Iranian Journal of Numerical Analysis and Optimization
Vol. 7, No. 2, (2017), pp 87–114
DOI:10.22067/ijnao.v7i2.67470

The state of the art in Dynamic Relaxation methods for structural mechanics

Part 2: Applications

M. Rezaiee-pajand*, J. Alamatian, and H. Rezaee

Abstract

The most significant advances over the sixty years relative to the Dynamic Relaxation applications in structural engineering are summarized. Emphasized are given towards plates, form finding, cable structures, dynamic analysis, and other applications. The role of DR solver in the linear and non-linear analyses is discussed. This investigation is undertaken to explain the application of the solution technique to the static, dynamic and stability problems. The influence of the methods on the use of isotropic and composite materials, such as orthotropic and laminated ones, is briefly covered. Critical analyses and suggestions regarding future research and applications will be presented.

Keywords: Dynamic Relaxation method; Application; Survey; Solver; Review; Iterative technique; State of the art.

1 Introduction

The first part of the paper entitled "The state of art in Dynamic Relaxation methods" devoted entirely to the DR formulations and its fictitious param-

*Corresponding author
Received 25 November 2016; revised 12 May 2017; accepted 23 April 2017
M. Rezaiee-pajand
Department of Civil Engineering, Factualy of Engineering, Ferdowsi Unversity of Mashhad, Mashhad, Iran. e-mail: rezaiee@um.ac.ir

J. Alamatian
Civil Engineering Department, Mashhad Branch, Islamic Azad Unversity, Mashhad, Iran. e-mail: alamatian@mshadiau.ac.ir

H. Rezaee
Department of Civil Engineering, Factualy of Engineering, Ferdowsi Unversity of Mashhad, Mashhad, Iran.

eters. For this purpose, the main two categories of the DR methods was reviewed; i.e., the viscous damping and the kinetic damping formulations. In spite of several similarities in formulation and iterative relationships, the viscous and kinetic approaches have the different physical basis. Furthermore, the proposed formulations for fictitious parameters are different. However, the parameters such as fictitious time step and initial displacement have unique form in both methods.
It is worth emphasizing that these solvers have been exploited in a vast area of the structural analyses. In other words, most of the DR research topics conducted so far have been related to their applications. There is only a little fraction of the papers which deal with DR formulations and their efficiencies, such as accuracy, stability and convergence rate. Therefore, the important issues of the DR applications require special consideration. Based on this fact, the applications of the DR schemes in the different structural analysis are reviewed. It should be noted that these strategies have been used for the different kind of analyses, such as static, dynamic, stability, form-finding and, etc. Verities of investigators have found the solution of diverse structures, like: truss, frame, shell, plate and, etc. The reasons for these wide ranges of applications can be found in some interesting DR specifications, which create unique potentials in engineering problems. The main specifications of DR algorithms will be listed in the following lines:

(a) Full vector operations

(b) Calculations are based on internal force rather than the secant stiffness matrix

(c) Simple numerical algorithm

(d) Iterative process

(e) Not sensitive to critical points in structural behavior

(f) Full automatic procedure (personal judgments have no effect on the solution efficiency)

The aforementioned advantages cause numerous usages in different structural engineering problems. The existing restrictions have not affected the vast applications of these solvers. So far, the analysts have found that there are verities of interesting and attractive applications of the DR methods. In the following sections, some of them are briefly reviewed.

2 Plates

Rushton applied Dynamic Relaxation to non-linear systems by analyzing variable thickness plates with geometrical non-linear behavior [101]. He also used this technique for solving the large deflection equations resulted from analyzing the post-buckling of variable thickness plates [103]. In another paper, this researcher analyzed the small and large deflection of simply supported plates with corners free to lift [104]. Moreover, DR method was utilized to analyze the post-buckling of variable thickness flat rectangular plates with mixed boundary conditions by Rushton [105]. This investigator used the DR method to study the large deflection of plates with small initial curvature [106]. By utilizing of the DR technique, Rushton found large deflection of rectangular plates with unsupported edges [107]. In another research, he also examined the buckling of laterally loaded plates having initial curvature [108].

In 1972, Alami analyzed the elastic large deflection behavior of plates under transverse loading by a method based on Dynamic Relaxation [1]. The DR approach was applied to non-linear analysis of clamped orthotropic skew plates having a constant thickness subjected to a uniformly-distributed transverse load by Alwar and Ramachandra [5]. Turvey and Wittrick utilized the DR method to solving flexural and stability problems of elastic geometrically behavior of laminated plates [158]. In another study, this solver was used to analyze the large deflection of skew sandwich plates with clamped edges by Alwar and Ramachandra [6].

Tuma and Galletly described the use of Dynamic Relaxation for analyzing large-deflection of circular and rectangular plates with various boundary conditions [128]. This technique was also employed for the analysis of non-linear behavior of clamped isotropic skew plates of constant thickness subjected to a uniformly distributed transverse load by Alwar and Rao [7]. Moreover, Murthy et al. adopted the DR scheme for analyzing the non-linear bending of circular plates of the linear elastic material and variable profiles subjected to a uniformly distributed lateral loading [78]. Dowling and Bawa used Dynamic Relaxation to solve the finite difference form of orthotropic small deflection plate equations. They took advantage from an analytical method of producing influence surfaces for stiffened steel bridge decks [33]. Utilizing this iterative approach, Turvey found the large deflections of square plates, obeying the Foeppl and Von Karman theories [129]. In another study, Chaplin used the DR approach to analyze the large elastic deformations of clamped skewed plates [21].

Turvey calculated the large deflection of circular plates with variable thickness utilizing two different DR methods. In the first technique, fictitious densities were chosen arbitrarily and in the second one, they were calculated using Gerschgorin bounds. By comparing the results, it was demonstrated that the latter improves the convergence characteristics. This researcher also implemented the second procedure for solving variable thickness annular plates subjected to a uniform lateral load [130]. In the other application, Frieze et al. took advantage of the DR algorithm to analyze the large deflection of elasto-plastic plates. They extended the strategy to include both geometrical and material non-linear effects in plates. Particularly, these investigators discussed the iteration parameters which control convergence [37]. Frieze and Dowling proposed a process based on Dynamic Relaxation for the exact interactive buckling analysis of plates forming box sections subjected to a generalized loading [38]. Using the DR approach, Turvey analyzed the axisymmetric large deflection behavior of circular plates with linear thickness tapers and affine imperfections [131]. He also applied the mentioned strategy to the analysis of the axisymmetric, elasto-plastic large deflection response of laterally loaded circular plates [132]. Moreover, Turvey utilized the method of Dynamic Relaxation for solving the set of non-incremental equations describing the axisymmetric, elasto-plastic, large deflection response of laterally loaded, thickness-tapered, circular plates [133].

There have been a lot of more studies related to the plate problems. Sherbourne and Murthy applied the DR method to analyze bending of circular plates with variable profile [120]. Webb and Dowling utilized Dynamic Relaxation to solve the governing equations from the material and geometric non-linear analysis of eccentrically stiffened plates, subjected to the combined or separate actions of lateral and in-plane loading [161]. By using this solver, Turvey analyzed the axisymmetric elastic large deflection behavior of glass-steel and glass-aluminum circular plates [137]. He also employed this kind of algorithm for analyzing axisymmetric small deflection equations for shear deformation of circular plates [138]. It should be added that Turvey and Lim applied several refinements to the DR method in order to improve computational efficiency. Then, they solved the equations coming from the analysis of axisymmetric full-range response of transverse pressure loaded, clamped and simply supported, circular plates [147]. Turvey and Der Avanessian used the DR method for a numerical solution of the governing equations for the axisymmetric elastic large deflection analysis of ring-stiffened plates [140]. In another research, Lim and Turvey adopted the DR approach to solve the plate equations for examining the elastic integrity degradation of uniform flat simply supported, and clamped circular plates subjected to uniform transverse pressure [148]. They also utilized this way in solution of the incremental equations governing the axisymmetric elasto-plastic large deflection response of circular steel plates with initial deflections [71].

