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Using Directional Distance Functions to Determine Ranking Ranges in Cross-efficiency Evaluations

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Abstract

Having a ranking range for each unit allows for assessing the extent to which the choice of DEA weights may affect the ranking derived from the cross-efficiency evaluation and can deal with the most important difficulty of cross-efficiency evaluations (i.e., the possible existence of alternative optimal solutions for the DEA weights, which may lead to different cross-efficiency scores depending on the choice that is made). This may, furthermore, yields some useful information in the case of real life problems. This paper develops the ranking ranges by the use of directional distance functions in calculating cross-efficiency evaluations. In this case, by simultaneously accounting for the inefficiency in inputs and outputs, more comprehensive ranking ranges were achieved in cross-efficiency evaluations. A numerical example is presented to show the validity of the proposed procedure by comparing the results with those of the previous approaches.

Keywords: Cross-efficiency Evaluation; Ranking; Data Envelopment Analysis; Directional Distance Functions.

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1. Introduction

Data Envelopment Analysis (DEA), is a powerful quantitative and analytical tool based on nonparametric linear programming methods for evaluating the performance of a set of peer entities called Decision Making Units (DMUs), which convert multiple inputs to multiple outputs. The first DEA model known as CCR model [4]. CCR, like other classical DEA models, in a self-evaluation process allows each DMU to has complete freedom in the selection of the most favorable weights, (which are often dissimilar with those of the other DMUs) to achieve its best possible relative efficiency. It is possible for the DMU under evaluation to ignore or assign a lower weight to the other inputs and outputs. Consequently it assigns a higher weight to few favorable measures from its own view. This so-called unrestricted weight flexibility problem often leads to unreasonable outcomes, since the weights provided are recurrently inconsistent with the prior knowledge or accepted views on the process, under evaluation. According to the efficiency scores, very often more than one DMU with the best relative efficiency of one is evaluated as DEA efficient. These DMUs cannot, however, be further discriminated.

Sexton et al. [8], addressed the issue of cross-efficiency as a DEA extension tool that can be used to rank and identify the best performing DMUs based on evaluating the total efficiency of a given DMU which is undergo self-and peer-evaluation in either input or output oriented approaches. A cross-efficiency matrix is formed based on these results. In this matrix, leading diagonal and the off-diagonal cells, covers the DEA efficiency scores of the DMUs and the cross-efficiency scores, respectively. This method, however, suffer the shortcoming of non-uniqueness of the DEA optimal weights. Hence, Doyle and Green [5], proposed two aggressive and benevolent approaches to overcome this shortcoming. According to these aggressive and benevolent formulations, the self-evaluation score of the unit under assessment, is maintained as a primary goal while the cross-efficiencies of the rest of the DMUs, as a secondary goal, are minimized and maximized, respectively. Further to these two strategies, other secondary-goal techniques have been suggested and evaluated ([6], [9]). The application of the benevolent or aggressive formulations still needs improvement, since the two formulations do not necessarily lead to identical rankings. Another interesting fact about the two formulations is that in most of the existing applications the aggressive formulation is used, but this choice has not been based on any theoretical evidence. Additionally, sometimes the weight sets induced by the aggressive or benevolent formulation are still non-unique [10].

To avoid some of the above-mentioned shortcomings Alcaraz et al. [1], proposed a procedure for cross-efficiency evaluations, without the need for any specific choices of DEA weights, which instead of a single ranking, yielded a ranking range for each unit. Ruiz [7], also developed models for cross-efficiency evaluation, which apply the benevolent and aggressive criteria to the choice of weights, based on directional distance functions.

In this paper, we report developing Alcaraz et al.'s proposed ranking ranges [1] through the application of Ruiz's idea [7], by the use of directional distance functions in calculating cross-efficiency evaluations. This way we obtained new ranking ranges that account for the inefficiencies both in inputs and in outputs simultaneously and yielded more complete ranking ranges of the DMUs as compared to the results of either of the oriented models. In the following lines we shall first provide a brief description of ranking ranges in cross-efficiency evaluations in the context of the oriented DEA models (sections 2), next we shall develop the proposed procedure using directional distance functions in section 3, before providing a numerical example in section 4 and finally concluding the work.

