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The Evaluation of Commercial Banks Performance and Market Risk by using of Fuzzy DEA

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Abstract
This study presents a reasonable program for large commercial banks in order to supply bank resources by the long term bank deposits and investments, making balance between financial commitments and investments, enhancing the value at risk by maintaining market and bank high liquidity, management of crisis in condition of liquidity shortage and funds, assessment of value at risk index by using determination bank interval efficiency, ranking the set of commercial big bank by using of the fuzzy data envelopment analysis (DEA) models. In the following, we extend fuzzy slack-based model (SBM) for fuzzy inputs and outputs data. We are determined the risk factors in bank operating process by using of inefficiency concept. In this study, we use the data of seven banks which were accepted in Tehran Stock Exchange (Eghtesad novin bank, Parsian, Tejarat, Sina, Karafarin, Melat and Saderat) over a 4 years' period from 2012 to 2015. We use the fuzzy DEA for assessment of value at risk index for Banks listed on the Tehran Stock Exchange.

Keywords: Data envelopment analysis, Assessment performance, Value at risk index, Efficiency; Fuzzy set.

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1. Introduction

Creating a profitable organization without consideration and awareness of the progress and achievement of objectives without identifying the challenges faced by organization and obtaining feedback and notice of the implementation of formulation of policies and identify issues that need serious improvement will not be possible. In general, we can say that all issues which are measurable cannot be controlled and what you cannot control it, is not manageable. The main subject in all organizational analysis, performance measurement and improvement requires organization and Thus, it is not conceivable without the performance assessment system. Professor Lotfi Asgar Zadeh introduced the theory of fuzzy sets [20]. Fuzzy set theory has been entered in many branches of science. In this regard, linear programming and data envelopment analysis (DEA) also have drawn attention to this branch of science. Zimmerman [21], Saati et al. [18]. Evaluating performance in organizations such as banks that have many branches or multiple parts of organization, they need to have additional control. Therefore, these organizations cannot use traditional methods of performance evaluation that focuses on financial aspects, to meet the organization's needs. In this study, we try to determine the index of market risk and performance assessment of commercial banks to understand the relationship between the two. In today's competitive world performance evaluation plays an important role in the survival and continuity of companies and organizations and so it is also important to evaluate and introduce performance appraisal criteria. In total, whatever the changes and the complexity and its acceleration, then we need to evaluate organizational performance is more than ever before. In management science, the process of evaluation is referred to in form of phrases such as: efficiency, effectiveness, significance, performance evaluation [14].

2. Literature review

Dadgar et al. [7] evaluated economic efficiency of Tejarat bank supervisors using data envelopment analysis in the period 2001 to 2003. They assumed that the outputs and inputs of Tejarat bank supervisors not optimize and by modifying combination of factors, their effectiveness increased. Finally, the conclusion is that the supervisory areas (three, four, and five) of Tehran are efficient and supervisory of Qom, Zanjan, East and West Azarbaijan are inefficient. Abbasian, et al. [1], in an article titled measure of factor productivity sectors with data envelopment analysis estimate the values of productivity and efficiency by comparing the relative economic sectors based on data values and output them. The results show that although the economic efficiency of the process is slightly increased, however, overall performance due to many economic activities that have had a significant material and human resources are not justified. The service sector is facing more problemsdue to the large number of people working, scope extent, scope of activities. The continued growth and dynamism of the service sector, characterized by major economic systems is managed and developed in the contemporary world. So ignoring the problems of this sector, such as low relative productivity in addition to many opportunities disclaims growth and economic development can provide lots of problems and social, political and cultural issues in future. Hadian and Azimi [11] evaluated iran's banking system efficiency for the ten commercial banks by using data envelopment analysis in the period 1997 to 1999. They concluded that in three years by assuming variable returns to scale three banks Meli, Keshavarzi and Sanatmadan are technical, allocative and economic efficient and Export Development Bank was only technically efficient and overall efficiency of specialized banks was higher than commercial banks. Fadaeinezhad and Aghbalnya [9] modeled the risk of investment in the Tehran Stock Exchange by using the VAR model. The results showed that the model is designed using both simple and exponential moving average at 95% reliable, but at higher confidence levels are not appropriate. The research was conducted in the context of market risk, all based on bankruptcy and the capital provided [2]. In 2000, Altunbas et al. [3] introduced the return of unpaid loans to banks as an indicator of risk or danger. In 1986, Hunter and Timme [12] introduced indicator of risk or danger based on field-scale economic concepts. Data envelopment analysis method initially was introduced by Charnes et al. [5]. In 2001, Tone [19] provided slack-based model (SBM) for evaluation of the set decision making units, those were efficient when the value of the objective function SBM is equal to one, and it means that all inputs and outputs slacks are zero. The most important feature of this model is that, it was unit invariant than to the change units of inputs and outputs. The fuzzy theory was originally developed by an Iranian scientist named Lotfi Asgar Zadeh and professor of Berkeley University. This theory today issued as a mighty tool in the mathematical sciences, computer and electrical engineering. This theory is to action under uncertainty, it is capable for mathematical formulation of many variables and concepts and systems that are inaccurate and grounds for reasoning, inference control and decision-making under uncertainty provided. Given that the risk index is an imprecise score, we introduce it as an imprecise number based on the concept the bank efficiency, in this paper; we use fuzzy data envelopment analysis models. In this area, see: Cooper et al. [6] and Despotis and Smirlis [8] and Guo and Tanaka [10] and Jahanshahloo et al. [13] and Kao and Liu [15]. Miller and Noulas [16] evaluated the efficiency of large commercial banks in England using data envelopment analysis in the period 1982-1995. They concluded that the mean inefficiency of England banks is at a low level in the period considered, the average efficiency has decreased in all Bank during the period considered. Pastor [17], by attention to Sufian researches, in an article titled Singapore banking efficiency and its relation to stock returns using data envelopment analysis to evaluate changes in the efficiency of commercial banks in the period 1993 to 2003 in Singapore. He estimated the average efficiency of commercial banks in Singapore 95.4% as a result 4.6% of inputs is wasted. He also points out that small commercial banks had better performance in terms of efficiency than larger banks. In addition, changes in stock indexes and stock prices had little impact on cost efficiency. The rest of the paper is organized as follows: Section 2 presents some basic definitions and notation relating the research. In section 3, we propose methodology research methodology. In section 4, we assessment performances and value at risk index of commercial banks that listed on the Tehran Stock Exchange and present our results in the end.

2.1. Terms and expressions of defined

Definition 2.1.1. Performance evaluation: evaluation of performance process can be defined to quantify the efficiency and effectiveness of operations of each organization.

Definition 2.1.2. Stock market risk: Value at Risk(VaR) represents the maximum expected loss on the portfolio or investment portfolio over a given time horizon (e.g. one day, one month or one year) at the confidence level in normal market conditions.

