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RBFs meshless method of lines based on adaptive nodes for Burgers' equations

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Abstract

We introduce a RBFs mesheless method of lines that decomposes the interior and boundary centers to obtain the numerical solution of the time dependent PDEs. Then, the method is applied with an adaptive algorithm to obtain the numerical solution of one dimensional problems. We show that in the problems in which the solutions contain region with rapid variation, the adaptive RBFs methods are successful so that the PDE solution can be approximated well with a small number of basis functions. The method is described in detail, and computational experiments are performed for one-dimensional Burgers' equations.

Keywords: Method of Lines; Radial basis functions; Adaptive Method; Burgers' Equations.

1 Introduction

The radial basis functions (RBFs) methods are one of the most attractive meshless methods. These methods are easy to implement, very suitable for problems in irregular geometries and the formulation for different dimensional problems are similar. Also, this method can be spectrally accurate [11]. A set of points called centers are needed to define the RBFs. Therefore, a RBF can be defined anywhere in a given domain, independently to the other RBFs.

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Both the approximation quality and the stability of the RBFs interpolation depend on the positions of the centers set [9].

The condition number of RBFs collocation methods becomes large when the number of centers increases, while reducing the number of centers improves the conditioning [9, 13]. In order to obtain numerical solution with the minimal numbers of centers, we can use a set of adaptive nodes rather than uniform ones. Especially in problems whose solutions contain regions of rapid variation, adaptive methods are preferred over fixed grid methods, [17]. The goal of an adaptive method is to obtain a numerical solution such that the error is less than a prescribed accuracy but with the minimal number of grid points. By using adaptive methods, the computational grid should reflect the profile of the solution. Clearly, grids with finer spacing should be concentrated in regions, where high variations occur, and much coarser grids can be used in other regions.

Some methods have been constructed to select centers of RBFs. In [6, 26], the power function is used to iteratively obtain an optimal set of nodes. In [25], an adaptive algorithm so-called residual sub-sampling is introduced such that nodes can be added or removed based on residuals evaluated at a finer set of nodes. Our goal is to move a fixed numbers of nodes in such a way that nodes move with time and concentrate in region of domain that the solution has rapid variations. To this goal in this paper, we use a simple adaptive nodes generation method that is used for finite difference computations [24] and RBFs method [23]. Also we introduce a RBFs meshless method of lines to solve time dependent PDE with adaptive centers. In this method, we divide centers to interior and boundary data centers and obtain the expansion coefficients of boundary centers as a function of interior ones. This gives an ODEs system that is only related to the expansion coefficients of the interior data centers instead of all data centers. Actually after approximation spatial derivatives of equation and boundary condition with RBFs, we have a system of differential algebraic equations (DAEs) [5]. By decomposing centers and replacing boundary coefficients as a function of interior ones we obtain a smaller system of ODEs. The resultant system of ODEs can be solved with a proper ODE solver. We use the function *ode15s* in Matlab for solving the resulting system of ordinary differential equations.

In this paper, in order to combine the adaptive method and the RBFs method of lines, we start with a set of uniform centers, then the adaptive method is used to obtain new centers for initial condition. After obtaining the adaptive centers, the PDE is advanced for a small time step. The *ode15s* in Matlab is used for solving the resultant ODEs system. Then, the numerical solution of the PDE is used to obtain adaptive centers for next time. The procedure is repeated until the final time. We perform computational experiment for unsteady Burgers' equations and demonstrate the benefits of adaptation in the numerical experiments.

The rest of the paper is organized as follows. In Section 2 at first the RBFs method of lines is introduced, then adaptive method is extended for

time dependent PDEs. Numerical experiment are given in Section 3. Finally, the conclusion is given in Section 4.

2 Meshless method of lines with adaptive RBFs

In this section, we introduce a RBFs mesheless method of lines that decomposes the interior and boundary centers to obtain the numerical solution of the time dependent PDEs. Then, apply the method with an adaptive algorithm to obtain the numerical solution of one dimensional Burgers' equations.