2. Ranking Ranges in Cross-efficiency Evaluations

Let there are n DMUs with m inputs and s outputs and the input and output vectors for $\mathrm{DMU_j}$, $j=1,\dots,n$, are illustrated by $\mathbf{x_j}=(x_{1j},x_{2j},\dots,x_{mj})'$ and $\mathbf{y_j}=(y_{1j},y_{2j},\dots,y_{sj})'$ respectively. These values are considered nonnegative, however at least one component of every input and output vector is positive. In standard cross-efficiency evaluation, the weights provided by the oriented CCR models are used to calculate the cross-efficiencies. While evaluating any given unit $(\mathrm{DMU_o}, o\in\{1,\dots,n\})$, the input oriented CCR efficiency score, θ_o, is obtained through the optimization problem as below:

$$\begin{aligned} \text{Max}\ & \theta_o=\frac{\mathbf{u_o'y_o}}{\mathbf{v_o'x_o}} \\ \text{s.t.}\ & \frac{\mathbf{u_o'y_j}}{\mathbf{v_o'x_j}}\le 1 \qquad j=1,\dots,\mathrm{n} \\ & \mathbf{v_o}\ge\mathbf{0_m},\ \mathbf{u_o}\ge\mathbf{0_s} \end{aligned} \tag{1}$$

Where $\mathbf{v_o}=(v_{1o},\dots,v_{mo})'$ and $\mathbf{u_o}=(u_{1o},\dots,u_{so})'$ represent vectors for the input and output weights, respectively, of $\mathrm{DMU_o}$. The $m\times n$ and the $s\times n$ matrices of the input weight vectors and output weight vectors are illustrated by $V=(\mathbf{v_1},\dots,\mathbf{v_n})$ and $U=(\mathbf{u_1},\dots,\mathbf{u_n})$ respectively, based on the choice of the DEA weights that each of the DMUs makes.

Using the Charnes and Cooper's transformation [3], the given fractional programming model is then converted into the equivalent linear programming (LP) as follows:

$$\begin{aligned} \text{Max}\ & \theta_o=\mathbf{u_o'y_o} \\ \text{s.t.}\ & \mathbf{v_o'x_o}=1 \\ & -\mathbf{v_o'x_j}+\mathbf{u_o'y_j}\le 0 \qquad j=1,\dots,\mathrm{n} \\ & \mathbf{v_o}\ge\mathbf{0_m},\ \mathbf{u_o}\ge\mathbf{0_s} \end{aligned} \tag{2}$$

One can obtain a set of optimal weights $(\mathbf{v_d},\mathbf{u_d})$, for each $\mathrm{DMU_d}$, $d=1,\dots,n$, through solving model (2). Further the cross-efficiency of each $\mathrm{DMU_j}$, i.e. E_{dj}, can be calculated using the weights of $\mathrm{DMU_d}$, as below:

$$E_{dj}=\frac{\mathbf{u_d'y_j}}{\mathbf{v_d'x_j}} \tag{3}$$

Next the cross-efficiency score for $\mathrm{DMU_j}$ shall be given by :

$$\bar{E}_j=\frac{1}{n}\sum_{d=1}^{n}E_{dj} \qquad j=1,\dots,n \tag{4}$$

Alcaraz et al. [1] considered all the possible choices of weights that can be adopted by all the DMUs achieving a range of possible rankings for each unit. These rankings are determined by the best and worst rankings each unit can attain.

To do this, they described two sets i.e., $H_o(V,U)=\{DMU_j,\ j=1,\dots,n\mid \bar{E}_j>\bar{E}_o\}$ and $L_o(V,U)=\{DMU_j,\ j=1,\dots,n\mid \bar{E}_j<\bar{E}_o\}$ for each (V,U). These are the sets of DMUs with cross-efficiency scores strictly higher and lower than that of $\mathrm{DMU_o}$, respectively (when (V,U) are the DEA weights chosen by the DMUs). This way the best and the worst rankings were found through using a couple of models that simultaneously allow for all the DEA weights of all the DMUs.

The best ranking of a given $\mathrm{DMU_o}$ is defined in terms of the minimum numbers of DMUs that perform better than $\mathrm{DMU_o}$, which is given by:

$$r_o^b=\text{Min}_{(V,U)}\left\{\left|H_o(V,U)\right|\right\}+1 \tag{5}$$

where $\left|H_o(V,U)\right|$ is the cardinality of the set $H_o(V,U)$ and r_o^b is achieved through the fallowing proposition