Definition 2.1.3. Data Envelopment Analysis: Data envelopment analysis is a nonparametric method to evaluate performance of the set of decision-making units. In this method decision-making units are independent units which use similar inputs to produce the same outputs. Homogeneity of necessary inputs and outputs of the units in the first condition is evaluated.

2.2. Scope of research

Due to the limited specialty society and the subject of the investigation, no sampling has been done and all banks listed in Tehran stock exchange information and financial statements are available on the Tehran Stock Exchange. Seven banks have been used in this study. They are Eghtesad novin bank, Parsian, Tejarat, Sina, Karafarin, Melat and Saderat. According to the above explanation, see study population included 27 (6 bank listed on the Tehran Stock Exchange for 4 years and a bank for 3 years). The study period is from 2012 to 2015. Banks accepted in Tehran Stock Exchange place in this paper.

3. Research Method

3.1. Efficiency of decision making units using a non-radial model

Assume that we have n DMUs, with the input and output vectors

$(X_j, Y_j),\ j = 1, \ldots, n.$

$X_j = (x_{1j}, x_{2j}, \ldots, x_{mj})^T$,

$Y_j = (y_{1j}, y_{2j}, \ldots, y_{sj})^T$.

Also, consider, $s_i^-, i = 1, \ldots, m$ and s_r^+, $r = 1, \ldots, s$ are the input (output) slacks.

$\lambda'_j,\ j = 1, \ldots, n,$ are intensity variables.

Assuming that $\tilde{X}_j = (\tilde{x}_{1j}, \ldots, \tilde{x}_{mj})$ and $\tilde{Y}_j = (\tilde{y}_{1j}, \ldots, \tilde{y}_{sj})$ represents the input vector and output vector corresponding to the j-th DMU is in fuzzy state. These can be represented by membership functions $\mu_{\tilde{X}_{ij}}(\tilde{x}_{ij}),\ \mu_{\tilde{Y}_{rj}}(\tilde{y}_{rj})$ in the convex fuzzy set. In this paper we will assume that they are fuzzy triangular numbers.

In 2001, Tone [19] introduce famous model slack based (SBM) model for evaluation efficiency the set of decision making units. By attention to SBM model, in the fuzzy environment, the Fuzzy-SBM formula can therefore be written as:

$$\begin{aligned}
\min\quad & q - \frac{1}{m}\sum_{i=1}^{m} s_i^- / \tilde{x}_{ik} \\
\text{s.t.}\quad & q + \frac{1}{s}\sum_{r=1}^{s} \frac{s_r^+}{\tilde{y}_{rk}} = 1, \\
& \sum_{j=1}^{n} \lambda'_j \tilde{x}_{ij} + s_i^- = q\tilde{x}_{ik}, \quad i = 1, \ldots, m, \\
& \sum_{j=1}^{n} \lambda'_j \tilde{y}_{ij} - s_r^+ = q\tilde{y}_{rk}, \quad r = 1, \ldots, s, \qquad (1) \\
& \sum_{j=1}^{n} \lambda'_j = q \\
& \lambda'_j \geq 0,\ j = 1, \ldots, n, s_i^- \geq 0, i = 1, \ldots, m, \\
& s_r^+ \geq 0,\ r = 1, \ldots, s,\ q > 0.
\end{aligned}$$

In model (1) all inputs and outputs are fuzzy data. If any input or output amounts is an exact value, the exact data can be expressed as degenerated membership functions.

3.2. Measuring the efficiency of using fuzzy SBM model.

Assuming that $\tilde{X}_j = (\tilde{x}_{1j}, \ldots, \tilde{x}_{mj})$ and $\tilde{Y}_j = (\tilde{y}_{1j}, \ldots, \tilde{y}_{sj})$ represents the input vector and output vector corresponding to the j-th DMU is in fuzzy state. These can be represented by membership functions $\mu_{\tilde{X}_{ij}}(\tilde{x}_{ij}),\ \mu_{\tilde{Y}_{rj}}(\tilde{y}_{rj})$ in the convex fuzzy set. If the $S(\tilde{x}_{ij})$ and $S(\tilde{y}_{rj})$ represent the support of fuzzy numbers $\tilde{x}_{ij}$ and $\tilde{y}_{rj}$ respectively. The support is the set of elements with membership functions larger than 0. Using the concept of α-cut in fuzzy theory. We can put α-cut collection for each of the above numbers defined as follows.

$$\begin{aligned}
&(X_{ij})_\alpha = \left\{x_{ij} \in S(\tilde{x}_{ij}) \middle| \mu_{\tilde{x}_{ij}}(x_{ij}) \geq \alpha\right\}, \\
&j = 1, \ldots, n,\ i = 1, \ldots, m, \qquad (2) \\
&(Y_{rj})_\alpha = \left\{Y_{rj} \in S(\tilde{y}_{rj}) \middle| \mu_{\tilde{y}_{rj}}(y_{rj}) \geq \alpha\right\}, \\
&j = 1, \ldots, n, \quad r = 1, \ldots, s,
\end{aligned}$$

It should be noted that the above sets are crisp sets and α-level set can be corresponding to each alpha value, the interval numbers corresponding to α-level sets is presented as follows.

$$\begin{aligned}
&(x_{ij})_\alpha = \left\{x_{ij} \in S(\tilde{x}_{ij}) \middle| \mu_{\tilde{x}_{ij}}(x_{ij}) \geq \alpha\right\} = \left[{x_{ij}}^l_\alpha, {x_{ij}}^u_\alpha\right] \\
&= \Big[\min_{x_{ij}} \left\{x_{ij} \in S(\tilde{x}_{ij}) \middle| \mu_{\tilde{x}_{ij}}(x_{ij}) \geq \alpha\right\}, \\
&\max_{x_{ij}} \left\{x_{ij} \in S(\tilde{x}_{ij}) \middle| \mu_{\tilde{x}_{ij}}(x_{ij}) \geq \alpha\right\}\Big] \qquad (3) \\
&(y_{rj})_\alpha = \left\{y_{rj} \in S(\tilde{y}_{rj}) \middle| \mu_{\tilde{y}_{rj}}(y_{rj}) \geq \alpha\right\} =
\end{aligned}$$

$\left[y_{rj}{}^{l}_{\alpha}, y_{rj}{}^{u}_{\alpha}\right]$
$=\left[\min_{y_{rj}}\left\{y_{rj} \in S(\tilde{y}_{rj}) \middle| \mu_{\tilde{y}_{rj}}(y_{rj}) \geq \alpha\right\}, \max_{y_{rj}}\left\{y_{rj} \in S(\tilde{y}_{rj}) \middle| \mu_{\tilde{y}_{rj}}(y_{rj}) \geq \alpha\right\}\right]$