2.1 RBFs meshless method of lines

There are two classes of RBFs, known as globally supported and locally supported [22, 16]. Globally supported RBFs are infinitely smoothed and contain a free parameter ϵ, called shape parameter. This parameter affects both accuracy of the solutions and conditioning of the collocation matrix. As ϵ decreases, numerical solution of PDEs gets more accurate and the condition number of the resulting matrix gets larger. If the shape parameter becomes too small, the ill-conditioned matrix leads to numerical instabilities and loss of precision. Thus it is important to select a good values for ϵ. There are some paper related to select an optimal value for RBFs shape parameter [21, 1, 14].

Generally a radial basis function is a function $\phi_i(x,\epsilon) = \phi\left(\epsilon\|x - x_i\|_2\right)$, which depends solely on the distance between $x \in \mathbb{R}$ and a fixed center $x_i \in \Omega$. $\phi_i : \mathbb{R}^+ \to \mathbb{R}$ is a continuous function and $\|\cdot\|_2$ represents the Euclidean norm. The multiquadrics (MQ) RBF proposed by Hardy [3], is one of the most used globally supported RBFs because of its spectral convergence property. In [4], Franke showed that the MQ RBF is one of the best methods among 29 scattered data interpolation schemes. We here use MQ RBF defined as $\phi(r,\epsilon) = \sqrt{1 + (\epsilon r)^2}$.

Let a set of N distinct centers $\{x_i\}_{i=1}^N$ is given in $\Omega \bigcup \partial\Omega$, where Ω is a bounded domain in $\mathbb{R}$. We assume that the arrangement of the centers is in such a way that the first N_I centers and the last N_B centers lie in Ω and $\partial\Omega$, respectively, $N = N_I + N_B$. Consider the following time dependent PDE of the general form

$$\frac{\partial u(x,t)}{\partial t} - \mathcal{L}u(x,t) = f(x,t), \quad x \in \Omega, \qquad \mathcal{B}u(x,t) = g(x,t), \qquad x \in \partial\Omega, \tag{1}$$

with the initial condition

$$u(x,0) = u_0(x). \tag{2}$$

$\mathcal{L}$ and $\mathcal{B}$ are differential and boundary operators respectively. We approximate the solution of equation (1) by

$$u^N(x) = \sum_{i=1}^{N_I} c_i(t)\phi\left(\|x - x_i\|\right) + \sum_{i=N_{I+1}}^{N} c_i(t)\phi\left(\|x - x_i\|\right). \tag{3}$$

Using collocation method to ensure that the approximation $u^N(x)$ satisfies in equations (1), one obtains the following system of equations for the expansion coefficients

$$A_{1,1}\dot{C}_1 + A_{1,2}\dot{C}_2 = F + \mathcal{L}_\phi(C_1, C_2), \tag{4}$$
$$0\,\dot{C}_1 + 0\,\dot{C}_2 = G(t) - \left(A_{2,1}C_1 + A_{2,2}C_2\right), \tag{5}$$

where

$$A_{1,1}(i,j) = \phi(\|x_i - x_j\|), \quad i = 1,\ldots,N_I, j = 1,\ldots,N_I,$$
$$A_{1,2}(i,j) = \phi(\|x_i - x_j\|), \quad i = 1,\ldots,N_I, j = N_{I+1},\ldots,N,$$
$$A_{2,1}(i,j) = \mathcal{B}\phi(\|x_i - x_j\|), \quad i = N_{I+1},\ldots,N, j = 1,\ldots,N_I,$$
$$A_{2,2}(i,j) = \mathcal{B}\phi(\|x_i - x_j\|), \quad i = N_{I+1},\ldots,N, j = N_{I+1},\ldots,N,$$
$$\mathcal{L}_\phi(C_1,C_2)^T = [\mathcal{L}_{1\phi}(C_1,C_2),\ldots,\mathcal{L}_{N_I\phi}(C_1,C_2)],$$
$$\mathcal{L}_{i\phi}(C_1,C_2) = \sum_{j=1}^{N_I} c_j(t)\mathcal{L}\phi\left(\|x_i - x_j\|\right) + \sum_{j=N_{I+1}}^{N} c_j(t)\mathcal{L}\phi\left(\|x_i - x_j\|\right),$$
$$F^T = [f(x_1,t),\ldots,f(x_{N_I},t)],$$