Proposition 2.1. For every $\mathrm{DMU_o}$

$$r_o^b = n - LE_o^* \quad (6)$$

Where LE_o^* is the optimal value of the problem

$$\text{Max } LE_o = \sum_{j \neq o} I_j$$

$$\text{s.t. } \frac{\mathbf{u}_d'\mathbf{y}_d}{\mathbf{v}_d'\mathbf{x}_d} = \theta_d^* \quad d = 1,...,n \quad (7.1)$$

$$\frac{\mathbf{u}_d'\mathbf{y}_j}{\mathbf{v}_d'\mathbf{x}_j} \leq 1 \quad j = 1,...,n\,;\, d = 1,...,n \quad (7.2)$$

$$E_{dj} = \frac{\mathbf{u}_d'\mathbf{y}_j}{\mathbf{v}_d'\mathbf{x}_j} \quad j = 1,...,n\,;\, d = 1,...,n \quad (7.3)$$

$$\bar{E}_j = \frac{1}{n}\sum_{d=1}^{n} E_{dj} \quad j = 1,...,n \quad (7.4)$$

$$\bar{E}_j - \bar{E}_o \leq M\left(1 - I_j\right) \quad j = 1,...,n,\; j \neq o \quad (7.5)$$

$$\mathbf{v}_d \geq \mathbf{0}_m,\, \mathbf{u}_d \geq \mathbf{0}_s,\, \forall d; I_j \in \{0,1\},\, \forall j \neq o \quad (7)$$

where θ_d^* is the efficiency score of DMU_d provided by (1), M is a big positive quantity and I_j, $j \neq o$ are binary variables that, at optimum, indicate whether DMU_o outperforms DMU_j or not.

Proof. See Alcaraz et al. [1].

It should be noted that when the output oriented model is used in DEA, a given unit performs better than the others if its efficiency score is lower. Hence, in that case, the number of DMUs with a cross-efficiency score higher than or equal to that of DMU_o should be maximized in (7) and to do so, one just needs to replace (7.5) with $\bar{E}_o - \bar{E}_j \leq M\left(1 - I_j\right), j = 1,...,n, j \neq o$.

The worst ranking that DMU_o that could be attained is defined in terms of the maximum numbers of DMUs that perform worse than that unit. This is given by

$$r_o^w = n - \text{Min}_{(V,U)}\left\{\left|L_o\left(V,U\right)\right|\right\} \quad (8)$$

where $\left|L_o\left(V,U\right)\right|$ is the cardinality of the set $L_o\left(V,U\right)$ and r_o^w is achieved through the fallowing proposition.

Proposition 2.2. For every DMU_o,

$$r_o^w = HE_o^* + 1 \quad (9)$$

where HE_o^* is the optimal value of the problem

$$\text{Max } HE_o = \sum_{j \neq o} I_j$$

$$\text{s.t. } \frac{\mathbf{u}_d'\mathbf{y}_d}{\mathbf{v}_d'\mathbf{x}_d} = \theta_d^* \quad d = 1,...,n \quad (10.1)$$

$$\frac{\mathbf{u}_d'\mathbf{y}_j}{\mathbf{v}_d'\mathbf{x}_j} \leq 1 \quad j = 1,...,n\,;\, d = 1,...,n \quad (10.2)$$

$$E_{dj} = \frac{\mathbf{u}_d'\mathbf{y}_j}{\mathbf{v}_d'\mathbf{x}_j} \quad j = 1,...,n\,;\, d = 1,...,n \quad (10.3)$$

$$\bar{E}_j = \frac{1}{n}\sum_{d=1}^{n} E_{dj} \quad j = 1,...,n \quad (10.4)$$

$$\bar{E}_o - \bar{E}_j \leq M\left(1 - I_j\right) \quad j = 1,...,n,\; j \neq o \quad (10.5)$$

$$\mathbf{v}_d \geq \mathbf{0}_m,\, \mathbf{u}_d \geq \mathbf{0}_s,\, \forall d; I_j \in \{0,1\},\; \forall j \neq o \quad (10)$$

Where θ_d^* is still the efficiency score of DMU_d provided by (1), M is a big positive quantity and I_j, $j \neq o$ are binary variables that, at optimum, indicate whether DMU_j outperforms DMU_o or not.

Proof. See Alcaraz et al. [1].

Again in an output orientation we should replace (10.5) in (10) with $\bar{E}_j - \bar{E}_o \leq M\left(1 - I_j\right), j = 1,...,n, j \neq o$.