Numerical solutions of the equations governing the elastic post-buckling response of flat and imperfect polar orthotropic circular and annular plates was carried out using a finite-difference version of the DR algorithm by Turvey and Drinali [145]. By utilizing weighting factors for mass and damping, Al-Shawi and Mardirosian used Dynamic Relaxation in the finite element analysis of plate bending [118]. Method of Dynamic Relaxation was successfully exploited to large deflection analysis of circular plates by Turvey and Der Avanessian [141]. Turvey and Lim utilized the DR procedure in numerical solution of the incremental equations governing the full-range axisymmetric flexural response of thickness-tapered circular plates [149]. For ensuring the numerical stability of DR procedure, Xiao-qing and Wan-lin proposed a method for calculation of fictitious densities. Then, they applied the DR technique to the non-linear bending problems of rectangular plates laminated of bimodular composite materials [164]. Machacek employed this solution process in the analysis of thin steel isotropic, and stiffened plates loaded in compression and bending [73]. In the other study, Turvey and Osman utilized the DR method in solution of first-order orthotropic shear deformation equations for the non-linear elastic bending response of rectangular plates [150].

Using Dynamic Relaxation, Machacek analyzed large deflection elasto-plastic behavior of thin steel eccentrically stiffened plates subjected to the lateral and/or in-plane load [74]. The large-deflection elasto-plastic behavior of flat plates and attached flat-bar stiffeners, including the effects of interaction was also presented using Dynamic Relaxation by Caridis and Frieze [16]. Turvey and Der Avanessian utilized the DR method in solution of the governing system of equations for the axisymmetric elasto-plastic large deflection analysis of ring-stiffened circular plates [143]. The application of the Dynamic Relaxation to study the post-buckling path of cylindrically curved panels of laminated composite materials during loading and unloading was also described by Wan-lin and Xiao-qing [160]. Using the DR technique in conjunction with graded finite-differences for a numerical solution of the stiffened plate equations, Turvey and Der Avanessian suggested a discretely stiffened plate theory for the elasto-plastic large deflection analysis of pressure loaded ring-stiffened circular plates [144]. In the other research, the DR procedure was coupled with an interlacing central finite-difference discretization scheme by Turvey and Salehi. They utilized the aforementioned way in solving equations for elastic large deflection of uniformly loaded sector plates [153]. A finite-difference implementation of the DR process was developed to solve the non-axisymmetric elastic large deflection equations of annular sector plate by Salehi and Turvey [114]. In another study, Turvey and Osman utilized finite differences in conjunction with the DR procedure for numerical solutions of the Mindlin laminated plate equations in large deflection initial failure analysis of square angle-ply laminated plates subjected to uniform pressure loading [151].

Combining DR scheme with finite-difference discretization, Salehi et al. analyzed geometrically non-linear behavior of sector plates [111]. In the other study, Salehi and Shahidi utilized the mentioned technique for solving the governing equations for the elastic large deflection of uniformly pressure loaded sector Mindlin plates [112]. Caridis employed the DR method on an interlacing finite difference grid for solving the large-deflection elasto-plastic equations of equilibrium of flat and stiffened plates [17]. Turvey and Salehi proposed a discretely stiffened circular plate theory. They obtained the solution of the large deflection plate equations with the aid of a finite-difference implementation of the DR algorithm [154]. The large deflection axisymmetric circular plate equations were solved using a finite-difference implementation of the DR method by Turvey and Drinali [146]. In the other application, Kadkhodayan et al. utilized this solution approach to predict the bifurcation point of elastic plates subjected to in-plane and transverse external forces [56]. Turvey and Salehi described a theory for the non-axisymmetric elastic large deflection analysis of sector plates stiffened by a single eccentric rectangular cross-section radial stiffener. They used the DR procedure for solving the discrete system of equations [155]. Using a finite difference implementation of the DR scheme, Turvey and Salehi also solved the governing equations for an elasto-plastic large deflection analysis of pressure loaded sector plates. The material of structure obeyed the Ilyushin full-section yield criterion and the flow theory of plasticity [156].

Kadkhodayan investigated the implementation of interlacing, and irregular rectangular meshes used in conjunction with the DXDR method, which is a modified DR technique, to study the large deflection behavior of elastic plates [53]. Moreover, this solver together with the finite difference discretization technique were utilized in analyzing the plate structure for geometrically linear and non-linear elastic. The plate material was assumed to be thick composite sector plates by Salehi and Sobhani [113]. Salehi and Aghaei took advantage of the DR technique together with the finite difference discretization method in non-linear shear analysis of non-axisymmetric circular viscoelastic plates [110]. The elasto-plastic large deflection equations for annular sector plates were solved by the DR procedure. This study was performed by Turvey and Salehi [157]. Furthermore, the DXDR method, which is a modified DR strategy, was utilized for solving equations of large deflection of rectilinear orthotropic square and circular plates by Kadkhodayan [54]. In another investigation, combination of the DR method with the finite difference technique was used to solve the discretized equations in the analysis of the non-linear bending of annular functionally graded plate (FGM) under mechanical loading [41]. In a recent research, Rezaiee and Estiri graded sixteen well-known DR procedures based on their performance in terms of the number of iterations and the analysis time in solving several bending plate examples with geometrical nonlinear behavior [96].

3 Form-finding

One of the extensive uses of DR algorithms is in the structural form-finding. In 1988, Barnes performed the form-finding analysis and fabrication patterning of wide span cable net and pre-stressed membrane roofs by taking advantage of the DR method with kinetic damping [9]. A procedure for numerical form-finding of the stable minimal surfaces represented by soap-film models, based on the DR technique, was also suggested by Lewis and Gosling [65]. In another study, Gosling and Lewis applied the matrix-based finite element method to the vector-based DR algorithm for form-finding of minimal surface membranes [42]. Moreover, the form-finding of lightweight tension structures comprising pre-stressed cable nets and fabric membranes was performed by

Lewis WJ and Lewis TS. They utilized DR technique for solving the resulting non-linear set of equilibrium equations [66]. Adriaenssens and Barnes developed a Dynamic Relaxation based method for form-finding and load analysis of a class of tensegrity structures with continuous tubular compression booms forming curved splines. This structure could be deployed from straight by pre-stressing a cable bracing system [2].

In the other research, Zhang and Shan utilized the DR method for form-finding of the pre-stressed cable and membrane structures [166]. By Dynamic Relaxation with kinetic damping, Brew and Lewis presented a computational methodology for form-finding of tension membrane structures (TMS), or fabric structures, used as roofing forms [13]. Li and Wei used the DR method in the formulation of the form-finding under uniform pre-tensioned force and the behavior analysis under static loads [70]. Han et al. proposed a form-finding strategy for tension structures with elastic supports with the aid of DR method [43]. The aforementioned algorithm was also applied to form-finding of a grid shell in composite materials by Douthe et al. They modeled the large structural rotations by means of the DR technique [31]. It should be noted that this kind of rotations may occur by bending of pre-stressed constructions during the erection process. Furthermore, the form-finding of non-regular tensegrity structures by the aid of a scheme, which was based on the DR method with kinetic damping, and was described by Zhang et al. [167].

Using the DR process, Baudriller et al. suggested a form-finding method for generating non-regular tensegrity shapes of higher diversity and complexity [11]. Lu and Ren utilized the DR method for the form-finding of three-dimensional cable structures in simulation of the cable mesh reflector for the large radio telescope [72]. In another investigation, Wu and Deng exploited the DR method for form-finding of slack cable-bar assembly [162].

Douthe and Baverel proposed a simple way using the DR algorithm to find the form and perform a structural analysis of multi-reciprocal frame systems. This technique is also called nexorades [30]. Furthermore, Ye and Zhou modified the DR method in two aspects. The first was a simple procedure to gain the damp value directly, which is close to that of the critical damp. In the second feature is an extension of the classic DR scheme, which was suggested [165] to obtain the desired membrane shapes. In another research, Lee and Han use DR method for geodesic shape finding of membrane structure with geodesic string [62].

4 Cable structures

The DR strategy is suitable for finding the response of the flexible structures, such as cable. This solution technique was employed for the analysis of cable structures by Day and Bunce [28]. Moreover, Lewis et al. developed a

DR method for the analysis of the non-linear static response of pre-tensioned cable roofs [68]. Using the aforementioned procedure, Lewis et al. investigated cladding-network stiffening effects in pre-tensioned cable roofs [69]. In another study, Lewis and Shan obtained the solution of the governing equilibrium and compatibility equations in the analysis of cladding-network interaction in pre-tensioned cable roof structures under static loads with the aid of the DR scheme with kinetic damping [67]. The DR method was also used in the analysis of cable net systems with many loads relieving devices by Shan et al. [117]. Form-finding, analysis and fabrication patterning of wide-span cable nets and grid shells, uniform or variably pre-stressed fabric membranes and battened membrane roofs was carried out using a Dynamic Relaxation with kinetic damping based technique by Barnes [10]. Chen et al. applied DR method to static analysis of cable nets [25]. They also employed this solver for finding equilibrium state of construction procedures of cable-domes by controlling the original length simulation of the construction process [24]. In order to analyze the nonlinear behavior of cable domes, which is affected by creep over time, a modified dynamic relaxation procedure was introduced by Kmet and Mojdis [59].