It should be noted that $\left[(x_{ij})^{l}_{\alpha}, (x_{ij})^{u}_{\alpha}\right]$ and $\left[(y_{rj})^{l}_{\alpha}, (y_{rj})^{u}_{\alpha}\right]$ are interval numbers according to a set of fuzzy numbers $\tilde{y}_{rj}$ and $\tilde{x}_{ij}$, respectively. Using different amounts of alpha, α-level sets can be presented as follows.
$\left[(x_{ij})\alpha | 0 < \alpha \leq 1\right], \left[(y_{rj})\alpha | 0 < \alpha \leq 1\right].$

By means of α-cut concept, we can convert fuzzy data envelopment analysis models (FDEA) into the Crisp-DEA model. By using of the Extension Principle fuzzy theory (Zadeh, [20], Zimmerman [21]), the efficiency membership function for the DMU can be defined as:

$$\mu_{\tilde{E}_k}(z) = \sup_{x,y} \min \left\{\left(\mu_{\tilde{x}_{ij}}(x_{ij}), \mu_{\tilde{y}_{rj}}(y_{rj}), \forall j, r, i \middle| z = E_k(x, y)\right)\right\} \quad (4)$$

In the above formula $E_k(x, y)$ is the efficiency score calculated by using of the traditional SBM model for the inputs and outputs set, for any efficiency score corresponding to amounts x_{ij}, y_{rj}, of z, its minimum degree of membership equals to the membership of $\tilde{E}_k$ in point z.
Now, in accordance with the concept of Pareto optimal solution and method of solving the interval problems, to calculate the lower bound of efficiency, we put under evaluation unit at worst condition and other units in the best condition. The efficiency lower bound for a certain amount of α with membership function $\mu_{\tilde{E}_k}$ is presented below.

$$\begin{aligned} &\min \quad q - \frac{1}{m}\sum_{i=1}^{m}(s_i^-)^L / (x_{ik})^L_\alpha \\ &s.t \quad 1 = q + \frac{1}{s}\sum_{r=1}^{s}(s_r^+)^U / (y_{rk})^U_\alpha \\ &\sum_{j=1,\neq k}^{n}(x_{ij})^U_\alpha \lambda'_j + (x_{ik})^L_\alpha \lambda'_k + (s_i^-)^L = q\,(x_{ik})^L_\alpha, \quad i = 1, \dots, m, \\ &\sum_{j=1,\neq k}^{n}(y_{rj})^L_\alpha \lambda'_j + (y_{rk})^U_\alpha \lambda'_k - (s_r^+)^U = q\,(y_{rk})^U_\alpha, \quad r = 1, \dots, s, \\ &\sum_{j=1,\neq k}^{n}\lambda'_j = q, \ \lambda'_j \geq 0, \quad j = 1, \dots, n, \\ &(s_i^-)^L \geq 0, i = 1, \dots m, \ (s_r^+)^U \geq 0, \\ &r = 1, \dots s, \quad q > 0. \end{aligned} \quad (5)$$

Similarly, to calculate the upper bound of efficiency, we put under evaluation unit at the best condition and other units in the worst condition. The efficiency upper bound for a certain amount of α with membership function $\mu_{\tilde{E}_k}$ is presented below.

$$\begin{aligned} &\min \quad q - \frac{1}{m}\sum_{i=1}^{m}(s_i^-)^U / (x_{ik})^U_\alpha \\ &s.t \quad 1 = q + \frac{1}{s}\sum_{r=1}^{s}(s_r^+)^L / (y_{rk})^L_\alpha \\ &\sum_{j=1,\neq k}^{n}(x_{ij})^L_\alpha \lambda'_j + (x_{ik})^U_\alpha \lambda'_k + (s_i^-)^U = q\,(x_{ik})^U_\alpha, \quad i = 1, \dots, m, \\ &\sum_{j=1,\neq k}^{n}(y_{rj})^U_\alpha \lambda'_j + (y_{rk})^L_\alpha \lambda'_k - (s_r^+)^L = q\,(y_{rk})^L_\alpha, \quad r = 1, \dots, s, \\ &\sum_{j=1,\neq k}^{n}\lambda'_j = q, \ \lambda'_j \geq 0, \ j = 1, \dots, n, \\ &(s_i^-)^U \geq 0, i = 1, \dots m, \ (s_r^+)^L \geq 0, \\ &r = 1, \dots s, \quad q > 0. \end{aligned} \quad (6)$$

Using the two models, we can evaluate an interval efficiency corresponding to each decision-making unit.

3.3. Super-efficiency with fuzzy SBM model.

Anderson and Peterson [4] presented the super-efficiency model for ranking units in case of a large number of efficient units. They have removed the unit under assessment of the possibility production set and its impact on the performance of other units investigated. If the inputs and outputs be fuzzy numbers according to the concept of α-cutting and membership functions for each fuzzy number in the convex fuzzy set. The SBM super-efficiency model is presented as follows.

$$\begin{aligned} &\min \quad \frac{1}{m}\sum_{i=1}^{m}\bar{x}'_i / \tilde{x}_{ik} \\ &s.t \quad \frac{1}{s}\sum_{r=1}^{s}\bar{y}'_r / \tilde{y}_{rk} = 1 \\ &\sum_{j=1,\neq k}^{n}\tilde{x}_{ij}\lambda'_j \leq \bar{x}'_i \quad i = 1, \dots, m \\ &\sum_{j=1,\neq k}^{n}\tilde{y}_{ij}\lambda'_j \geq \bar{y}'_r \quad r = 1, \dots, s \\ &\sum_{j=1,\neq k}^{n}\lambda'_j = q \\ &\lambda'_j \geq 0, j = 1, \dots, n, \neq k, \end{aligned} \quad (7)$$

$\overline{x}_i' \geq q\tilde{x}_{ik}, \ i = 1, \dots, m,$
$\overline{y}_r' \leq q\tilde{y}_{ij}, \ \overline{y}_r' \geq 0, r = 1, \dots, s, \ q > 0.$

According to the definitions (3) and (4) and method of solving the problems interval, Super SBM model to calculate lower bounds for super-efficiency are presented as following.

$$\begin{aligned} \min \quad & \frac{1}{m}\sum_{i=1}^{m}(\overline{x}_i')^L/(x_{ik})_\alpha^L \\ s.t \quad & \frac{1}{s}\sum_{r=1}^{s}(\overline{y}_r')^U/(y_{rk})_\alpha^U = 1 \\ & \sum_{j=1,\neq k}^{n}(x_{ik})_\alpha^L\lambda_j' \leq (\overline{x}_i')^L, \ i = 1, \dots, m, \quad (8) \\ & \sum_{j=1,\neq k}^{n}(Y_{ik})_\alpha^U\lambda_j' \geq (\overline{y}_r')^U, \quad r = 1, \dots, s, \\ & \sum_{J=1,\neq k}^{n}\lambda_j' = q \\ & \lambda_j' \geq 0, \ j = 1, \dots, n, \neq k, \ (\overline{x}_i')^L \geq q(x_{ik})_\alpha^L, \ i = 1, \dots, m, \\ & (\overline{y}_r')^U \leq q(y_{ik})_\alpha^U, \ (\overline{y}_r')^U \geq 0, r = 1, \dots, s, \ q > 0. \end{aligned}$$