3. Ranking the ranges in cross-efficiency evaluations through the Directional Distance Function (DDF)

In the light of the above mentioned, we tend to propose more comprehensive ranking ranges in cross-efficiency evaluations, based on Ruiz's idea [7], for applying directional distance functions in cross-efficiency evaluations. This way the resulting efficiency measures, have been found to reflect the potential of a given DMU_o for increasing outputs while simultaneously reducing inputs along a given direction determined by the vector $\left(\mathbf{g}^{\mathbf{x}},\mathbf{g}^{\mathbf{y}}\right)$ (more comprehensive information can be found in [2]). Ruiz [7], chose a given direction $\left(\mathbf{g}^{\mathbf{x}},\mathbf{g}^{\mathbf{y}}\right)$ as $\left(-\mathbf{x}_{\mathbf{o}},\mathbf{y}_{\mathbf{o}}\right)$ and explored the duality relations regarding the models of directional distance functions and found some relationships that define the cross-efficiencies with the form of a ratio. In particular, it has been shown that the cross-efficiencies obtained in this new context can be calculated equivalently with the optimal weights of the CCR model.

The developments below are intended to provide ranking ranges in cross-efficiency evaluations based on directional distance functions. We start with the following problem which has the same optimal solutions as model (1).

$$\begin{aligned} \text{Max } & \eta_o = \frac{1}{2}\left(1+\frac{\mathbf{u}'_{\mathbf{o}}\mathbf{y}_{\mathbf{o}}}{\mathbf{v}'_{\mathbf{o}}\mathbf{x}_{\mathbf{o}}}\right) \\ \text{s.t. } & \frac{\mathbf{u}'_{\mathbf{o}}\mathbf{y}_{\mathbf{j}}}{\mathbf{v}'_{\mathbf{o}}\mathbf{x}_{\mathbf{j}}} \le 1 \qquad j=1,\dots,n \\ & \mathbf{v}_{\mathbf{o}} \ge \mathbf{0}_{\mathbf{m}},\ \mathbf{u}_{\mathbf{o}} \ge \mathbf{0}_{\mathbf{s}} \end{aligned} \tag{11}$$

According to Ruiz's procedure [7], the value of the directional distance function, β_o, which falls in the range of $[0,1)$, is calculated as using:

$$\beta_o = \frac{\mathbf{v}'_{\mathbf{o}}\mathbf{x}_{\mathbf{o}} - \mathbf{u}'_{\mathbf{o}}\mathbf{y}_{\mathbf{o}}}{\mathbf{v}'_{\mathbf{o}}\mathbf{x}_{\mathbf{o}} + \mathbf{u}'_{\mathbf{o}}\mathbf{y}_{\mathbf{o}}} \tag{12}$$

and the cross-efficiency of a given $\mathrm{DMU}_{\mathrm{j}}$, $j=1,\dots,n$, obtained with such weights of $\mathrm{DMU}_{\mathrm{d}}$ is defined as

$$E_{dj}^{\beta} = \frac{\mathbf{v}'_{\mathbf{d}}\mathbf{x}_{\mathbf{j}} - \mathbf{u}'_{\mathbf{d}}\mathbf{y}_{\mathbf{j}}}{\mathbf{v}'_{\mathbf{d}}\mathbf{x}_{\mathbf{j}} + \mathbf{u}'_{\mathbf{d}}\mathbf{y}_{\mathbf{j}}}, \quad j=1,\dots,n \tag{13}$$

Where $\left(\mathbf{v}_{\mathbf{d}},\mathbf{u}_{\mathbf{d}}\right)$, is an optimal solution of model (11) for the weights of any $\mathrm{DMU}_{\mathrm{d}}$, $d=1,\dots,n$.

The cross-efficiency score of $\mathrm{DMU}_{\mathrm{j}}$, $j=1,\dots,n$ is defined as usual, as the average of the cross-efficiencies obtained with the weights of all the DMUs as fallow

$$\bar{E}_j^{\beta} = \frac{1}{n}\sum_{d=1}^{n} E_{dj}^{\beta}, \quad j=1,\dots,n \tag{14}$$

3.1. The best ranking of a DMU

In cross-efficiency evaluations with directional distance functions, a given unit performs better than others if its cross-efficiency score is lower. In this case, the most favorable scenario for a given $\mathrm{DMU}_{\mathrm{o}}$ is associated with a choice of DEA weights for all the DMUs, which gives rise to the minimum number of DMUs with lower cross-efficiency score than that of $\mathrm{DMU}_{\mathrm{o}}$. Thus the best ranking of a $\mathrm{DMU}_{\mathrm{o}}$ is given by

$$\left(r_o^b\right)^{\beta} = \mathrm{Min}_{(V,U)}\left\{\left|H_o^{\beta}(V,U)\right|\right\}+1 \tag{15}$$