5 Dynamic analysis

Utilizing DR method in the dynamic structural analysis is a new application, which has been introduced since 2008. There are only a few papers, which deal with this interesting application of the DR method, most of them were presented by Rezaiee and Alamatian [3,93]. In this approach, DR algorithm is combined with numerical time integrations. It is well known that numerical integration procedures are appropriate for dynamic analysis of structures. These schemes are commonly applied to both linear and non-linear structural behaviors. The aforementioned techniques can calculate the dynamic responses via step by step process. Numerical integration strategies are divided to the implicit and explicit scheme. In the implicit formulation, the dynamic equilibrium equation of structure at time $t^{k+1}$is converted to the below equivalent static system [3, 93]:

$$[S]_{EQ}^{k+1}\{D\}^{k+1} = \{P\}_{EQ}^{k+1} \tag{1}$$

where $[S]_{EQ}^{k+1}$, $\{D\}^{k+1}$ and $\{P\}_{EQ}^{k+1}$ are the equivalent stiffness matrix, the displacement vector and equivalent load vector at $k+1^{th}$ step of numerical integration formulation, respectively. The equivalent stiffness matrix and equivalent load vector are dependent to the implicit dynamic algorithm. Rezaiee-Pajand and Alamatian [93] used the DR method to solve Eq. (1). The proposed technique led to a considerable reduction in errors of nonlinear dynamic analysis. Moreover, Sarafrazi presented a different scheme to analyze the structures. He assumed real and also fictitious time axes. In this strategy, real and fictitious times were considered simultaneously [115]. Recently, an explicit dynamic analysis was performed by the Dynamic Relaxation method [79].

6 Other applications

Because of interesting specifications of the DR method, it has a great potential ability to be used in other kinds of structural engineering problems. For instance, Hussain Khan and Lafitte employed a finite difference based strategy exploiting the DR procedure for performing design analysis of a cylindrical pre-stressed concrete pressure vessel [52]. Moreover, Rushton discussed the determination of the critical damping and optimum time increment. He utilized the DR method for stress analysis problems [102]. The DR method was also developed for time dependent stress analysis of the pre-stressed concrete pressure vessels by Cederberg and David [20]. Cassell introduced the concept of fictitious densities to the DR method. He analyzed doubly curved shells under arbitrary surface tractions and temperature loading [18]. Furthermore, Brew and Brotton formulated the DR method using the first-order dynamic equilibrium relationship and studied the stability conditions of the frame structures [12]. Shmuel and Alterman proposed a scheme for analyzing the dynamic stress distribution close to a crack which cuts through the finite thickness of a given plate. They utilized the aforementioned technique to rapidly improve the convergence of their suggested strategy [121]. The DR procedure was also applied to the analysis of bridge slabs by Cassell et al. [19].

In another application, Hansson used this solver for calculation of plane problems with general shape and non-linear material behavior [45]. Bunce and Brown utilized the DR approach in solving the equilibrium equations resulted from analyzing the finite deflection of plane frames [14]. The DR method was also exploited to computation of stresses, and displacements induced by underground excavations by Crouch and Fairhurst [27]. In addition, Hansson and Schnellenbach analyzed some special problems of local stress conditions of pre-stressed concrete reactor pressure vessels with non-linear material behavior. They performed calculations by the DR process [46]. By use of this solution strategy, Rushton and Hook analyzed the large deflections of rectangular plates and beams obeying a non-linear stress-strain law [109]. The DR approach was also used for analyzing disks with anisotropic material, complying with Ramberg-Osgood stress-strain relations with variable profiles by Sherbourne and Murthy [119]. In the other research, Hook and Rushton investigated beams and plates problems, which deflect until contact is made with internal restraints, using a modified form of the DR method [51]. Swan took advantage of the DR scheme for the dynamic stress analysis of certain linear elastic fracture problems [124]. Moreover, the DR methodology was utilized in calculations for the analysis of two-dimensional behavior of the liners in pre-stressed concrete reactor pressure vessels by Oberpichler et al. [83].

Hansson and Stoever described a process based on the DR algorithm for three-dimensional analysis of local crack development in pre-stressed concrete pressure vessels [47]. On the other hand, Bunce and Brown proposed a technique based on Dynamic Relaxation for analyzing the non-linear behavior of large deflection of thin ideally elastic rods [15]. Using the method of Dynamic Relaxation, Frieze and Dowling analyzed the elastic post-buckling interaction behavior of box sections [38]. In another investigation, Frieze applied Dynamic Relaxation to the analysis of interactive buckling in the elasto-plastic range of thin-walled rectangular sections [37]. Harding used the DR method to solve governed equations from elasto-plastic analysis of initially imperfect thin cylinders subjected to axial compression and external pressure [48]. The DR method was also utilized to solve the axisymmetric, large deflection response of a uniformly loaded, circular membrane subjected to an initial pre-stress/pre-strain by Turvey [134]. In another study, Tsotsos and Hatzigogos adopted the DR procedure for the analysis of the foundations of dams and embankments [127].

Oberpichler performed calculations for designing the liner-anchor system of concrete vessels by the DR method [81]. Moreover, Turvey combined the Dynamic Relaxation with the Tsai-Hill failure criterion and obtained a non-linear elastic solution for the large deflection of laminated strip equations [135]. In another research, Oberpichler used the DR approach for design calculations of steel-liners (in Pre-stressed Concrete Reactor Pressure Vessel (PCRV) or a Concrete Containment Vessel (PCCV)) and their anchorage with regard to non-linear behavior of liner-material and anchorage [82]. Turvey also utilized the DR method to solve the equations for anti-symmetrical laminated, cross-ply strips [136]. In the other paper, Papadrakakis developed DR scheme for the post-buckling behavior of spatial structures [84]. Numerical studies were carried out on the elastic large deflection behavior of cross-ply laminated bi-modular strips employing the DR method by Turvey [139]. In 1982, an implicit DR relationship exploring general increment control was introduced by Felippa [36]. Moreover, Estefen and Harding described a finite difference DR program for the analysis of ring-stiffened shells, including the behavior of the rings as discrete elements [34]. The DR method was also applied to the solution of the non-linear equilibrium equations governing the analysis of the post-critical ultimate load conditions of space trusses by Papadrakakis and Manolis [85].

Krings et al. performed the requisite non-linear calculations for the design of concrete tunnel linings with non-linear material behavior by the DR scheme [61]. Zienkiewicz et al. suggested an accelerated procedure for improvement of the convergence rate [172]. Using the DR technique, a method was proposed for finding the optimum layout of modularly constrained truss structures subjected to multiple load conditions by Topping [125]. Estefen and Harding also investigated the inter-ring buckling of cylindrical leg members by use of the DR scheme [35]. Analysis of a small-deflection beam was carried out with the aid of DR technique by Turvey and Der Avanessian [142]. In order to facilitate static analysis of non-linear problems, Rericha modified the DR method. He exploited continuous loading in time instead of the ordinary step function of time. Inertia and damping forces arising during the loading process were kept at a minimum using an optimum load time history [92]. In another study, Chou and Wu proposed a Dynamic Relaxation finite element method for metal forming processes [26]. Using a natural combination of Lyaponov's dynamic criterion on stability and the modified adaptive DR method, Zhang et al. obtained an approach for predicting the bifurcation points of elastic-plastic buckling of plates and shells [169].

Qiang determined fictitious time and damping by Rayleigh's principle [87]. In another investigation, Lewis used the DR process in the analysis of the non-linear static response of pre-tensioned cable net and pin-jointed frame structures [64]. The modified adaptive DR method (maDR method) was applied to study of the axisymmetric deformation mechanism of work-pieces in conical cup tests by Zhang et al. [170]. Chen et al. also developed a numerical method for modeling plane strain rolling based on DR technique [22]. Furthermore, Rao et al. investigated the use of the DR scheme in the problems with geometric and material non-linearities. It was found from numerical studies that in the case of geometric non-linearity, obtained steady-state solution compares well with the static solution. In this technique, the process of convergence to steady state can be accelerated by choosing higher mass densities and an appropriate mass proportional damping. On the other hand, when both non-linearities were existed, diverse steady-state solutions were obtained for different values of artificial damping [91]. The application of the DR method to study the post-buckling path of cylindrically curved panels of laminated composite materials during loading and unloading was also described by Wan-lin and Xiao-qing [160]. In another paper, Moncrieff and Topping introduced a new procedure for generating cutting patterns for membrane structures, by means of a two-dimensional Dynamic Relaxation analysis for each cloth [77].