and

$$G(t)^T = [g(x_{N_{I+1}},t),\ldots,g(x_N,t)].$$

Equations (4) and (5) are distinct from ODEs because the coefficient matrix of the $\dot{C}^T = [\dot{C}_1, \dot{C}_2]$ is singular and are referred to as differential-algebraic equations (DAEs). DAEs differ in many ways from ordinary differential equations and there are some problems to be expected in solving these systems. More information about differential-algebraic equations can be found in [10, 12]. In order to reach a system of ODEs, we obtain C_2 and $\dot{C}_2$ from equation (5) as follows:

$$C_2 = A_{2,2}^{-1}\left(G(t) - A_{2,1}C_1\right), \tag{6}$$
$$\dot{C}_2 = A_{2,2}^{-1}\left(\dot{G}(t) - A_{2,1}\dot{C}_1\right). \tag{7}$$

Note that unlike the interpolation problem the invertibility of $A_{2,2}$ may failed for some special centers arrangements. However, numerical experiments show that the cases of singularity for Kansa method is rare [19]. We substitute C_2

and $\dot{C}_2$ into equation (4) to obtain a $N_I \times N_I$ nonlinear system of ordinary differential equation for C_I as follows:

$$\left(A_{1,1} - A_{1,2}A_{2,2}^{-1}A_{2,1}\right)\dot{C}_1 = F + \mathcal{L}_\phi(C_1) - A_{1,2}A_{2,2}^{-1}\dot{G}(t), \tag{8}$$

where

$$\mathcal{L}_\phi(C_1)^T = [\mathcal{L}_{1\phi}(C_1), \ldots, \mathcal{L}_{N_I\phi}(C_1)],$$
$$\mathcal{L}_{i\phi}(C_1) = \sum_{j=1}^{N_I} c_j(t)\mathcal{L}\phi\left(\|x_i - x_j\|\right) + \sum_{j=N_{I+1}}^{N} d_j(t)\mathcal{L}\phi\left(\|x_i - x_j\|\right),$$

and $d_j(t)$ is jth component of the vector $C_2 = A_{2,2}^{-1}\left(G(t) - A_{2,1}C_1\right)$.

After solving the reduced system using a proper ODE solver, its solution vector C_1 is applied to obtain C_2 and C, using the relations $C_2 = A_{2,2}^{-1}\left(G(t) - A_{2,1}C_1\right)$ and $C = [C_1, C_2]$. This method can be used for high dimensional problems. In case of one dimensional problem, we have only two boundary nodes x_1 and x_N.

2.2 Adaptive method

In this section, the proposed mesheless method of lines that decomposes the interior and boundary centers to reach a smaller system of equations is combined with an adaptive algorithm that is used for finite difference and RBFs computations [24, 23]. In this method, at first the arclength of the numerical solution is computed. Then, the total length is divided into $(N-1)$ equal part and the projection of each part onto x-axis determines the position of adaptive centers. The selected nodes on x-axis are such that the variation of the solution is equi-distributed on each section.

Suppose that the approximate solution and the centers are given at the time step t_n. The adaptive method is generalized for RBFs and introduced in the following algorithm:

1) $S_1 = 0, S_j = S_{j-1} + \sqrt{(h_j^n)^2 + (u_j^n - u_{j-1}^n)^2}, j = 2...N,$
$\left(u_j^n = u(x_j^n, t^n), h_j^n = x_j^n - x_{j-1}^n\right).$
This step compute the arclength of solution u at time step t^n.

2) $\delta = \frac{S_N}{N-1}, k = 2, \bar{x}_1^n = x_1^n, \bar{x}_N^n = x_N^n.$
In this step the total length is divided into $(N-1)$ equal part.

3) For $j = 2, \cdots, N-1, \Delta = (j-1)\delta.$

- while $\Delta > S_k$ put $k = k+1$,
- $\bar{x}_j^n = x_{k-1}^n + \frac{(\Delta - S_{k-1})h_k^n}{S_k - S_{k-1}}$, Next j.

These steps project each part on solution onto x-axis.

The set $\bar{x}_j, j = 2, \cdots, (N-1)$ are adaptive interior nodes and $\bar{x}_1, \bar{x}_N$ are the boundary nodes which are fixed. In using adaptive centers in region with rapid variations, nodes are close to each other and hence a larger value of shape parameter is needed. In order to obtain results with a smaller shape parameter, the final set of centers are selected as $.9\bar{x}_j + .1x_j$.