Super SBM model to calculate upper bounds for super-efficiency are presented as following.

$$\begin{aligned} \min \quad & \frac{1}{m}\sum_{i=1}^{m}(\overline{x}_i')^U/(x_{ik})_\alpha^U \\ s.t \quad & \frac{1}{s}\sum_{r=1}^{s}(\overline{y}_r')^L/(y_{rk})_\alpha^L = 1 \\ & \sum_{j=1,\neq k}^{n}(x_{ik})_\alpha^U\lambda_j' \leq (\overline{x}_i')^U, i = 1, \dots, m, \quad (9) \\ & \sum_{j=1,\neq k}^{n}(Y_{ik})_\alpha^L\lambda_j' \geq (\overline{y}_r')^L, \quad r = 1, \dots, s, \\ & \sum_{J=1,\neq k}^{n}\lambda_j' = q \\ & \lambda_j' \geq 0, \ j = 1, \dots, n, \neq k, \ (\overline{x}_i')^U \geq q(x_{ik})_\alpha^U, \ i = 1, \dots, m, \\ & (\overline{y}_r')^L \leq q(y_{ik})_\alpha^L, \ (\overline{y}_r')^L \geq 0, r = 1 \dots, s, \ q > 0. \end{aligned}$$

If the membership function is unknown for different values of α and numbers corresponding interval values, we can have used Chen method to rank the fuzzy numbers. Assuming $E_k^\alpha=[(E_k)_\alpha^l, (E_k)_\alpha^u]$ be efficiency interval of models corresponding to the value of α (7 and 8) and h is the maximum amount available to the membership function corresponding to the fuzzy numbers. Put $\alpha_i = \frac{ih}{m}, \ i = 1, \dots, m.$ The ranking index that provided by Kao and Liu [14] are provided below.

$$I = \frac{\sum_{i=0}^{m}[(E_k)_{\alpha_i}^U - c]}{\sum_{i=0}^{m}[(E_k)_{\alpha_i}^U - c] - \sum_{i=0}^{m}[(E_k)_{\alpha_i}^L - d]}, \ m \to \infty (10)$$

In this case, $c = \min_{i,k}\{(E_k)_{\alpha_i}\}$ and $d = \max_{i,k}\{(E_k)_{\alpha_i}\}$. As we can use of lower and upper bound average of the super-efficiency for ranking of DMUs.

4. Assessment performances of commercial banks

In this study, the required information by examining the financial statements and descriptive report on the bank site and the Stock Exchange site has been collected. First, in order to analyze the results, we used of CCR and BCC models and obtained efficiency scores of bank branches. In the following, in order to earn the rank of units, we solved super-efficiency CCR and BCC models.

4.1. Research data:

Input and output variables in the bank data are as follows.

4.1.1. Input variables:

Research input variables, including number of staff, total deposits, value at risk, which is defined below each separately.

4.1.1.1. Number of staff:

The number of staff of each sample is equal to sum of all staff in the different branches of banks across the country.

4.1.1.2. Total deposit:

Total deposits of the following can be obtained.

1. Investment deposit at the central bank
2. The legal deposit in the liberated areas: legal deposit equal to 10% of deposits is in free zones.
3. Legal deposit within the country: in accordance with paragraph 3 of article 14 of the monetary and financial law, approved 1972 in determining the interest rate legal deposit banks at the central bank may the ratio various different for it to be determined, in terms of composition and activity of banks, however, the ratio is less than 10% and not more than 30%.

Now the legal deposit at the central bank, according to the type of deposit banks is between 10% and 70% respectively. These resources are subject to legal deposit: Demand deposits (Deposit loan monetary and currency, check bank sold, Currency transfers), Loan Savings, Short-term deposits, long-term deposits, Deposit guarantee, Housing, credit the payment ago.

4.1.1.3. Total fixed assets:
Fixed assets are recorded in the accounts based costing. These assets include land, buildings, upholstery and computer equipment, vehicles; buildings leased optimization and asset prepayment. Optimization and overhaul costs that cause a significant increase in capacity or useful life of fixed assets or fundamental improvement in the quality of their efficiency considered as a capital expenditure and depreciated over the remaining useful life of the asset. The cost of maintenance and minor repairs in order to maintain or restore the economic benefits expected standard of performance is evaluated based on the entity's primary assets are done and in the event regarded as the current cost and profit and loss account in the period.

4.1.1.4. VaR index:
In this study, we use of data envelopment analysis to calculate the value at risk associated with the banks listed on the Tehran Stock Exchange.
One of the inputs to the banks is the amount of overdue loans by customers which is considered as a fuzzy number. In this study, we consider it efficiency interval that is obtained of models (5), (6). This amount represents the amount of market risk. In following sensitivity to changes attributed considers other inputs and outputs. Decision making units in this research are banks listed on the Tehran Stock Exchange in the years 2012 to 2015.

4.1.2. Output variables:
Output variables research includes total loans, total investment and wage costs.
The following is a separately defined.

4.1.2.1. Total loans:
Total loans are calculated as sum of loans to customers at all branch banks.

4.1.2.2. Total investments:
Investments include: stock investments and investments in other stocks. Short term investments in listed stock exchange companies that are quick transaction based on the total market value of above investments, are evaluated. Other short-term investments are evaluated at the lower of cost and net sales value of each investment. Long-term investments at finished price are evaluated after a permanent decline in value of investments. Investment income of subsidiary and affiliated company shares at the time of the adoption of profits through the general assembly equity investee companies (until the date of approval of the financial statements) and other long-term investments and current income at the time of profit approval by the general assembly equity of investee companies (as of the balance sheet) is detected.

4.1.2.3. Costs, Banking Wages
Total costs, bank wages can be achieved through the Table (1):

Table 1: costs, bank wages (commission)

Wages paid to brokers
Wages concern to bonds trust paid to other banks
Wages paid concern to mechanized systems
Other
Net wages pay

4.2. Results

First, we obtained efficiency of branches with regard to the data relating to the branches in Table (2) using conventional models such as CCR and BCC models in input orientation. In following, we solved CCR and BCC super-efficiency models for ranking branches efficient. By attention to the fourth input of branches is a fuzzy number, we use from the middle it for solving CCR and BCC models. The results in the Table (3) are presented.