Using an improved adaptive DR method, Zhang analyzed the shape effects on the tensile strength of axisymmetric rock specimens [168]. Shugar proposed the automated Dynamic Relaxation (ADR) algorithm. He determined the initial equilibrium configuration and pre-stressed state of compliant structures by this scheme [123]. Moreover, the application of DR method to model discrete crack propagation was described by Gerstle et al. [40]. Sauve and Badie combined the shell formulation with the DR technique, a three-dimensional contact algorithm and generalizations of the constitutive equations for time-dependent deformation to yield accurate simulations of creep response in nuclear reactor fuel channels [116]. In an interesting application, a variable-arc-length approach was introduced by Ramesh and Krishnamoorthy, which can trace snap-through or snap-back regions of the load-deflection path, automatically. They proposed a displacement incrementation procedure to handle multiple loadings in the post-buckling analysis of structures by Dynamic Relaxation [88]. DR approach was also used to analyze equilibrium configurations of partly wrinkled membranes by Haseganu and Steigmann [49].

New schemes for Elastic Post-Buckling (EPB) analysis and Inelastic Post-Buckling (IEPB) analysis of truss structures using the DR method were introduced by Ramesh and Krishnamoorthy. These researchers used the variable-arc-length technique for tracing the post-buckling paths for elastic, EPB and IEPB analyses [89]. In the other research, a parallel algorithm for the DR strategy was presented by Topping and Khan [126]. Moreover, Kommineni and Kant described a unified approach for linear and geometrically non-linear analyses of composite and sandwich shells by means of the DR process [60]. The DR iteration was also adopted for the geometrically non-linear analysis of plates and shells involving large deflections, small rotations and strains by Ramesh and Krishnamoorthy [90]. Kadkhodayan and Zhang combined two vital components: the DXDR algorithm and a stable consistent algorithm for the integration of the constitutive equations of plasticity. They used this strategy for analyzing general elastic-plastic problems [55]. Furthermore, the DR method was adopted for solving delayed hydride cracking (DHC) problems by Metzger and Sauvé [75].

Using the DR process, Turvey and Osman analyzed small and large deflection Mindlin plate equations for unsymmetrical laminated cross-ply strips subjected to uniform lateral pressure [152]. Oakley and Knight Jr suggested a parallel adaptive Dynamic Relaxation (ADR) algorithm for non-linear structural analysis [80]. By means of finite differences together with DR method, Atai and Steigmann developed an equilibrium theory for the coupled finite deformations of elastic curves and surfaces [8]. The DR procedure was also employed in the analysis the fracture of materials by Metzger [76]. Moreover, Wakefield presented the application of DR approach to the analysis of tensile structures [159]. Xiaoqing and Hong utilized the DR solver to investigate the influence of compression-bending coupling on non-linear behavior of cylindrically slightly curved panels of unsymmetrical laminated composite materials subjected to uniform uniaxial compression during loading and unloading [163]. Han and Lee employed DR procedure for stabilizing of unstable structures [44]. In addition, the Dynamic Relaxation was combined with neural networks to increase model accuracy of tensegrity structures by Domer et al. [29]. Shoukry et al. employed DR approach for analyzing large transportation structures as dowel jointed concrete pavements and 306-m-long, reinforced concrete bridge superstructure under the effect of temperature variations [122]. In another study, Chen et al. used the DR method and obtained a technique to find the internal force of the elements, and the nodal coordinates of the structures composed of cables in tension and struts in compression in the equilibrium state by controlling the original length [23].

Hegyi et al. employed the DR method for membrane analysis by an eight-node quadrilateral element [50]. Kan and Ye described a way based on modified Dynamic Relaxation for solving the problem of rigid displacement [57]. In another study, Douthe et al. exploited the DR method in numerical modeling of grid shell with composite materials [32]. Plotzitza et al. employed the DR technique in the numerical simulation of concrete under explosive loading [86]. Moreover, the DR method was utilized to obtain the analytical solution of free damped vibrations of a linear viscoelastic oscillator, based on the fractional derivative model involving more than one different fractional parameter and several relaxation (retardation) times by Rossikhin et al. [100]. For reducing the calculation cost of the DR technique, Kashiwa and Onoda combined this method with an iterative technique via two switching rules. In this strategy, the numerical scheme used in the analysis is selected as the situation of the analysis. An unstable region of the analysis, in which the static iterative method cannot obtain the converged solution, is performed by the robust DR method. Then, a stable region that is costly to be solved by the DR technique is performed by the efficient iterative method. The performance of the suggested process was verified through a comparison of numerical analyses for the wrinkled membrane with the new way and conventional ones [58].

Based on the fact that a static problem has an equivalent wave speed of infinity, and a dynamic problem has a wave speed of finite value, an effective loading algorithm associated with the explicit DR procedure was presented to produce meaningful numerical solutions for static problems by Zhao et al. [171]. Rezaiee-Pajand and Alamatian proposed a method for tracing static path of truss structures using DR technique [94]. Furthermore, an automatic DR method was proposed for tracing the static path of structures, which have snap-through or snap-back regions, by these researchers [95]. In a recent research by Rezaiee and Estiri, a variable load factor, obtained from minimization of the unbalanced displacement increment, was employed in DR algorithm. Consequently, their proposed process was capable of tracing the equilibrium path at snap-through and snap-back points in addition to possessing high accuracy and efficiency [96]. In another paper, they minimized the work increment by the external loads and obtained a new formulation of the load factor [96]. Utilizing the structural equilibrium path, which was obtained from the DR method, these researchers calculated the buckling limit load. They set the work increment of the external forces to zero and found a new equation to derive the load factor [96]. Moreover, the first structural buckling load was recently calculated by simple DR algorithm [4]. Lee et al. presented an explicit arc-length method based on cylindrical arc-length method using Dynamic Relaxation with kinetic damping for tracing the post-buckling equilibrium path of structures [63].

7 Conclusion

This paper was devoted to the applications of the DR methods. According to all the evidence presented throughout this study, these techniques fit quite well with the linear and also non-linear behaviors of structures. They were successfully utilized in static and dynamic problems. A variety of the structures, such as trusses, frames, plates, membranes, shells and cables, can be analyzed by the aforementioned formulations. This survey showed that the applications of DR methods in structural engineering problems are very vast. The enormous usage of these solvers in different branches of structural analysis is due to simplicity, full explicit vector operations and no sensitivity to non-linear behaviors, which are the main specifications of DR procedure. In this paper, a wide range of DR applications was analyzed, and very useful reference sources were introduced. Due to important roles of these strategies, providing a reasoned and evidence critique and also more practical ways of function are still remained to be found. Based on the broad features of presented studies, it is suitable to make some suggestions for the future research and applications of these non-linear solvers. From the relevance point of view, the DR method has a great potential of abilities to be use in different engineering problems. These applications could not be limited to the structural analysis, so that the DR algorithms may be utilized for other branches of civil engineering, such as hydraulics, numerical solution of water flows, and soil mechanics. It should be noted that there in no limitation for using DR tactic in the other engineering and applied mathematics branches.

References

1. Aalami, B. *Large deflection of elastic plates under patch loading*, ASCE Journal of the Structural Division 98 (ST11) (1972), 2567-2586.

2. Adriaenssens, S.M.L.a and Barnes, M.R. *Tensegrity spline beam and grid shell structures*, Engineering Structures 23(1) (2001), 29-36.

3. Alamatian, J. *New implicit higher order time integration for dynamic analysis*, Structural Engineering and Mechanics 48(5) (2013), 711-736.

4. Alamatian, J. and Hosseini Nejad Goshik, M. *An efficient explicit framework for determining the lowest structural buckling load using Dynamic Relaxation method*, Mechanics Based Design of Structures and Machines (2016), 1-12.

5. Alwar, R.S. and Rao, N.R. *Nonlinear analysis of orthotropic skew plates*, AIAA Journal 11(4) (1973-a), 495-498.

6. Alwar, R.S. and Rao, N.R. *Nonlinear analysis of skew sandwich plates*, proceeding of the Canadian Congress of Applied Mechanics (CANCAM), Montreal, Canada, (1973-b).

7. Alwar, R.S. and Rao, N.R. *Large elastic deformations of clamped skewed plates by dynamic relaxation*, Computers and Structures 4(2) (1974), 381-398.