Where $\left|H_o^{\beta}(V,U)\right|$ is the cardinality of the set

$$H_o^{\beta}(V,U) = \left\{DMU_j, j=1,\dots,n \mid \bar{E}_j^{\beta} < \bar{E}_o^{\beta}\right\}$$

Based on proposition 2.1, for every $\mathrm{DMU}_{\mathrm{o}}$

$$\left(r_o^b\right)^{\beta} = n - LE_o^{\beta*} \tag{16}$$

Where $LE_o^{\beta*}$ is the optimal value of the problem

$$\begin{aligned} & \text{Max } LE_o^{\beta} = \sum_{j\ne o} I_j \\ & \text{s.t. } \frac{1}{2}\left(1+\frac{\mathbf{u}'_{\mathbf{d}}\mathbf{y}_{\mathbf{d}}}{\mathbf{v}'_{\mathbf{d}}\mathbf{x}_{\mathbf{d}}}\right) = \eta_d^* \quad d=1,\dots,n \quad (17.1) \\ & \frac{\mathbf{u}'_{\mathbf{d}}\mathbf{y}_{\mathbf{j}}}{\mathbf{v}'_{\mathbf{d}}\mathbf{x}_{\mathbf{j}}} \le 1 \quad j=1,\dots,n;\ d=1,\dots,n \quad (17.2) \\ & E_{dj}^{\beta} = \frac{\mathbf{v}'_{\mathbf{d}}\mathbf{x}_{\mathbf{j}} - \mathbf{u}'_{\mathbf{d}}\mathbf{y}_{\mathbf{j}}}{\mathbf{v}'_{\mathbf{d}}\mathbf{x}_{\mathbf{j}} + \mathbf{u}'_{\mathbf{d}}\mathbf{y}_{\mathbf{j}}} \quad j=1,\dots,n;\ d=1,\dots,n \quad (17.3) \\ & \bar{E}_j^{\beta} = \frac{1}{n}\sum_{d=1}^{n} E_{dj}^{\beta} \quad j=1,\dots,n \quad (17.4) \\ & \bar{E}_o^{\beta} - \bar{E}_j^{\beta} \le M\left(1-I_j\right) \quad j=1,\dots n,\ j\ne o \quad (17.5) \\ & \mathbf{v}_{\mathbf{d}} \ge \mathbf{0}_{\mathbf{m}},\ \mathbf{u}_{\mathbf{d}} \ge \mathbf{0}_{\mathbf{s}}, \forall d; I_j \in \{0,1\}, \forall j \ne o \quad (17) \end{aligned}$$

Where η_d^* is the efficiency score of $\mathrm{DMU_d}$ given by (11).

3.2. The worst ranking of a DMU

The worst ranking of a $\mathrm{DMU_o}$ is also given by

$$\left(r_o^w\right)^\beta = n - \mathrm{Min}_{(V,U)}\left\{\left|L_o^\beta(V,U)\right|\right\} \qquad (18)$$

Where $\left|L_o^\beta(V,U)\right|$ is the cardinality of the set

$$L_o^\beta(V,U) = \left\{DMU_j,\ j=1,\ldots,n \mid \bar{E}_j^\beta > \bar{E}_o^\beta\right\}$$

According to the proposition 2.2, for every $\mathrm{DMU_o}$

$$\left(r_o^w\right)^\beta = HE_o^{\beta*} + 1, \qquad (19)$$

Where $HE_o^{\beta*}$ is the optimal value of the problem

$$\mathrm{Max}\ HE_o^\beta = \sum_{j\neq o} I_j$$

$$\text{s.t. } \frac{1}{2}\left(1+\frac{\mathbf{u'_d y_d}}{\mathbf{v'_d x_d}}\right) = \eta_d^* \quad d=1,\ldots,n \quad (20.1)$$

$$\frac{\mathbf{u'_d y_j}}{\mathbf{v'_d x_j}} \le 1 \quad j=1,\ldots,n;\ d=1,\ldots,n \quad (20.2)$$

$$E_{dj}^\beta = \frac{\mathbf{v'_d x_j} - \mathbf{u'_d y_j}}{\mathbf{v'_d x_j} + \mathbf{u'_d y_j}} \quad j=1,\ldots,n;\ d=1,\ldots,n \quad (20.3)$$

$$\bar{E}_j^\beta = \frac{1}{n}\sum_{d=1}^n E_{dj}^\beta \quad j=1,\ldots,n \quad (20.4)$$

$$\bar{E}_j^\beta - \bar{E}_o^\beta \le M\left(1-I_j\right) \quad j=1,\ldots n, j\neq o \quad (20.5)$$

$$\mathbf{v_d} \ge \mathbf{0_m},\ \mathbf{u_d} \ge \mathbf{0_s}, \forall d; I_j \in \{0,1\}, \forall j \neq o \quad (20)$$

Where η_d^* is the efficiency score of $\mathrm{DMU_d}$ provided by (11).