We solved CCR model for obtaining the efficiency of decision making units in a state of constant returns to scale. The results are presented in Table (3), As can be seen units 1, 6, 7, 8, 9, 10, 11, 15, 16, 19, 22, 23 and 27 are branches efficient according to the first column of Table (3). We solved BCC model for obtaining the efficiency of decision making units in a state of variable returns to scale.

The results are presented in fifth column of Table (3), as can be seen in the case of variable returns to scale, units 1, 3, 4, 5, 7, 8, 9, 10, 11, 15, 16, 18, 19, 20, 22, 23, 25 and 27 are efficient and other units are inefficient. We compared the results of the CCR, BCC, SBM models in Figure (1). Given that the number of efficient units is determined by CCR, BCC models are more than one unit to distinguish between efficient units, we used of super-efficiency models. The third column of Table (3) shows the scores of CCR super efficiency. Rank units in the second column of Table (3) is specified, as can be seen units 19, 22, 15 and 1 have the highest rank. For non-extreme units and inefficient units, the scores of efficiency and super efficiency are the same. The sixth column of Table (3) shows the amounts of super-efficiency of the BCC model. Units 19, 22, 11 and 5 have the most rank and it indicates its importance in comparison with other branches of the branch. These units can be considered as benchmark of other branches. Also, we can use of SBM model for evaluation efficiency of branches. The second column of Table (4) shows the efficiency scores of SBM model. We compared the super efficiency scores of CCR, BCC, SBM models in Figure (2).

Table 2: Data for the 27 commercial banks in Tehran Stock Exchange

DMU	Staff (person)	Total fixed assets (NT dollar)	Total deposits (NT dollar)	VaR (NT dollar)			Total loans (NTdollar)	Total investments (NT dollar)	Handling fees and commissions (NT dollar)
1	20768	11921763199736	104363651000000	30603098600000	34194947130392	36855606768719	153015493000000	53461000000	163089000000
2	21041	12299222558614	218992707000000	34733860400000	38810531428096	41830323039968	173669302000000	53950000000	203943000000
3	21236	12594956972553	268675466000000	47593341000000	53179313654654	57317119538529	237966705000000	54370000000	300078000000
4	21039	12873410000000	337703219000000	55528310400000	62045600780046	66873279708800	277641552000000	55870000000	410759000000
5	2151	3196039063257	61855891000000	422710000000	472322959528	509073729456	53611917000000	1101242000000	33665000000
6	2798	3242505887466	84561442000000	4263682000000	4764105180212	5134793349941	64164451000000	1391040000000	55779000000
7	2693	4600850000000	96416932000000	3405829000000	3805567015039	4101672709230	85727097000000	1453894000000	76200000000
8	2970	2532448000000	115640348000000	4818284000000	5383800143663	5802705886913	100896057000000	2204333000000	122010000000
9	3429	3420546323258	140218488000000	2968126000000	3316491345303	3574542765288	101424169000000	2082204000000	84374000000
10	3878	3363301000000	162009443000000	4607745000000	5148550436828	5549151738855	122393578000000	2469913000000	112775000000
11	4482	4293146786834	190145025000000	7967089000000	8902176563851	9594842114301	157247408000000	6063614000000	111507000000
12	30929	18325218000000	117205865000000	12882800000000	14394838596227	15514880276864	89387130000000	12241147000000	62771000000
13	30790	18502789000000	134955138000000	8703772000000	9725323153224	10482036555494)	106260752000000	7860614000000	71303000000
14	29218	19306690000000	144912094000000	14686570000000	16410314891342	17687177882741	127838715000000	14644181000000	53538000000
15	29739	20306982000000	143072637000000	12580573000000	14057139580142	15150905386201)	136870592000000	24938085000000	151844000000
16	1897	367292000000	22106562000000	729399000000	815007675137	878422249749	19698702000000	1067472000000	1096000000
17	1998	638314000000	24394401000000	1127081000000	1259365128689	1357354517444	19316086000000	1135282000000	6841000000
18	2067	749084000000	32139623000000	1559111000000	1742101965303	1877652412778	24497596000000	3028029000000	25640000000
19	2179	969141000000	41847528000000	2804486000000	3133645117162	3377469535205)	34936604000000	4399375000000	43113000000
20	1222	380421676883	21313010000000	1408145000000	1573417269156	1695842603119)	17345093000000	388922000000	12399000000
21	1306	591257515571	28582650000000	2469958000000	2759853971921	2974594238743	22046998000000	398934000000	13264000000
22	1350	613873745138	29263088000000	2347246000000	2622739413454	2826810993755)	22467788000000	562379000000	100525000000
23	1388	853098000000	31453459000000	1229213000000	1373484237540	1480353069967	27106152000000	754426000000	61415000000
24	25089	13416385000000	266087178000000	15175451000000	16956575260809	18275941985625	208015879000000	1569310000000	155844000000
25	25065	13983159000000	277902468000000	15205786000000	16990470642932	18312474718665	225954131000000	1821251000000	268546000000
26	24737	13914075000000	384359074000000	15398815000000	17206155222324	18544941404863)	262868778000000	963965000000	604118000000
27	23997	16067563000000	450326332000000	(11295251000000	12620960897388	13602979706440)	334072013000000	951427000000	890377000000

Table 3: Results of the CCR and BCC models

DMU	CCR	super CCR	rank	BCC	super BCC	RANK
DMU01	1	1.538	4	1	1.563	6
DMU02	0.883	0.883	21	0.887	0.887	24
DMU03	0.995	0.995	14	1	1.06	14
DMU04	0.934	0.934	16	1	1.042	16
DMU05	1	1.442	5	1	2.29	4
DMU06	0.857	0.857	24	0.858	0.858	25
DMU07	1	1.019	13	1	1.019	17
DMU08	1	1.123	9	1	1.171	10
DMU09	1	1.025	12	1	1.088	13
DMU10	1	1.034	11	1	1.099	12
DMU11	1	1.218	7	1	2.535	3
DMU12	0.791	0.791	27	0.791	0.791	27
DMU13	0.832	0.832	25	0.846	0.846	26
DMU14	0.927	0.927	17	0.928	0.928	22
DMU15	1	1.658	3	1	1.673	5
DMU16	1	1.315	6	1	1.491	7
DMU17	0.889	0.889	20	0.939	0.939	21
DMU18	0.911	0.911	18	1	1.048	15
DMU19	1	7.064	1	1	7.505	1
DMU20	0.982	0.982	15	1	1.29	8
DMU21	0.878	0.878	22	0.999	0.999	19
DMU22	1	2.275	2	1	2.559	2
DMU23	1	1.05	10	1	1.164	11
DMU24	0.868	0.868	23	0.951	0.951	20
DMU25	0.91	0.91	19	1	1.013	18
DMU26	0.795	0.795	26	0.891	0.891	23
DMU27	1	1.212	8	1	1.246	9

All inputs and outputs are crisp numbers. By attention to the fourth input of branches is a fuzzy number, we use from the middle it for solving SBM model. Table (4) shows the results corresponding to the model SBM. The third column of Table (4) shows the scores of rank units. Given that the number of efficient units is determined by SBM model is more than one unit to distinguish between efficient units, we used of super-efficiency models. The fourth column of Table (4) shows the scores of SBM super efficiency. The sixth column of Table (4) shows the scores of rank units obtained from the SBM super efficiency model. As can be seen units 19, 20, 22, 23, 27 is the highest rating, and other organizations are next in place.