In solving PDE problems, at first we apply the above adaptive algorithm for the initial condition to obtain the adaptive centers at $t = 0$. Then, adaptive centers are used for the RBFs method of lines to advance the PDE for a small time step. Next, the approximate solution at this time is used to obtain the adaptive centers again. Note that in each step we need to interpolate u at the adaptive centers to obtain initial condition for next time. The procedure is repeated until approximate solution is obtained at the final time.

3 Numerical experiments

In this section, the proposed method is applied to obtain numerical solution of Burgers' equation as follows:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = \frac{1}{Re}\frac{\partial^2 u}{\partial x^2}, \quad x \in (0,1), \tag{9}$$

where Re is the Reynolds number. Equation (9) has shock wave behavior when the coefficient of kinematic viscosity $\nu = 1/Re$ is small. Also, it is a useful model for many interesting physical problems such as modeling of fluid dynamics, turbulence, boundary layer behavior, shock wave formation, traffic flow and is an interesting test problem for establishing the efficiency of different methods [8, 20].

Example 1. We consider Equation (9) with the following exact solution [15]

$$u(x,t) = \frac{\alpha + \mu + (\mu - \alpha)exp(\eta)}{1 + exp(\eta)}, \tag{10}$$

where $\eta = \alpha.Re.(x - \mu t - \beta), \alpha, \mu$ and β are arbitrary constant.

In this example α, μ and β are .4, .6 and .125 respectively. The boundary conditions are

$$u(0,t) = 1, \quad u(1,t) = .2 \quad t > 0. \tag{11}$$

Initial condition is taken from the exact solution. In order to measure the error, root mean square error (rms) is computed at M evaluation nodes z_i as:

$$rms \;\; error = \sqrt{\frac{\sum_{i=1}^{M} \left(u^N(z_i) - u(z_i)\right)^2}{M}}.$$

Table 1 shows the rms error at $t = .2, .4, .6, .8$ and $t = 1$ for $Re = 100$ and $Re = 500$. The results are computed for $N = 50$ adaptive centers. When Re increases, the gradient of solution become sharper and consequently a larger values of shape parameter is needed. In this example, the values of shape parameters for $Re = 100$ and $Re = 500$ are 50 and 150, respectively.

The numerical solution in Example 1 at $t = .1, t = .5$ and $t = 1$ for $Re = 100$ and $Re = 500$ are shown in Figures 1.a and 2.a respectively. Figures 1.b and 2.b show the corresponding nodes trajectories. Figures show that the nodes move with time and are concentrated in region with rapid variations. When Re increases, the gradient become sharper and the nodes are more concentrated in region with rapid variations.

The numerical and exact solutions of Example 1 at $t = 1$ are plotted in Figure 3. In order to obtain numerical solution with a set of uniform centers a larger number of nodes is needed [23]. Figure 4 shows the numerical solutions and absolute errors for $N = 50$ uniform and adaptive centers. In the case of using uniform centers, the numerical solution with some oscillations is obtained for $Re = 500$ and $\epsilon = 50$ at $t = .1$. As Figure 5 shows in order to obtain an acceptebale solution at this time, we need to use more uniform nodes or a set of adaptive centers.

Table 1: rms error values corresponding to Example 1

Re	rms error (t=.2)	rms error (t=.4)	rms error (t=.6)	rms error (t=.8)	rms error (t=1)
100	2.121018e-003	3.149610e-003	4.023452e-003	4.826538e-003	5.599954e-003
500	1.485532e-003	1.840686e-003	2.517397e-003	4.152420e-003	6.497818e-003

Example 2. We consider Burgers' equation (9) with the initial condition

$$u(x, 0) = sin(\pi x),$$

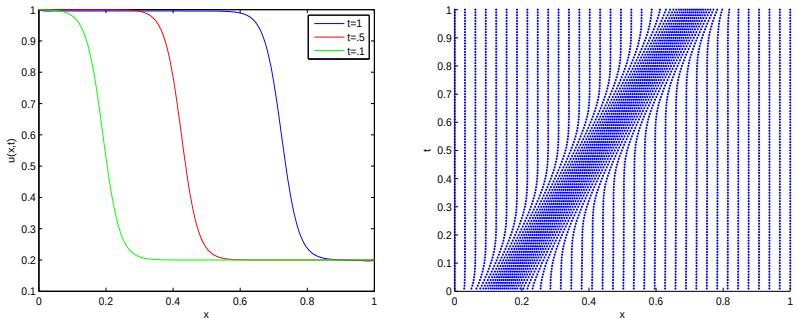
and the boundary conditions

$$u(0, t) = u(1, t) = 0, \quad t > 0.$$

The exact solution for this example is given by [15]