8. Atai, A.A. and Steigmann, D.J. *Coupled deformations of elastic curves and surfaces*, International Journal of Solids and Structures 35(16) (1998), 1915-1952.

9. Barnes, M.R. *Form-finding and analysis of prestressed nets and membranes*, Computers and Structures 30(3) (1988), 685-695.

10. Barnes, M.R. *Form finding and analysis of tension structures by Dynamic Relaxation*, International Journal of Space Structures 14(2) (1999), 89-104.

11. Baudriller, H., Maurin, B., Cañadas, P., Montcourrier, P.H., Parmeggiani, A. and Bettache, N. *Form-finding of complex tensegrity structures: application to cell cytoskeleton modeling*, Comptes Rendus–Mecanique 334(11) (2006), 662-668.

12. Brew, J.S. and Brotton, M. *Non-linear structural analysis by Dynamic Relaxation method*, International Journal of Numerical Methods in Engineering 3(4) (1971), 463-483.

13. Brew, J.S. and Lewis, W.J. *Computational form-finding of tension membrane structures - Non-finite element approaches: Part 3. Comparison of mesh constraint methods*, International Journal of Numerical Methods in Engineering 56(5) (2003), 685-697.

14. Bunce, J.W. and Brown, E.H. *The finite deflection of plane frames*, International Journal of Mechanical Sciences 15(3) (1973), 189-198.

15. Bunce, J.W. and Brown, E.H. *Non-linear bending of thin, ideally elastic rods*, International Journal of Mechanical Sciences 18(9-10) (1976), 435-441.

16. Caridis, P.A. and Frieze, P.A. *Flexural-torsional elasto-plastic buckling in flat stiffened plating using dynamic relaxation. Part 1: Theory*, Thin-Walled Structures 6(6) (1988), 453-481.

17. Caridis, P. *Panel 2D - a program for the static and dynamic full-range analysis of unstiffened and stiffened plates*, International Conference on Computational Structures Technology – Proceedings, Napier University, Edinburgh, UK, (1994), 131-140.

18. Cassell, A.C. *Shells of revolution under arbitrary loading and the use of fictitious densities in dynamic relaxation*, Institute of Civil Engineers, ICE proceedings Journal, USA 45 (1970), 65-78.

19. Cassell, A.C., Hobbes, R.E., Bridgett, M.S. and Awotwi, P.A.K. *Dynamic Relaxation analysis of multiple span slabs*, Structural Engineering 49(12) (1971), 549-559.

20. Cederberg, H. and David, M. *Computation of creep effects in prestressed concrete pressure vessels using Dynamic Relaxation*, Nuclear Engineering and Design 9(4) (1969), 439-448.

21. Chaplin, T.K. *Large elastic deformations of clamped skewed plates by Dynamic Relaxation*, Computers and Structures 8(2) (1978), 257-258.

22. Chen, B.K., Choi, S.K. and Thomson, P.F. *Analysis of plane strain rolling by the dynamic relaxation method*, International Journal of Mechanical Sciences 31(11-12) (1989), 839-851.

23. Chen, L.M., Dong, S.L. and Yuan, X.F. *Theoretical and experimental study on the construction of cable domes*, Tumu Gongcheng Xuebao/China Civil Engineering Journal 39(11) (2006), 33-36.

24. Chen, L.M., Dong, S.L. and Yuan, X.F. *Theoretical analysis of construction procedures of cable-domes*, Gongcheng Lixue/Engineering Mechanics 25(4) (2008), 134-139.

25. Chen, S., Lin, W., Xu, G. and Huang, J. *Dynamic Relaxation method for static analysis of cable nets*, Yingyong Lixue Xuebao/Chinese Journal of Applied Mechanics 19(2) (2002), 34.

26. Chou, P.C. and Wu, L. *A Dynamic Relaxation finite-element method for metal forming processes*, International Journal of Mechanical Sciences 28(4) (1986), 231-250.

27. Crouch, S.L. and Fairhurst, C.F. *Analysis of rock mass deformations due to excavations*, ASME AMD 3(1973), 203-210.

28. Day, A.S. and Bunce, J.W. *Analysis of cable networks*, Civil Engineering Published Works Revelation 65(765) (1970), 383-386.

29. Domer, B., Fest, E., Lalit, V. and Smith, I.F.C. *Combining Dynamic Relaxation method with artificial neural networks to enhance simulation of tensegrity structures*, ASCE Journal of Structural Engineering 129(5) (2003), 672-681.

30. Douthe, C. and Baverel, O. *Design of nexorades or reciprocal frame systems with the dynamic relaxation method*, Computers and Structures 87(21-22) (2009), 1296-1307.

31. Douthe, C., Baverel, O. and Caron, J.F. *Form-finding of a grid shell in composite materials*, Journal of the International Association for Shell and Spatial Structures 47(150) (2006), 53-62.

32. Douthe, C., Baverel, O. and Caron, J.F. *Gridshell in composite materials: Towards wide span shelters*, Journal of the International Association for Shell and Spatial Structures 48(155) (2007), 175-180.

33. Dowling, P.J. and Bawa, A.S. *Influence surfaces for orthotropic steel bridge decks*, Institute of Civil Engineers, ICE proceedings Journal, USA, 59(1975), 149-168.

34. Estefen, S.F. and Harding, J.E. *Ring stiffener behavior and its interaction with cylindrical panel buckling*, Institute of Civil Engineers, ICE proceedings Journal, USA, 75(1983), 243-264.

35. Estefen, S.F. and Harding, J.E. *Effect of ring stiffeners on inter-ring cylindrical panel buckling*, *Offshore Engineering, Volume 5: Proceedings of the 5th International Symposium on Offshore Engineering. Rio De Janeiro, Brazil*; Code 10983 (1986), 460-476.

36. Felippa, C.A. *Dynamic Relaxation under general increment control*, Mathematical Programming 24 (1982), 103-133.

37. Frieze, P.A. *Elasto-plastic buckling in short thin-walled beams and columns*, Institute of Civil Engineers, ICE proceedings Journal, USA, 65 (1978) 857-874.

38. Frieze, P.A. and Dowling, P.J. *Interactive buckling analysis of box sections using dynamic relaxation*, Computers and Structures 9(5) (1978), 431-439.

39. Frieze, P.A., Hobbs, R.E. and Dowling, P.J. *Application of Dynamic Relaxation to the large deflection elasto-plastic analysis of plates*, Computers and Structures 8(2) (1978), 301-310.

40. Gerstle, W.H. and Xie, M. *FEM modeling of fictitious crack propagation in concrete*, Journal of Engineering Mechanics 118(2) (1992), 416-434.

41. Golmakani, M.E. and Kadkhodayan, M. *Nonlinear bending analysis of annular FGM plates using higher-order shear deformation plate theories*, Composite Structures 93(2) (2011), 973-982.

42. Gosling, P.D. and Lewis, W.J. *Optimal structural membranes - II. Form-finding of prestressed membranes using a curved quadrilateral finite element for surface definition*, Computers and Structures 61(5) (1996), 885-895.

43. Han, D.J., Song, X.B. and Wang, H.T. *Dynamic Relaxation for integer form-finding analysis of tension structures with elastic supporting bars*, Huanan Ligong Daxue Xuebao/Journal of South China University of Technology (Natural Science) 33(11) (2005), 66-71.

44. Han, S.E. and Lee, K.S. *A study of the stabilizing process of unstable structures by dynamic relaxation method*, Computers and Structures 81(17) (2003), 1677-1688.

45. Hansson, V. *Calculation of plane problems of random shapes and nonlinear material behavior*, Konstr Ingenieurbau Ber 14 (1973), 70-78.

46. Hansson, V. and Schnellenbach, G. *Some special problems regarding the analysis of prestressed concrete reactor pressure vessels with due consideration towards non-stationary temperature fields and nonlinear behavior of the material*, Konstr Ingenieurbau Ber 19 (1974), 30-38.

47. Hansson, V. and Stoever, R. *Three-dimensional analysis of cracked concrete in prestressed concrete pressure vessels and comparisons with experimental results*, International Conference on Structural Mechanics in Reactor Technology. 3rd Transaction; London, UK, (1975), 10-22.

48. Harding, J.E. *Elasto-plastic analysis of imperfect cylinders*, Institute of Civil Engineers, ICE proceedings Journal, USA, 65 (1978), 875-892.

49. Haseganu, E.M. and Steigmann, D.J. *Analysis of partly wrinkled membranes by the method of dynamic relaxation*, Computational Mechanics 14(6) (1994), 596-614.

