tran and Duckstein (2002).

A GLRFN is $\tilde{A}$ denoted as $\tilde{A}=(a_1,a_2,a_3,a_4)$ and an α- level interval of fuzzy number $\tilde{A}$ as:

$$[\tilde{A}]^{\alpha}=\left[A^{l}(\alpha), A^{r}(\alpha)\right]=\begin{bmatrix} a_2-(a_2-a_1)L_A^{-1}(\alpha), \\ a_3+(a_4-a_3)R_A^{-1}(\alpha) \end{bmatrix} \tag{3}$$

Definition 1. For $\tilde{A},\tilde{B} \in F$, define the signed distance of $\tilde{A},\tilde{B}$ as follows see [11]:

$$d(\tilde{A},\tilde{B})=\int w(\alpha)\begin{bmatrix} \left(A^{l}(\alpha)-B^{l}(\alpha)\right)+ \\ \left(A^{u}(\alpha)-B^{u}(\alpha)\right) \end{bmatrix} d\alpha \tag{4}$$

Here, $w(\alpha)$ is weighting function that w: [0,1] $\rightarrow$ [0,1]. If $\int w(\alpha)d\alpha=\frac{1}{2}$ then we say that $w(\alpha)$ is a regular function.

3. New Metric D_{TM}

3.1. Distance for interval numbers D_{TM}

Let $f(x)=(a-b)x+b$ and $g(x)=(c-d)x+d$. The distance of two intervals [a,b] and [c,d], $(a \le b, c \le d)$ denoted by $d_{TMI}^{(p)}([a,b],[c,d])$ and defined by:[10]

$$d_{TMI}^{(p)}=\left(D_{TMI}^{(p)}([a,b],[c,d])\right)^{1/p} \tag{5}$$

and

$$D_{TMI}^{(p)}([a,b],[c,d])=\left\|f(x)-g(x)\right\|_{L_p}^{p} \tag{6}$$

Where $\|.\|$ is the usual norm in the L_p space on the interval [0,1] (p>1). this distance is a metric distance on interval numbers. For metric properties see [10].

3.2. Metric for fuzzy number D_{TM}

Using the metric proposed in later section, a distance between two fuzzy number $(\tilde{A} \text{ and } \tilde{B})$ can be defined as:

$$d_{TMF}^{(p)}(\tilde{A},\tilde{B},s)=\left(D_{TMF}^{(p)}(\tilde{A},\tilde{B},s)\right)^{\frac{1}{p}} \quad (7)$$

Such that

$$D_{TMF}^{(p)}(\tilde{A},\tilde{B},s)=\frac{\int_0^1 s(\alpha)D_{TMI}^{(P)}\left(\left[\tilde{A}\right]^{\alpha},\left[\tilde{B}\right]^{\alpha}\right)d\alpha}{\int_0^1 s(\alpha)d\alpha} \quad (8)$$

Here s is a continuous positive weight function on [0,1]. It can be proved that $d_{TMF}^{(p)}(\tilde{A},\tilde{B},s)$ on GLRFNs. See the metric properties in [10].

4. Metric for fuzzy number D_{TMF}

An important topic in DEA interpretation is the ranking of DMUs. The ranking of number and fuzzy number evaluted by several ways. With the new metric D_{TMF}, proposed the ranking in this section. In this new metric D_{TMF} used from degree of distance with crisp $\max(M)$ and crisp $\min(m)$ as: [10]

$$M \geq \max\left(\sup\tilde{A} \cup \sup\tilde{B}\right) \text{ and } m \leq \min\left(\sup\tilde{A} \cup \sup\tilde{B}\right) \quad (9)$$

Denoted the degree of distance between $\tilde{A}$ and crisp numbers max(M) and min(m) by $\gamma_d^{(p)}(\tilde{A},M)$ and $\gamma_d^{(p)}(\tilde{A},m)$, respectively and is defined as follows:

$$\gamma_d^{(p)}(\tilde{A},M)=\frac{d_{TMF}^{(p)}(\tilde{A},M,s)}{d_{TMF}^{(p)}(\tilde{A},M,s)+d_{TMF}^{(p)}(\tilde{A},m,s)},$$
$$\gamma_d^{(p)}(\tilde{A},m)=\frac{d_{TMF}^{(p)}(\tilde{A},m,s)}{d_{TMF}^{(p)}(\tilde{A},M,s)+d_{TMF}^{(p)}(\tilde{A},m,s)} \quad (10)$$

$d_{TMF}^{(p)}(\tilde{A},M,s)$ and $d_{TMF}^{(p)}(\tilde{A},m,s)$ are distances between fuzzy number $\tilde{A}$ and crisp number max(M) and min(m), respectively.