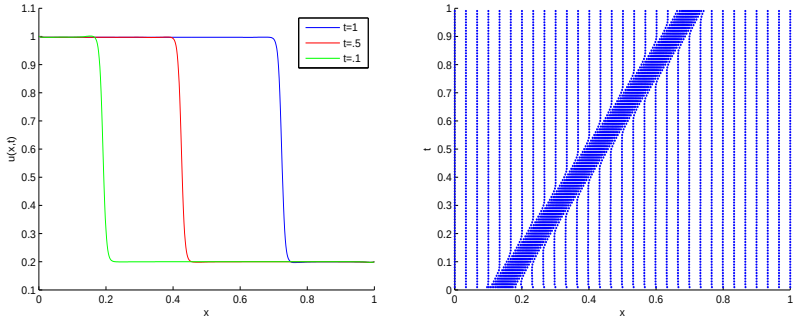
$$u(x, t) = \frac{2\pi\nu \sum_{i=1}^{\infty} iA_i \sin(i\pi x) exp(-i^2\pi^2\nu t)}{A_0 + \sum_{i=1}^{\infty} A_i \cos(i\pi x) exp(-i^2\pi^2\nu t)}, \tag{12}$$

with the Fourier coefficients



(a) (b)

Figure 1: The numerical solution and corresponding nodes trajectories for $N = 50$ and $Re = 100$



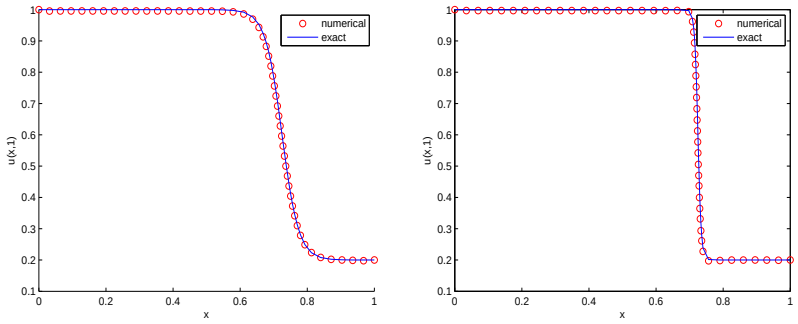
(a) (b)

Figure 2: The numerical solution and corresponding nodes trajectories for $N = 50$ and $Re = 500$

$$A_0 = \int_0^1 \exp\left\{-(2\pi\nu)^{-1}(1-\cos(\pi x)\right\} dx, \tag{13}$$

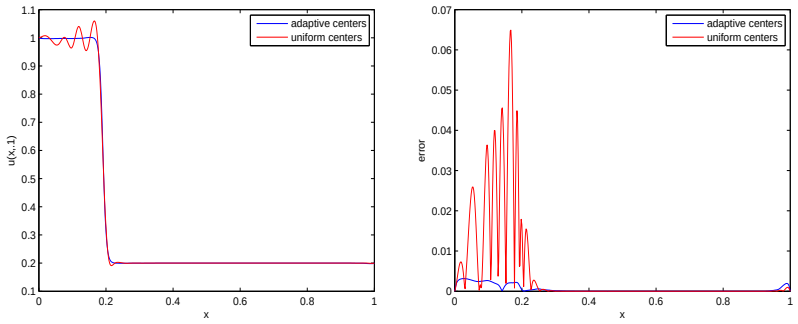
$$A_i = 2\int_0^1 \exp\left\{-(2\pi\nu)^{-1}(1-\cos(\pi x))\right\} \cos(i\pi x) dx, \quad i \geqslant 1. \tag{14}$$