4. Numerical example

This section tends to illustrate the proposed procedure and compare its results with ranking ranges proposed by Alcaraz et al. [1], using a small data set used by Sexton et al. [8], where six nursing homes were evaluated with two inputs and two outputs. The data together with the efficiency score provided by (11) are given in Table 1 (which has been solved by Lingo[1] 14). Table 2, further reports the ranking ranges with directional distance function as well as ranking ranges proposed by Alcaraz et al. [1]. The ranking ranges based on the directional distance functions are also depicted graphically in fig. 1.

Table 1. Data (Inputs, Outputs and Efficiency Scores)

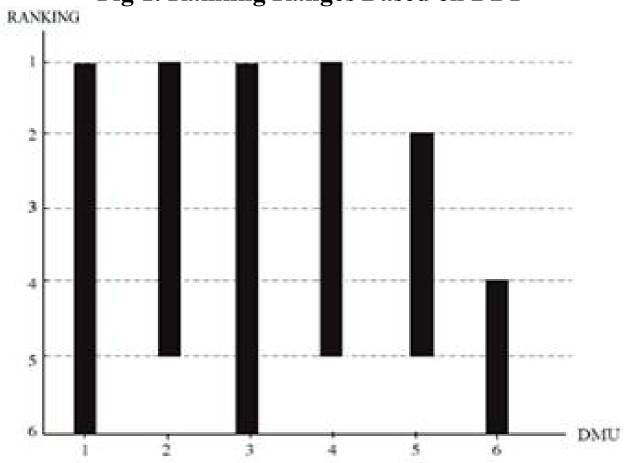
DMU	x_1	x_2	y_1	y_2	η_d^*
1	1.5	0.2	1.4	0.35	1
2	4	0.7	1.4	2.1	1
3	3.2	1.2	4.2	1.05	1
4	5.2	2	2.8	4.2	1
5	3.5	1.2	1.9	2.5	0.988749
6	3.2	0.7	1.4	1.5	0.933726

Table 2. Ranking ranges

DMU	Ranking ranges based on DDF		Ranking ranges	
	$\left(r_o^b\right)^\beta$	$\left(r_o^w\right)^\beta$	r_o^b	r_o^w
1	1	6	1	6
2	1	5	1	5
3	1	6	1	6
4	1	5	1	4
5	2	5	2	5
6	4	6	4	6

1. A Powerful Software Package

Fig 1. Ranking Ranges Based on DDF



In this example, ranges for the possible rankings of the DMUs are very wide and most of the DMUs could take almost any position in the rankings of DMUs that the cross-efficiency evaluation based on directional distance functions may yield, which is the same as the ranking ranges proposed by Alcaraz et al. [1]. From Table 2 it can be concluded that the best and worst rankings of DMUs 1, 2, 3, 5, 6 and the best ranking of DMU 4 are the same in the two mentioned ranking ranges (the procedure proposed here and that of Alcaraz et al. [1]), but the worst rankings of DMU 4 are different. In fact, the worst ranking of DMU 4 can change from 4 to 5, by simultaneously accounting for the inefficiency in inputs and in outputs.

5. Conclusions

Determining a ranking range for each unit in cross-efficiency evaluation, without any need to make any specific choice of DEA weights is an effective procedure to improve Sexton et al.'s method [8], which is the non-uniqueness of the factor weights obtained from the DEA models. Based on the proposed procedure the ranking ranges are developed based on directional distance functions. To do this, the peer-evaluation of DMUs was used based on measures that simultaneously account for the inefficiency in inputs and in outputs. These ranking ranges can also complete the cross-efficiency evaluation based on directional distance functions with information regarding the extent of confidence with the ranking obtained. This is because directional distance functions have the same difficulties as the classical DEA efficiency scores when used for ranking. Finally, by the numerical results, the advantages of the proposed method over the previous one have been shown.

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