Proposition 1 $\gamma_d^{(p)}(\tilde{A},M)+\gamma_d^{(p)}(\tilde{A},m)=1$

Definition 1 The ranking method is as follows:

$$\text{(a)} \quad \tilde{A}\precsim\tilde{B} \Leftrightarrow \begin{cases} \gamma_d^{(p)}(\tilde{A},M) \geq \gamma_d^{(p)}(\tilde{B},M) \\ \text{or} \\ \gamma_d^{(p)}(\tilde{A},m) \leq \gamma_d^{(p)}(\tilde{B},m) \end{cases} \quad (11)$$

Such that the degree of ranking is defined as follows:

$$\gamma_{\lambda M+(1-\lambda)m}^{(\tilde{A}<\tilde{B})}=\frac{\lambda.\gamma_d^{(p)}(\tilde{A},M)}{\gamma_d^{(p)}(\tilde{A},M)+\gamma_d^{(p)}(\tilde{B},M)}+\frac{(1-\lambda).\gamma_d^{(p)}(\tilde{B},M)}{\gamma_d^{(p)}(\tilde{A},m)+\gamma_d^{(p)}(\tilde{B},m)}, \ \lambda\in[0,1] \quad (12)$$

λ is chosen according to the decision-maker idea. If $\lambda=0$, this means that the ranking method is as Eq. 7 and if $\lambda=1$, it means that it is as Eq. 6 and if $0<\lambda<1$, this means that the ranking method works with crisp min(m) and crisp max(M).

$$\text{(b)} \quad \tilde{A}=\tilde{B} \Leftrightarrow d_{TMF}^{(p)}(\tilde{A},\tilde{B},s)=0 \quad (13)$$

Then the degree of ranking is defined as follows:

$$\gamma_{\lambda M+(1-\lambda)m}^{(\tilde{A}=\tilde{B})}=1 \text{ and } \gamma_{\lambda M+(1-\lambda)m}^{(\tilde{A}>\tilde{B})}=\gamma_{\lambda M+(1-\lambda)m}^{(\tilde{A}<\tilde{B})} \quad (14)$$

and

$$\text{(c)} \quad \tilde{A}=\tilde{B} \Leftrightarrow \begin{cases} \gamma_d^{(p)}(\tilde{A},M)=\gamma_d^{(p)}(\tilde{B},M)=\frac{1}{2} \\ \text{or} \\ \gamma_d^{(p)}(\tilde{A},m)=\gamma_d^{(p)}(\tilde{B},m)=\frac{1}{2} \end{cases} \quad (15)$$

In this case, we have a maximum ambiguity for ranking, therefore the degree of ranking is defined as follows:

$$\gamma_{\lambda M+(1-\lambda)m}^{(\tilde{A}=\tilde{B})}=\gamma_{\lambda M+(1-\lambda)m}^{(\tilde{A}>\tilde{B})}=\gamma_{\lambda M+(1-\lambda)m}^{(\tilde{A}<\tilde{B})}=\frac{1}{2} \quad (16)$$

In this method, for ranking n fuzzy numbers $\tilde{A}_1,...,\tilde{A}_n$ we use comparison of the degree of distance with crisp max (M) and any crisp min(m) where:

$$M \geq \max\left(\bigcup_{i=1}^{n} \sup\tilde{A}_i\right) \quad \text{and} \quad m \leq \left(\bigcap_{i=1}^{n} \sup\tilde{A}_i\right) \quad (17)$$

If we add the $\tilde{C}$ to the set then M, n and γ may change, but the ranking of $\tilde{A}_1,...,\tilde{A}_n$ will not change. [1]

Consider the following model:

$$\begin{aligned}
\min \quad & \left(\sum_{i=1}^{m} d\left(\tilde{x}_i,\tilde{x}_{ip}\right)\right)^{\frac{1}{p}}+\left(\sum_{r=1}^{s} d\left(\tilde{y}_r,\tilde{y}_{rp}\right)\right)^{\frac{1}{p}} \\
\text{s.t} \quad & \sum_{j=1,j\neq p}^{n} \lambda_j \tilde{x}_{ij} \leq \tilde{x}_i \qquad i=1,...,m \\
& \sum_{j=1,j\neq p}^{n} \lambda_j \tilde{y}_{rj} \leq \tilde{y}_r \qquad r=1,...,s \qquad (18) \\
& \tilde{x}_i \geq 0, \quad \tilde{y}_r \geq 0 \quad i=1,...,m, \ r=1,...,s \\
& \lambda_j \geq 0 \quad j=1,...,n, \quad j \neq p
\end{aligned}$$