In this example, $N = 50$ nodes are used. The computation are performed for a final time $t = 3$. The numerical solution at $t = .01, t = .1, t = 1, t = 2$ and $t = 3$ for $Re = 100$ and $Re = 500$ are shown in Figures 6.a and 7.a respectively. Initial condition in Example 2 does not have rapid variation, but the variation of the solution increases with time. The variation increases until a time t_0 less than $t = .75$. After this time, the variation of the solution decreases. Nodes trajectories also have such behavior. The nodes trajectories are shown in Figures 6.b and 7.b. Nodes are moved with time and concen-



(a) (b)

Figure 3: The numerical and exact solutions at $t = 1$ for (a) $Re = 100$, (b) $Re = 500$



(a) (b)

Figure 4: The numerical solutions (a) and absolute errors (b) in the case of using uniform and adaptive centers at $t = .1$ for $Re = 500$

trated in region with rapid variations. For $t < t_0$, the variation increases and nodes are concentrated in region with rapid variation. For $t \geqslant t_0$, the variation of the solution decreases with the time and hence the nodes trajectories diverge.

Figure 8 shows the numerical solution, exact solution and the absolute error at $t = 3$ when $Re = 100$, $\epsilon = 50$ and $N = 50$. We can see that, the error of the proposed method method is as small as 10^{-4}.

The numerical solution for $Re = 500$ are obtained for $\epsilon = 110$. In this case, obtaining numerical results with $N = 50$ and $Re = 500$ uniform centers is not possible as well.

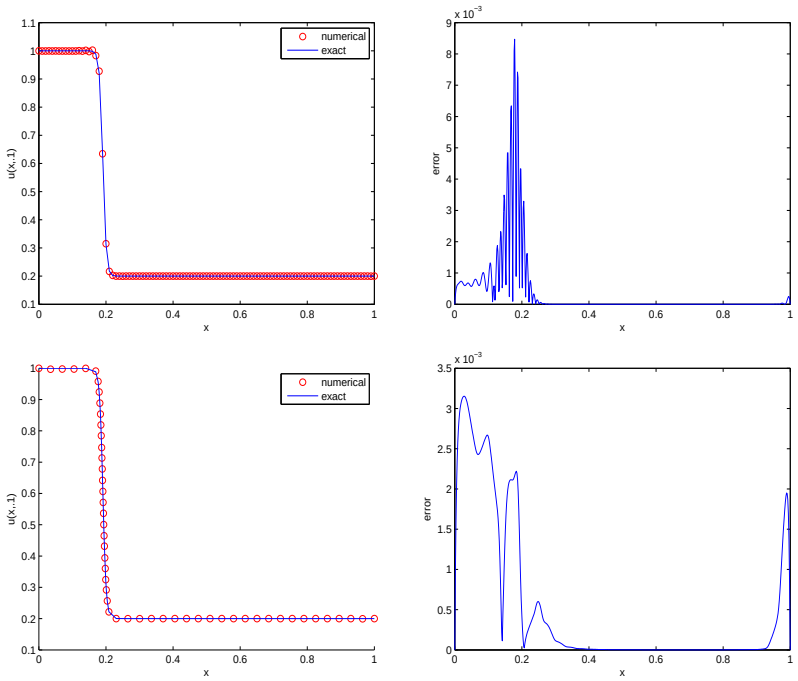
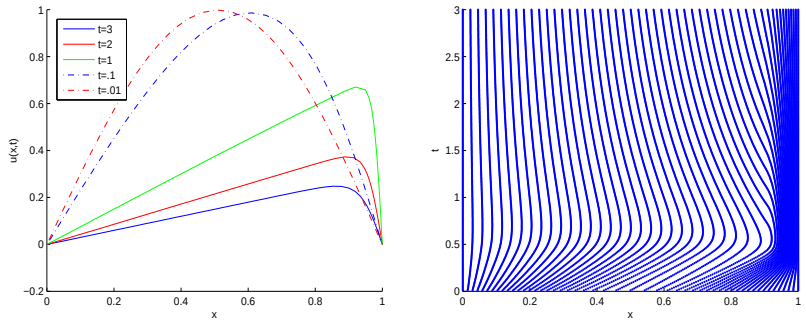


Figure 5: The numerical and exact solution and absolute error at $t = .1$ when $Re = 500$ for $N = 100$ uniform centers (top) and $N = 50$ adaptive centers (down)