In this study, we used fuzzy SBM model to determine VaR indicators related to Bank branches. We consider it efficiency interval that is obtained of models (5), (6). The results are different from the results of traditional DEA models. As previously mentioned, to solve models (5) and (6) can use different values of α. In this study, we used values 0, 0.3, 0.5, 0.7, 1.

When $\alpha = 0$, we will have the greatest risk and confidence interval is 0.99. The difference between the upper and lower efficiency of the model (6) is highest, in contrast, when $\alpha = 1$, there is no risk and market conditions is quite stable. So in this case the upper and lower bounds efficiency is equal.

Table 4: The empirical results

DMU	SBM (non-risk)	Rank	Super-SBM (non-risk)	Rank	Fuzzy-SBM							
						α=0	α=0.3	α=0.5	α=0.7	α=1	Fuzzy-SBM	Rank
1	1	1	1.5	27	L	1	1	1	1	1	1	1
					U	1	1	1	1	1		
2	0.010	27	1.5	27	L	0.009	0.009	0.009	0.009	0.01	1	1
					U	0.01	0.01	0.01	0.01	0.01		
3	1	1	1.5	27	L	0.024	0.025	0.025	0.025	0.025	1	1
					U	0.025	0.025	0.025	0.025	0.025		
4	1	1	1.5	27	L	0.023	0.023	0.023	0.023	0.023	1	1
					U	0.023	0.023	0.023	0.023	0.023		
5	1	1	1.5	27	L	1	1	1	1	1	1	1
					U	1	1	1	1	1		
6	0.398	25	1.5	27	L	0.438	0.427	0.419	0.411	0.398	0.981	20
					U	0.517	0.458	0.44	0.423	0.398		
7	1	1	1.5	27	L	1	1	1	1	1	1	1
					U	1	1	1	1	1		
8	1	1	1.5	27	L	1	1	1	1	1	1	1
					U	1	1	1	1	1		
9	1	1	1.5	27	L	0.897	0.924	0.947	1	1	1	1
					U	1	1	1	1	1		
10	1	1	1.5	27	L	0.857	0.896	1	1	1	1	1
					U	1	1	1	1	1		
11	1	1	3	2	L	1	1	1	1	1	1	1
					U	1	1	1	1	1		
12	0.402	20	3	2	L	0.39	0.394	0.397	0.4	0.402	0.883	23
					U	0.423	0.417	0.413	0.41	0.402		
13	0.344	22	3	2	L	0.351	0.35	0.349	0.347	0.344	0.814	27
					U	0.391	0.377	0.367	0.358	0.344		
14	0.393	21	3	2	L	0.379	0.383	0.386	0.388	0.393	0.915	21
					U	0.41	0.404	0.401	0.398	0.393		
15	0.027	26	3	2	L	1	1	1	1	1	1	1
					U	1	1	1	1	1		
16	1	1	1.911	9	L	1	1	1	1	1	1	1
					U	1	1	1	1	1		
17	0.497	19	1.5	27	L	0.286	0.274	0.268	0.259	0.244	0.856	24
					U	0.389	0.332	0.306	0.281	0.244		
18	1	1	1.518	12	L	1	0.741	0.708	0.674	0.621	1	1
					U	1	1	0.75	0.697	0.621		
19	1	1	4.973	1	L	1	1	1	1	1	1	1
					U	1	1	1	1	1		
20	1	1	1.731	10	L	0.604	0.606	0.607	0.607	0.607	1	1
					U	0.644	0.632	0.625	0.618	0.607		
21	0.519	18	1.5	27	L	0.295	0.285	0.279	0.272	0.261	1	1
					U	0.316	0.298	0.287	0.276	0.261		
22	1	1	2.092	7	L	1	1	1	0.001	0.001	1	1
					U	1	1	1	1	0.001		
23	1	1	1.625	11	L	1	1	1	1	1	1	1
					U	1	1	1	1	1		
24	0.252	24	1.5	27	L	0.244	0.246	0.248	0.25	0.252	0.831	26
					U	0.263	0.259	0.257	0.255	0.252		
25	1	1		27	L	0.316	0.318	0.32	0.322	0.325	0.84	25
					U	0.339	0.333	0.331	0.328	0.325		
26	0.329	23	1.5	27	L	0.318	0.321	0.323	0.326	0.329	0.887	22
					U	0.343	0.339	0.336	0.333	0.329		
27	1	1	1.954	8	L	1	1	1	1	1	1	1
					U	1	1	1	1	1		

We used different values of α in order to sensitivity analyze of the results and evaluate the potential impact of market risk on the efficiency analysis.

Figure 1: Comparison of the efficiency scores of units with different models.

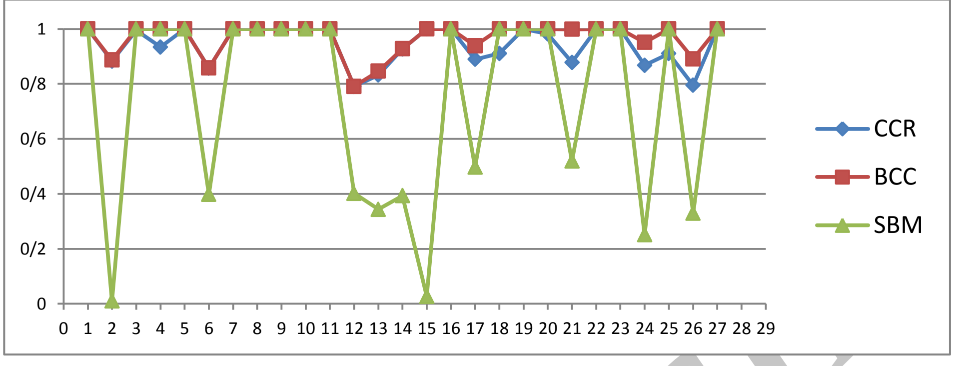
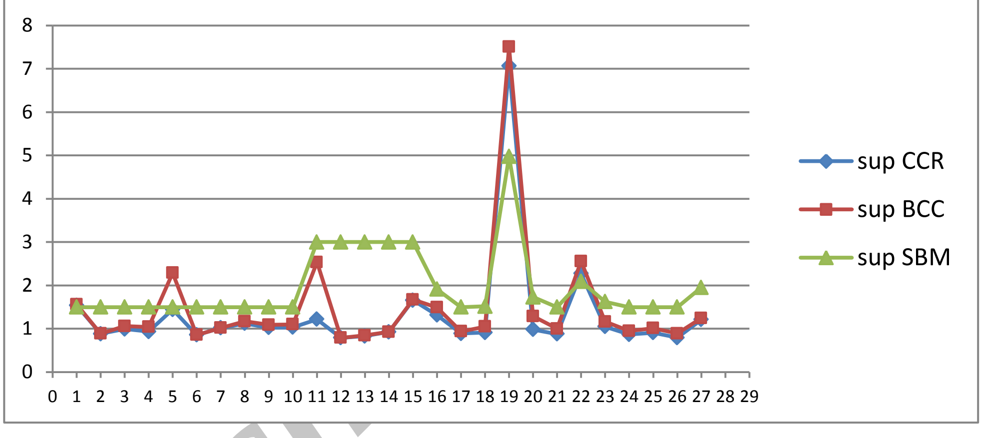