With using of equation (11) and choice the set of second constraint in all two constraint, we have:

$$\begin{aligned}
\min \quad & \left(\sum_{i=1}^{m} d\left(\tilde{x}_i,\tilde{x}_{ip}\right)\right)^{\frac{1}{p}}+\left(\sum_{r=1}^{s} d\left(\tilde{y}_r,\tilde{y}_{rp}\right)\right)^{\frac{1}{p}} \\
\text{s.t} \quad & \gamma_d^{(p)}\left(\sum_{j=1,j\neq p}^{n} \lambda_j \tilde{x}_{ij},m\right) \leq \gamma_d^{(p)}\left(\tilde{x}_i,m\right) \qquad i=1,...,m \\
& \gamma_d^{(p)}\left(\tilde{y},m\right) \leq \gamma_d^{(p)}\left(\sum_{j=1,j\neq p}^{n} \lambda_j \tilde{y}_{rj},m\right) \qquad r=1,...,s \\
& \tilde{x}_i \geq 0, \quad \tilde{y}_r \geq 0 \quad i=1,...,m, \ r=1,...,s \qquad (19) \\
& \lambda_j \geq 0 \quad j=1,...,n, \quad j \neq p
\end{aligned}$$

Considering the information is nonnegative in DEA, and concerning to select value of min for several fuzzy number, we can select the zero to min (m=0), and with locate the equation 10 we have:

$$\begin{aligned}
\min \quad & \left(\sum_{i=1}^{m} d\left(\tilde{x}_i,\tilde{x}_{ip}\right)\right)^{\frac{1}{p}}+\left(\sum_{r=1}^{s} d\left(\tilde{y}_r,\tilde{y}_{rp}\right)\right)^{\frac{1}{p}} \\
\text{s.t} \quad & \frac{d_{TMF}\left(\sum_{j=1,j\neq p}^{n} \lambda_j \tilde{x}_{ij},0,s\right)}{d_{TMF}\left(\sum_{j=1,j\neq p}^{n} \lambda_j \tilde{x}_{ij},M,s\right)+d_{TMF}\left(\sum_{j=1,j\neq p}^{n} \lambda_j \tilde{x}_{ij},0,s\right)} \leq \\
& \frac{d_{TMF}\left(\tilde{x}_i,0,s\right)}{d_{TMF}\left(\tilde{x}_i,M,s\right)+d_{TMF}\left(\tilde{x}_i,0,s\right)} \qquad i=1,...,m \\
& \frac{d_{TMF}\left(\tilde{y}_r,0,s\right)}{d_{TMF}\left(\tilde{y}_r,M,s\right)+d_{TMF}\left(\tilde{y}_r,0,s\right)} \leq \\
& \frac{d_{TMF}\left(\sum_{j=1,j\neq p}^{n} \lambda_j \tilde{y}_{rj},0,s\right)}{d_{TMF}\left(\sum_{j=1,j\neq p}^{n} \lambda_j \tilde{y}_{rj},M,s\right)+d_{TMF}\left(\sum_{j=1,j\neq p}^{n} \lambda_j \tilde{y}_{rj},0,s\right)} \\
& r=1,...,s \\
& \tilde{x}_i \geq 0, \quad \tilde{y}_r \geq 0 \quad i=1,...,m, \ r=1,...,s \qquad (20) \\
& \lambda_j \geq 0 \quad j=1,...,n, \quad j \neq p
\end{aligned}$$

With using of equation (6) and (7) obtained the following model for ranking the fuzzy numbers:

$$\begin{aligned}
\min \quad & \left(\sum_{i=1}^{m} d\left(\tilde{x}_i,\tilde{x}_{ip}\right)\right)^{\frac{1}{p}}+\left(\sum_{r=1}^{s} d\left(\tilde{y}_r,\tilde{y}_{rp}\right)\right)^{\frac{1}{p}} \\
\text{s.t} \quad & -\int_0^1 \sum_{j=1,j\neq p}^{n} \lambda_j \frac{\underline{x}_{ij}+\overline{x}_{ij}}{2} d\alpha \leq \int_0^1 \frac{\underline{x}_i+\overline{x}_i}{2} d\alpha \\
& -\int_0^1 \frac{\underline{y}_r+\overline{y}_r}{2} d\alpha \leq \int_0^1 \sum_{j=1,j\neq p}^{n} \lambda_j \frac{\underline{y}_{rj}+\overline{y}_{rj}}{2} d\alpha \qquad (21) \\
& \tilde{x}_i \geq 0, \quad \tilde{y}_r \geq 0 \quad i=1,...,m, \ r=1,...,s \\
& \lambda_j \geq 0 \quad j=1,...,n, \quad j \neq p
\end{aligned}$$

5. Conclusions

This paper, introduced the model for ranking the fuzzy data with new metric. We need to rewrite the DEA models, when we have the inaccurate data. Is the fuzzy DEA one of the models for solving the cases with inaccurate data. We used the ranking concept for comparison among DMUs.

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