4 Conclusion

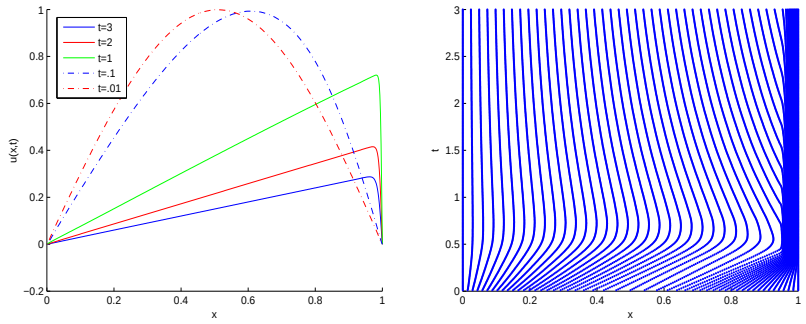
The adaptive MQ RBF method of lines has been proposed for obtaining the numerical solution of Burgers' equations. In the method of lines, centers in the domain were portioned into the interior and the boundary centers. By portioning centers and obtaining the expansion coefficients for boundary centers as a function of interior ones, the DAEs system was converted to a smaller ODEs system. The resulting ODE system was solved with *ode15s* in Matlab. Also, we have used a simple adaptive nodes generation method to enable the method for obtaining numerical solution of the problem with high gradient. In the adaptive method, the nodes moved with time and concentrated in region with rapid variation. When the gradient of solution increases the nodes become more closer in region with rapid variation. In this case numerical solution can be obtained with less number of centers in comparison with using uniform centers.

Numerical experiments have been performed for one-dimensional Burgers' equations. Numerical results show that the proposed adaptive method are preferred over fixed grid methods. For example, the adaptive method is able



(a) (b)

Figure 6: (a) The numerical solutions at different times for $Re = 100$ and (b) corresponding nodes trajectories in Example 2



(a) (b)

Figure 7: (a) The numerical solutions at different times for $Re = 500$ and (b) corresponding nodes trajectories in Example 2

to solve Burgers' equation for $Re = 500$ and $N = 50$ whereas the numerical solution could not be obtained for $N = 50$ uniform centers.

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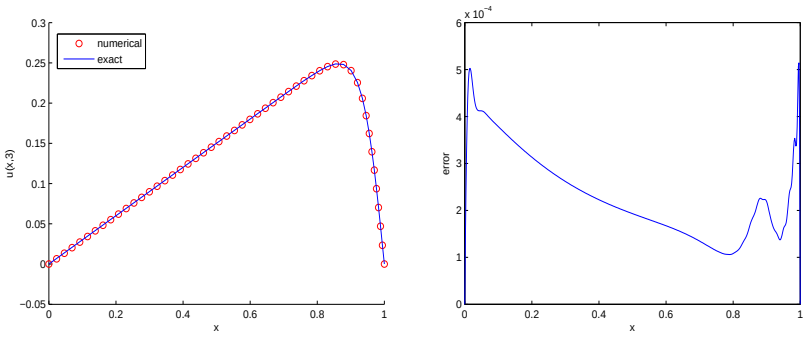

(a) (b)

Figure 8: (a) The numerical and exact solutions at $t = 3$ for $Re = 100$ and (b) corresponding absolute error in Example 2

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روش خطوط بدون شبکه توابع پایه شعاعي برمبناي نقاط تعدیل شده براي حل معادلات برگرز

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چکیده : یک روش خطوط بر مبناي توابع پایه شعاعي براي حل معادلات با مشتقات جزیي معرفي مي‌شود که براي بدست آوردن جواب، نقاط مرزي و دروني را جدا مي‌کند. این روش همراه با یک الگوریتم تعدیل نقاط بکار مي‌رود. نشان مي‌دهیم براي مسایلي که با تغییرات شدید در جواب مواجه هستند، نقاط تعدیل شده نسبت به نقاط یکنواخت عملکرد بهتري دارند به گونه‌اي که با تعداد نقاط کمتر مي‌توان جواب را با دقت مورد نظر بدست آورد. روش براي حل معادله برگرز بکار رفته است.

کلمات کلیدی : روش خطوط؛ توابع پایه شعاعي؛ روش‌هاي تعدیل؛ معادلات برگرز.