50. Hegyi, D., Sajtos, I., Geiszter, G. and Hincz, K. *Eight-node quadrilateral double-curved surface element for membrane analysis*, Computers and Structures 84(31-32) (2006), 2151-2158.

51. Hook, P.M. and Rushton, K.R. *Buckling of beams and plates onto an intermediate support studied by the Dynamic Relaxation method*, Journal of Strain Analysis 10(3) (1975), 153-158.

52. Hussain Khan, M. and Lafitte, R. *Analysis of a cylindrical prestressed concrete pressure vessel with a large central penetration in the slab*, Nuclear Engineering and Design 5(2) (1967), 191-198.

53. Kadkhodayan, M. *Using interlacing and irregular meshes in plate analysis*, Iranian Journal of Science and Technology, Transaction B: Technology 26(3) (2002), 419-430.

54. Kadkhodayan, M. *An investigation to the orthotropic plates by using irregular rectilinear mesh*, Proceedings of 26th IASTED International Conference on Modelling, Identification and Control, Innsbruck, Austria, 12 -14 February (2007), 259-264.

55. Kadkhodayan, M. and Zhang, L.C. *Consistent DXDR method for elastic-plastic problems*, International Journal of Numerical Methods in Engineering 38(14) (1995), 2413-2431.

56. Kadkhodayan, M., Zhang, L.C. and Sowerby, R. *Analyses of wrinkling and buckling of elastic plates by DXDR method*, Computers and Structures 65(4) (1997), 561-574.

57. Kan, Y. and Ye, J.H. *Form finding of cable domes by modified dynamic relaxation*, Gongcheng Lixue/Engineering Mechanics 24(9) (2007), 50-55.

58. Kashiwa, M. and Onoda, J. *Wrinkling analysis using improved dynamic relaxation method*, AIAA Journal 47(7) (2009), 1601-1607.

59. Kmet, S. and Mojdis, M. *Time-dependent analysis of cable domes using a modified dynamic relaxation method and creep theory*, Computers and Structures 125 (2013), 11-22.

60. Kommineni, J.R. and Kant, T. *Pseudo-transient large deflection analysis of composite and sandwich shells with a refined theory*, Computational Methods in Applied Mechanics and Engineering 123(1-4) (1995), 1-13.

61. Krings, W. and Meiswinkel, R. *Design of concrete tunnel linings with non-linear material behavior installed during driving*, Beton-und Stahlbetonbau 79(3) (1984), 63-69.

62. Lee, K. and Han, S. *Geodesic shape finding of membrane structure with geodesic string by the dynamic relaxation method*, Structural Engineering and Mechanics 39(1) (2011), 16-23.

63. Lee, K., Han, S. and Park, T. *A simple explicit arc-length method using the dynamic relaxation method with kinetic damping*, Computers and Structures 89 (2011), 216-233.

64. Lewis, W.J. *The efficiency of numerical methods for the analysis of prestressed nets and pin-jointed frame structures*, Computers and Structures 33(3) (1989), 791-800.

65. Lewis, W.J. and Gosling, P.D. *Stable minimal surfaces in form-finding of lightweight tension structures*, International Journal of Space Structures 8(3) (1993), 149-166.

66. Lewis, W.J. and Lewis, T.S. *Application of formian and dynamic relaxation to the form-finding of minimal surfaces*, Journal of the International Association for Shell and Spatial Structures 37(122) (1996), 165-186.

67. Lewis, W.J. and Shan, J. *Numerical modelling of the non-linear static response of clad cable net structures*, Computers and Structures 35(1) (1990), 15-22.

68. Lewis, W.J., Jones, M.S., Lewis, G. and Rushton, K.R. *Cladding-network interaction in retensioned cable roofs, studied by Dynamic Relaxation*, Computers and Structures 19(5-6) (1984), 885-897.

69. Lewis, W.J., Jones, M.S. and Rushton, K.R. *Dynamic Relaxation analysis of the non-linear static response of pretensioned cable roofs*, Computers and Structures 18(6) (1984), 989-997.

70. Li, C.Y. and Wei, D.M. *Analysis of cable-membrane structures by Dynamic Relaxation method*, Huanan Ligong Daxue Xuebao/Journal of South China University of Technology (Natural Science) 32(6) (2004), 80-84.

71. Lim, G.T. and Turvey, G.J. *Axisymmetric elasto-plastic large deflection behaviour of circular steel plates with initial deflections-A parametric study*, Computers and Structures 21(6) (1985), 1187-1196.

72. Lu, Y.J. and Ren, G.X. *Simulation of the cable mesh reflector for the large radio telescope fast*, Gongcheng Lixue/Engineering Mechanics 24(10) (2007), 165-169.

73. Machacek, J. *Analysis of thin plates with eccentric stiffeners*, Stavebnicky casopis 36(3) (1988-a), 191-212.

74. Machacek, J. *Study on imperfections of stiffened plating*, Acta Technica CSAV (Ceskoslovensk Akademie Ved) 33(5) (1988-b), 582-606.

75. Metzger, D.R. and Sauvé, R.G. *A self-induced stress model for simulating hydride formation at flaws*, American Society of Mechanical Engineers, Pressure Vessels and Piping Division (Publication) 326(1996), 137-142.

76. Metzger, D.R. *Explicit finite element formulation for a general strip yield model*, American Society of Mechanical Engineers, Pressure Vessels and Piping Division (Publication) 370(1998), 189-195.

77. Moncrieff, E. and Topping, B.H.V. *Computer methods for the generation of membrane cutting patterns*, Computers and Structures 37(4) (1990), 441-450.

78. Murthy, S. and Sherbourne, N.A. *Nonlinear bending of elastic plates of variable profile*, ASCE Journal of Engineering Mechanics Division 100(2) (1974), 251-265.

79. Namadchi, A.H. and Alamatian, J. *Explicit dynamic analysis using Dynamic Relaxation method*, Computers and Structures 175 (2016), 91-99.

80. Oakley, D.R. and Knight, Jr.N.F. *Non-linear structural response using adaptive dynamic relaxation on a massively parallel-processing system*, International Journal of Numerical Methods in Engineering 39(2) (1996), 235-259.

81. Oberpichler, R. *Design calculations for anchored steel linings of concrete vessels*, Konstruktiver Ingenieurbau Berichte 33 (1980), 1-52.

82. Oberpichler, R. *Design of steel-liners and their anchorage with regard to non-linear behavior of liner-material and anchorage*, International Conference on Structural Mechanics in Reactor Technology, London, UK (1981), 11-23.

83. Oberpichler, R., Schnellenbach, G. and Weber, R. *Stress analysis of liners for prestresssed concrete reactor pressure vessels with regard to non-linear behavior of liner material and of anchor-characteristics*, International Conference on Structural Mechanics in Reactor Technology, 3rd Transaction; London, UK (1975), 50-67.

84. Papadrakakis, M. *Post-buckling analysis of spatial structures by vector iteration methods*, Computers and Structures 14(5-6) (1981), 393-402.

85. Papadrakakis, M. *Inelastic post-buckling of trusses*, Journal of Structural Engineering 109(9) (1983), 2129-2147.

86. Plotzitza, A., Rabczuk, T. and Eibl, J. *Techniques for numerical simulations of concrete slabs for demolishing by blasting*, Journal of Engineering Mechanics 133(5) (2007), 523-533.

87. Qiang, S. *An adaptive Dynamic Relaxation method for non-linear problems*, Computers and Structures 30(1988), 855-859.

88. Ramesh, G. and Krishnamoorthy, C.S. *Post-buckling analysis of structures by dynamic relaxation*, International Journal of Numerical Methods in Engineering 36(8) (1993), 1339-1364.

89. Ramesh, G. and Krishnamoorthy, C.S. *Inelastic post-buckling analysis of truss structures by dynamic relaxation method*, International Journal of Numerical Methods in Engineering 37(21) (1994), 3633-3657.

90. Ramesh, G. and Krishnamoorthy, C.S. *Geometrically non-linear analysis of plates and shallow shells by Dynamic Relaxation*, Computational Methods in Applied Mechanics and Engineering 123(1-4) (1995), 15-32.

91. Rao, A., Rama, M. and Shantaram, D. *Non-linear finite element analysis by dynamic relaxation*, International Journal of Structures 10(1) (1990), 37-53.

92. Rericha, P. *Optimum load time history for non-linear analysis using Dynamic Relaxation*, International Journal of Numerical Methods Engineering 23(12) (1986), 2313-2324.

93. Rezaiee-Pajand, M. and Alamatian, J. *Nonlinear dynamic analysis by Dynamic Relaxation method*, Journal of Structural Engineering and Mechanics 28(5) (2008), 549-570.