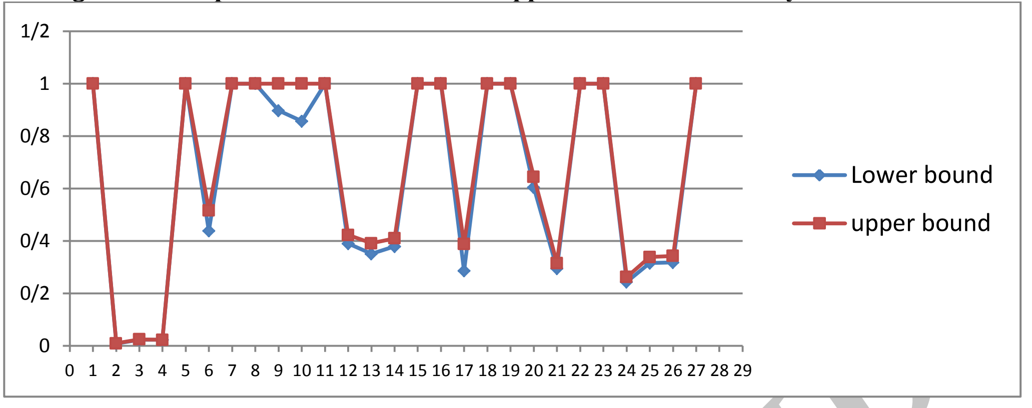
Figure 2: Comparison of the supper efficiency scores of units with different models



First, we consider α = 0. According to the column 7 Table (4) for α = 0 the lower bound of efficiency for units 2, 3, 4, 6, 9, 12, 13, 14, 17, 20, 21, 24, 25 and 26 is less than one, this means that these units are inefficient and risk indicators suggest that these units are inefficient in their current performance. Given that only the upper bound of units 9 and 10 of these units is equal to one. So the two units by changing market conditions can be efficient and market risk is effective on the two units. The upper bound of other above branches is less than one, this means that there is no risk about these and these units cannot efficient in present circumstances.

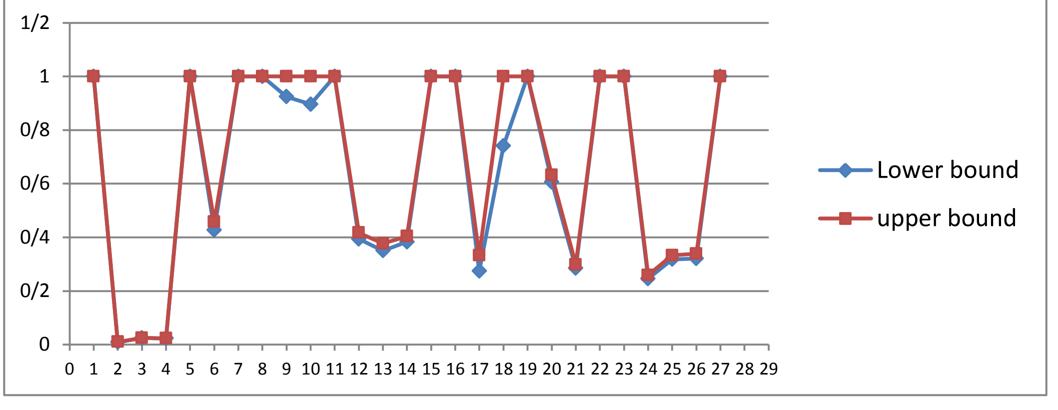
According to the column 6 Table (4) for α = 0 units 1, 2, 7, 8, 11, 15, 16, 18, 19, 22, 23 and 27 have the lower and upper bounds of efficiency equal to one. So these units are fully efficient and there is no risk in this case. We compared the lower and upper bounds of efficiency for α = 0 in Figure (3).

Figure 3: Comparison of the lower and upper bounds of efficiency scores for α = 0



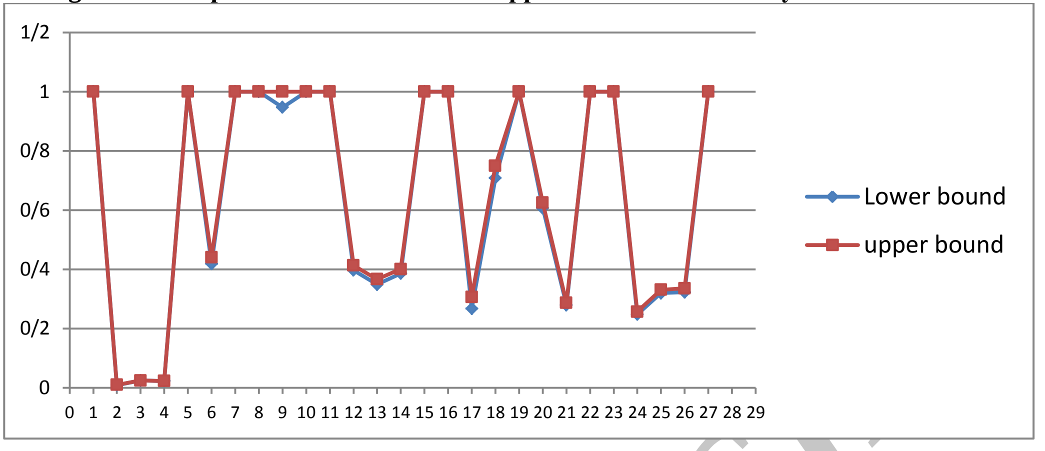
Now, we proposed results for α = 0.3. According to the column 7 Table (4), the lower bound efficiency units 2, 3, 4, 6, 9, 10, 12, 13, 14, 17, 18, 20, 21, 24, 25 and 26 is less than one. This means that these units are inefficient and in between these units' only units 9, 10 and 18 have upper bounds efficiency equal to one. So, these units will include market risk index and by changing market conditions can be efficient and market risk is effective on the units. The lower and upper bound efficiency of unit 18 is equal to one for α = 0 and is fully efficient, but the lower bound efficiency of this unit is less than one and the upper bound efficiency of this unit is equal to one for α = 0.3, in this state, this unit is quite inefficient. We compared the lower and upper bounds of efficiency scores for α = 0.3 in Figure (4).