94. Rezaiee-Pajand, M. and Alamatian, J. *Dynamic Relaxation method for tracing static path of truss structures*, Modeling in engineering 17 (2009), 27-39.

95. Rezaiee-Pajand, M. and Alamatian, J. *Automatic DR structural analysis of snap-through and snap-back using optimized load increments*, ASCE Journal of Structural Engineering 137(1) (2011), 109-116.

96. Rezaiee-Pajand, M. and Estiri, H. *Mixing dynamic relaxation method with load factor and displacement increments*, Computers and Structures 168 (2016), 78-91.

97. Rezaiee-Pajand, M. and Estiri, H. *Computing the structural buckling limit load by using dynamic relaxation method*, International Journal of Non-Linear Mechanics 81 (2016), 245-260.

98. Rezaiee-Pajand, M. and Estiri, H. *A comparison of large deflection analysis of bending plates by dynamic relaxation*, Periodica Polytechnica Civil Engineering 60(4) (2016), 619-645.

99. Rezaiee-Pajand, M. and Estiri, H. *Finding equilibrium paths by minimizing external work in dynamic relaxation method*, Applied Mathematical Modelling 40 (2016), 10300-10322.

100. Rossikhin, Yu, A., Shitikova, M.V. and Shcheglova, T.A. *Analysis of free vibrations of a viscoelastic oscillator via the models involving several fractional parameters and relaxation/retardation times*, Computers and Mathematics with Applications 59(5) (2010), 1727-1744.

101. Rushton, K.R. *Large deflection of variable-thickness plates*, International Journal of Mechanical Sciences 10 (1968-a), 723-735.

102. Rushton, K.R. *Dynamic Relaxation method used for stress analysis*, Conf. Recent Advances in Stress Analysis, Royal Aeronautical Society, London, UK (1968-b), 41-46.

103. Rushton, K.R. *Postbuckling of tapered plates*, International Journal of Mechanical Sciences 11(5) (1969-a), 461-480.

104. Rushton, K.R. *Simply supported plates with corners free to lift*, Journal of Strain Analysis 4(4) (1969-b), 306-311.

105. Rushton, K.R. *Postbuckling of rectangular plates with various boundary conditions*, Aeron Quart 21(1970-a), 163-181.

106. Rushton, K.R. *Large deflexion of plates with initial curvature*, International Journal of Mechanical Sciences 12(12) (1970-b), 1037-1051.

107. Rushton, K.R. *Large deflection of plates with unsupported edges*, Journal of Strain Analysis 7(1) (1972-a), 44-53.

108. Rushton, K.R. *Buckling of laterally loaded plates having initial curvature*, International Journal of Mechanical Sciences 14(10) (1972-b), 667-680.

109. Rushton, K.R. and Hook, P.M. *Large deflection of plates and beams obeying non-linear stress-strain laws*, Journal of Strain Analysis 9(3) (1974), 178-184.

110. Salehi, M. and Aghaei, H. *Dynamic Relaxation large deflection analysis of non-axisymmetric circular viscoelastic plates*, Computers and Structures 83(23-24) (2005), 1878-1890.

111. Salehi, M. and Shahidi, A. *Transverse shear deformability of sector plates: New results*, American Society of Mechanical Engineers, Petroleum Division (Publication) 64(8-3) (1994-a), 545-552.

112. Salehi, M. and Shahidi, A. *Large deflection analysis of elastic sector Mindlin plates*, Computers and Structures 52(5) (1994-b), 987-998.

113. Salehi, M. and Sobhani, A.R. *Elastic linear and non-linear analysis of fiber-reinforced symmetrically laminated sector Mindlin plate*, Composite Structures 65(1) (2004), 65-79.

114. Salehi, M. and Turvey, G.J. *Elastic large deflection response of annular sector plates-a comparison of DR finite-difference, finite element and other numerical solutions*, Computers and Structures 40(5) (1991), 1267-1278.

115. Sarafrazi, S.R. *Numerical integration for structural dynamic analysis*, PhD Dissertation, Department of Civil Engineering, Ferdowsi University, Mashhad, Iran, 2010.

116. Sauve, R.G. and Badie, N. *Non-linear shell formulation for time-dependent deformation*, ASME Pressure Vessels and Piping Division (Publication) 265(1993), 269-275.

117. Shan, J., Yi, J. and Zheng, Q. *Investigation into the behavior of load relieving systems under loading*, Gong Cheng Li Xue/Engineering Mechanics 16(3) (1999), 38-42.

118. Shawi, F.A.N. and Mardirosion, A.H. *An improved Dynamic Relaxation method for the analysis of plate bending problems*, Computers and Structures 27 (1987), 237-240.

119. Sherbourne, A.N. and Murthy, D.N.S. *Stresses in disks with variable profile*, International Journal of Mechanical Sciences 16(7) (1974), 449-459.

120. Sherbourne, A.N. and Murthy, D.N.S. *Bending of circular plates with variable profile*, Computers and Structures 11(4) (1980), 355-361.

121. Shmuely, M. and Alterman, Z.S. *Three dimensional numerical analysis of stress distribution in the vicinity of a crack tip*, Israel Journal of Technology, 9(5) (1971), 523-530.

122. Shoukry, S.N., William, G.W., Riad, M.Y. and McBride, K.C. *Dynamic Relaxation: A technique for detailed thermo-elastic structural analysis of transportation structures*, International Journal of Computational Methods in Engineering Sciences and Mechanics 7(4) (2006), 303-311.

123. Shugar, T.A. *Automated dynamic relaxation solution method for compliant structures*, ASME Computer Engineering Division 4 (1991), 1-12.

124. Swan, G. *The numerical solution of certain dynamic crack propagation problems*, International Journal of Rock Mechanics and Mining Sciences 12(10) (1975), 295-302.

125. Topping, B.H.V. *Intuitive form-finding of truss structures*, Civil Engineering for Practicing and Design Engineers 3(5) (1984), 491-509.

126. Topping, B.H.V. and Khan, A.I. *Parallel computation schemes for Dynamic Relaxation*, Engineering computations 11(6) (1994), 513-548.

127. Tsotsos, S. and Hatzigogos, T. *Dynamic Relaxation analysis of the foundations of dams and embankments*, Bulletin of the International Association of Engineering Geology 20 (1979), 237-239.

128. Tuma, F.Y. and Galletly, G.D. *Large-deflection of elastic plates using Dynamic Relaxation*, Transactions of the Canadian Society for Mechanical Engineering 2(3) (1973), 121-128.

129. Turvey, G.J. *Study of the behavior of square plates at large deflections according to the theories of FOEPPL and Von Karman*, Journal of Strain Analysis in Engineering Design 13(1) (1978-a), 11-16.

130. Turvey, G.J. *Large deflections of tapered annular plates by Dynamic Relaxation*, ASCE Journal of Engineering Mechanics Division 104(2) (1978-b), 351-366.

131. Turvey, G.J. *Large deflection of circular plates with affine imperfections*, International Journal of Solids and Structures 14(7) (1978-c), 559-569.

132. Turvey, G.J. *Axisymmetric elasto-plastic flexure of circular plates in the large deflexion regime*, Institute of Civil Engineers, ICE proceedings Journal, USA 67 (1979-a), 81-92.

133. Turvey, G.J. *Thickness-tapered circular plates–an elastoplastic large deflection analysis*, Journal of structural mechanics 7(3) (1979-b), 247-271.

134. Turvey, G.J. *Large deflecion of prestressed/prestrained circular membranes*, Institute of Civil Engineers. Part 1 - Design & Construction, ICE proceedings Journal, USA 67 (1979-c), 1121-1126.

135. Turvey, G.J. *Study of the effects of imperfections and lay-up antisymmetry on the initial flexural failure response of cross-ply strips*, Economic Computation and Economic Cybernetics Studies and Research 1 (1980), 291-304.

136. Turvey, G.J. *Large deflection cylindrical bending analysis of cross-ply laminated strips*, Journal of Mechanical Engineering Science 23(1) (1981), 21-29.

137. Turvey, G.J. *Axisymmetric elastic large deflection analysis of composite circular plates*, Fibre Science and Technology 16(3) (1982-a), 191-217.

138. Turvey, G.J. *Axisymmetric flexure of variable thickness circular mindlin plates*, International Journal of Structures 2 (1982-b), 81-88.

139. Turvey, G.J. *Study of the nonlinear elastic flexure of bimodular composite strips*, Fibre Science and Technology 16(2) (1982-c), 121-147.

140. Turvey, G.J. and Der Avanessian, N.G.V. *Elastic large deflection analysis of ring-stiffened circular plates using graded finite-difference*, International Conference on Numerical Methods in Engineering: Theory and Applications Swansea. Wales,UK 7–11 January, 3 (1985), 875-884.

141. Turvey, G.J. and Der Avanessian, N.G.V. *Elastic large deflection of circular plates using graded finite-differences*, Computers and Structures 23(6) (1986-a), 763-774.

142. Turvey, G.J. and Der Avanessian, N.G.V. *DR analysis of beams using graded finite differences*, Computers and Structures 22(5) (1986-b), 737-742.

143. Turvey, G.J. and Der Avanessian, N.G.V. *Axisymmetric elasto-plastic large deflection response of ring stiffened circular plates*, International Journal of Mechanical Sciences 31(11-12) (1989), 905-924.

144. Turvey, G.J. and Der Avanessian, N.G.V. *Full-range response of clamped ring-stiffened circular steel plates-comparisons between experiment and theory*, Computers and Structures 37(1) (1990), 55-70.

145. Turvey, G.J. and Drinali, H. *Elastic postbuckling of orthotropic circular and annular plates with imperfections*, The Third Conference of Composite Structures, Paisley, Scotland, Elsevier Applied Science, London, England (1985), 315-335.

146. Turvey, G.J. and Drinali, H. *Nonlinear first yield analysis of circular plates subjected to combined uniform pressure and edge compression*, Computers and Structures 63(4) (1997), 769-773.

147. Turvey, G.J. and Lim, G.T. *Axisymmetric full-range analysis of transverse pressure-loaded circular plates*, International Journal of Mechanical Sciences 26(9-10) (1984-a), 489-502.

148. Turvey, G.J. and Lim, G.T. *On the degradation of elastic integrity in transverse pressure loaded circular steel plates*, 20th Annual British Society for Strain Measurement Conference on Structural Integrity, Lancaster University, Lancaster, UK (1984-b).

149. Turvey, G.J. and Lim, G.T. *A parametric study of the axisymmetric full-range response of thickness-tapered circular steel plates*, Computers and Structures 22(3) (1986), 459-468.

150. Turvey, G.J. and Osman, M.Y. *Large deflection analysis of orthotropic Mindlin plates*, ASME, Petroleum Division (Publication) 24(1988), 163-172.

151. Turvey, G.J. and Osman, M.Y. *Large deflection initial failure analysis of angle-ply laminated plates*, Composite Structures 25(1-4) (1993), 529-539.

152. Turvey, G.J. and Osman, M.Y. *Elastic and initial flexural failure analysis of unsymmetrically laminated cross-ply strips*, Composites Part B: Engineering 7(5) (1996), 505-518.

153. Turvey, G.J. and Salehi, M. *DR large deflection analysis of sector plates*, Computers and Structures 34(1) (1990), 101-112.

154. Turvey, G.J. and Salehi, M. *Circular plates with one diametral stiffener - An elastic large deflection analysis*, Computers and Structures 63(4) (1997), 775-783.

155. Turvey, G.J. and Salehi, M. *Large deflection analysis of eccentrically stiffened sector plates*, Computers and Structures 68(1-3) (1998), 191-205.

156. Turvey, G.J. and Salehi, M. *Elasto-plastic response of uniformly loaded sector plates: Full-section yield model predictions and spread of plasticity*, Computers and Structures 79(26-28) (2001), 2335-2348.

157. Turvey, G.J. and Salehi, M. *Annular sector plates: Comparison of full-section and layer yield predictions*, Computers and Structures 83(28-30) (2005), 2431-2441.

158. Turvey, G.J. and Wittrick, W.H. *Large deflection and post-buckling behavior of some laminated plates*, Aeron Quart 24(2) (1973), 77-86.

159. Wakefield, D.S. *Engineering analysis of tension structures: Theory and practice*, Engineering Structures 21(8) (1999), 680-690.

160. Wan-lin, D. and Xiao-qing, H. *A study of the post-buckling path of cylindrically curved panels of laminated composite materials during loading and unloading*, Applied Mathematics and Mechanics 11(7) (1990), 651-657.

161. Webb, S.E. and Dowling, P.J. *Large-deflection elasto-plastic behavior of discretely stiffened plates*, Institute of Civil Engineers. Part 1 - Design & Construction, ICE proceedings Journal, USA 69(2) (1980), 375-401.

162. Wu, X.S. and Deng, H. *Form finding of slack cable-bar assembly by Dynamic Relaxation method*, Jisuan Lixue Xuebao/Chinese Journal of Computational Mechanics 25(2) (2008), 229-236.

163. Xiao-qing, H. and Hong, Z. *Influence of compression-bending coupling on the stability behavior of anisotropic laminated panels*, Applied Mathematics and Mechanics 20(1) (1990), 18-26.

164. Xiao-qing, H. and Wan-lin, D. *Effects of transverse shear on the nonlinear bending of rectangular plates laminated of bimodular composite materials*, Applied Mathematics and Mechanics 8(1) (1987), 81-88.

165. Ye, J.H. and Zhou, S.L. *The modified Dynamic Relaxation method for form-finding of membrane structures*, Gongcheng Lixue/Engineering Mechanics 25(12) (2008), 194-201.

166. Zhang, H. and Shan, J. *Form-finding of prestressed cable and membrane structures by Dynamic Relaxation*, Gongcheng Lixue/Engineering Mechanics 19(2) (2002), 41-44.

167. Zhang, L., Maurin, B. and Motro, R. *Form-finding of nonregular tensegrity systems*, Journal of Structural Engineering 132(9) (2006), 1435-1440.

168. Zhang, L.C. *Numerical analysis of the shape effect on the tensile strength evaluation of axisymmetric specimens*, International Journal of Numerical Analysis Methods in Geomechanics 15(4) (1991), 241-250.

169. Zhang, L.C., Yu, T.X. and Wang, R. *A new approach of predicting bifurcation points of elastic-plastic buckling of plates and shells*, Acta Mechanica Sinica 5(2) (1989-a), 145-151.

170. Zhang, L.C., Yu, T.X. and Wang, R. *Investigation of sheet metal forming by bending-Part I. Axisymmetric elastic-plastic bending of circular sheets pressed by cylindrical punches*, International Journal of Mechanical Sciences 31(4) (1989-b), 285-300.

171. Zhao, C.B., Peng, S.L., Liu, L.M., Hobbs, B.E. and Ord, A. *Effective loading algorithm associated with explicit Dynamic Relaxation method for simulating static problems*, Journal of Central South University of Technology 16(1) (2009), 125-130.

172. Zienkiewicz, O.C. and Lohner, R. *Accelerated Relaxation or direct solution future prospects for FEM*, International Journal of Numerical Methods Engineering 21(1) (1985), 1-11.

مروری بر روش های رهایی پویا در سازه های مکانیکی
بخش دوم : کاربردها

محمد رضایی پژند[1] ، *جواد علامتیان*[2] و *حسینه رضایی*[3]

[1] دانشگاه فردوسی مشهد، دانشکده مهندسی، گروه مهندسی عمران
[2] دانشگاه آزاد اسلامی، واحد مشهد، گروه مهندسی عمران
[3] دانشگاه فردوسی مشهد، دانشکده مهندسی، گروه مهندسی عمران

دریافت مقاله ۵ آذر ۱۳۹۵، دریافت مقاله اصلاح شده ۲۲ اسفند ۱۳۹۵، پذیرش مقاله ۳ اردیبهشت ۱۳۹۶

چکیده : در این مقاله، پیشرفت های مهم در کاربردهای روش های رهایی پویا طی شصت سال گذشته در مهندسی سازه خلاصه می شوند. تاکید بر روی صفحه ها، شکل یابی، سازه های ریسمانی، واکاوی پویا و دیگرکاربردها خواهد بود. همچنین، نقش روش های رهایی پویا درتحلیل خطی و ناخطی آشکار می گردد. از سوی دیگر، این پژوهش به شرح شیوه های حل مساله های ایستایی، پویایی و پایداری می پردازد. به طور خلاصه، اثر این فن ها درتحلیل ماده های همگن، مرکب، سه سانگرد و لایه ای بررسی می گردند. افزون براین ها، پیرامون کاربردهای روش های رهایی پویا، تحلیل های کمانشی و پیشنهاد پژوهش های آیندگان سخن به میان می آید.

کلمات کلیدی : روش های رهایی پویا؛ کاربرد؛ مرور؛ حل کننده؛ فن های تکراری.