Figure 4: Comparison of the lower and upper bounds of efficiency scores for α = 0.3



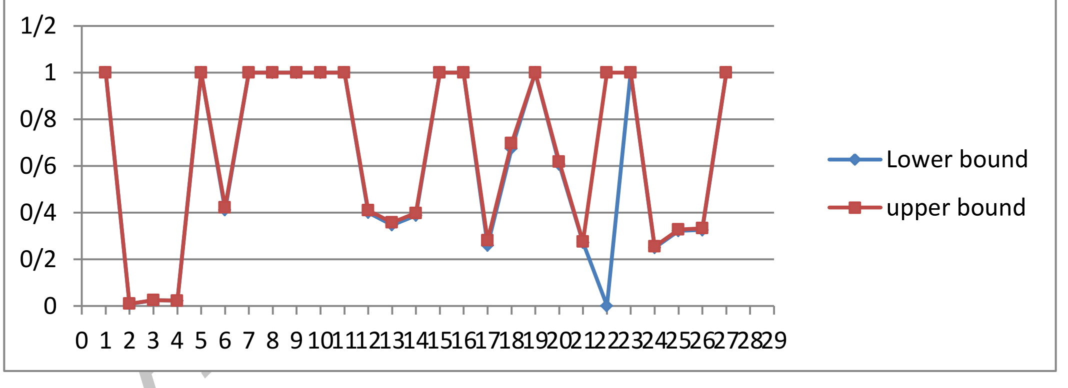
Now, we proposed the results of the models (5), (6) for α = 0.5. According to the column 8 Table (4) units 1, 5, 7, 8, 10, 11, 16, 19, 22, 23, 27 have the lower bounds efficiency less than one. This means that these units are inefficient. In between these units only unit 9 has the upper bound efficiency equal to one. The upper bound efficiency of other units is less than one. Therefore, unit 9 is in risk condition and can be efficient. The units 1, 5, 7, 8, 10, 11, 16, 19, 22, 23, 27 have lower and upper bound efficiency equal to one, then they are full efficient and the performance are not affected by market risk. We compared the lower and upper bounds of efficiency scores for α = 0.5 in Figure (5).

Figure 5: Comparison of the lower and upper bounds of efficiency scores for α = 0.5



Now, we proposed results for α =0.7. According to the column 9 Table (4), the lower bounds efficiency units 2, 3, 4, 6, 12, 13, 14, 17, 18, 20, 21, 22, 23, 25 and 26 is less than one. This means that these units are inefficient and in between these units' only unit 22 has upper bound efficiency equal to one. So, this unit will include market risk index and by changing market conditions can be efficient and market risk is effective on the units.

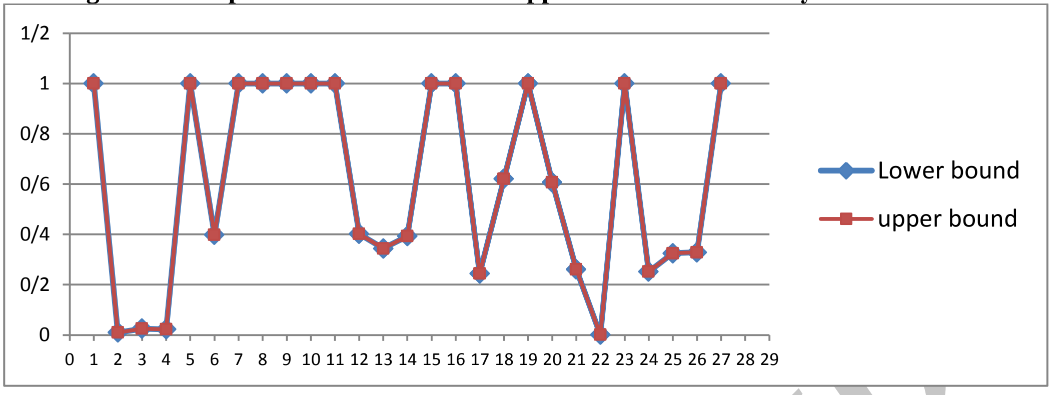
Figure 6: Comparison of the lower and upper bounds of efficiency scores for α = 0.7



The upper bound of other above branches is less than one, this means that there is no risk about these and these units cannot efficient in present circumstances. The units 1, 5, 7, 8, 9, 10, 11, 15, 16, 19, 23 and 27 have lower and upper bound efficiency equal to one, then they are full efficient and the performance are not affected by market risk. We compared the lower and upper bounds of efficiency scores for α = 0.7 in Figure (6).

Now, we proposed results for α =1. In this case there is no risk and lower and upper bound efficiency are equal. According to the tenth column Table (4), units 2, 3, 4, 6, 12, 13, 14, 17, 18, 20, 21, 22, 24, 25 and 27 are inefficient and their lower and upper bounds efficiency less than one. The other units are efficient and their lower and upper bounds efficiency equal to one. We compared the lower and upper bounds of efficiency scores for α = 1 in Figure (7).

Figure 7: Comparison of the lower and upper bounds of efficiency scores for α = 1



The eleventh column Table (4) shows the fuzzy efficiency scores for the units in the absence of risk and input orientation. The final column shows the rank of units according to these scores.

As, we can rank units according to different scores of α and lower and upper bounds efficiency scores in sixth to tenth columns Table (4) using Chen and Klein Index in [14].

5. Conclusions

Due to the great influence of commercial banks in the country in recent year's performance evaluation in a competitive market and pay attention to issues of market risk due to the financial performance of banks is important. In this study, we evaluated efficiency of the 27 branches of commercial banks in the Tehran Stock Exchange by using of fuzzy data envelopment analysis. We consider the fourth input of branches as an indicator of risk that is a fuzzy number.

Fuzzy SBM models and α-cut concept used to determine the lower and upper bounds efficiency of branches in order to determine the risk index. As was observed, the lower and upper bounds efficiency of branches not equal. For sensitivity analysis results, we solved models for different values of α and achieved market risk index based on the efficiency interval. We consider the amount of overdue loans as an input variable and investigated its impact on the performance of branches bank. We used of super efficiency models for ranking of efficient branches.

The method presented in the paper can be used to measure the performance the banks due to the competitive market in future years. We can develop models for other trapezoid fuzzy numbers such as LR and the other ranking methods of fuzzy numbers. Models were also used in state of constant return to scale; we can develop them for variable returns to scale technology. As well as other non-radial models like Russell's model.

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