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A limited resource assignment problem with shortage in the fire department

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Abstract

Assigning available resources to fire stations is a main task of fire department's administrator in a city. The importance of this problem increases when the number of available resources are inadequate. In this situation, the goal is to assign the limited available resources to fire stations such that the associated penalties of the shortages are minimized. Here, we first give a mathematical approach to consider some penalties for the shortage. Next, we give an integer program to minimize the sum of associated penalties. The proposed model can be used in many other problems arisen from health services, emergency management, and so on. We also propose a heuristic to efficiently solve the problem in a reasonable time. Our proposed heuristic has two phases. In the first phase, using a greedy approach, our proposed heuristic constructs a proper feasible solution. Next, in the second phase, we propose a local search to improve the quality of the solution constructed in the first phase. To show the efficiency of our proposed heuristic, we compare our proposed heuristic with CPLEX based on the running time and the quality of obtained solutions on two groups of problems (real-word problems and randomly generated problems). The numerical results show that on the 80% of benchmark problems, the obtained solution is the same as CPLEX's solutions. Also, the running time of our proposed algorithm is almost 10 times better than CPLEX's running time, in average.

Keywords: Resource assignment problem; Fire stations; Shortage; Integer programming; Heuristics.

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1 Introduction

There are many decision making problems for a fire department's administrator or even for a fire district's administrator (see [5,9,13]; also, [1,3,7,11,12]). One of these decision problems is to assign available resources (e.g., equipments or fire engines etc.) to fire stations (see [12], page 524, to find some references). In the real world situations, it is possible that there are not sufficient resources to satisfy all the requirements of each fire station. Thus, some of fire stations may lack some resources to extinguish or to service the fire accidents or events in their regions (other nearly problems in emergency management are studied by researchers, e.g., [2, 6, 8]). To decrease the influences of the shortage for a type of resource, some penalties are considered, based on its importance in each fire station. In a fire station, to compute the importance of a required resource, we estimate the possibility of the occurring of each event in the fire station's region. Next, based on the predefined impact of the events and the estimated possibility for arising of the events, the corresponding penalties are considered. In addition, when fire incidents occur or rescue services are needed, the responsible fire station usually requests additional resources or even firefighters from its neighboring fire stations (neighboring services). However, the aim of the problem is to assign the available resources to fire stations such that the sum of the corresponding penalties is minimized.

Here, we give an integer linear formulation to model the problem. We also propose a heuristic to efficiently solve the problem. We finally compare the performance of our algorithms with ILOG OPL(CPLEX solver)'s result on some randomly generated test problem and some real world problems.

The remainder of the paper is organized as follows: In section 2, we give an integer program to model the problem. The proposed algorithm is presented in section 3. In section 4, we show the efficiency of our proposed algorithm in comparison with ILOG OPL on some randomly generated test problems and some real world problems. We conclude in section 5.

2 Mathematical model

We here consider all resources are independent. It is obvious that when two or more types of resources are dependent, one can consider them as a package. Therefore, we can assume the shortage exists for a resource. It is obvious that the model and related solution approach can be used as a procedure within a successive approach for solving the problem with shortage in two or more types of resources. As mentioned in section 1, before providing the integer program, we must give some necessary estimation or computation to determine the corresponding penalty of the recourse's shortage in a fire station. Table 1 presents some parameters or given data of the problem.

Table 1: Parameters or given data

T:	the availability of the resource.
q:	the number of events.
n:	the number of fire stations.
f_k:	the required number of the resource to service the event k.
p_{kj}:	the frequency of the event k in the region of station j.
pr_{kj}:	the possibility for occurring the event k in the region of station j.
w_k:	the importance (weight) of the event k.
$N(j)$:	the set of the fire station j's neighboring fire stations.
β_j:	the importance (weight) of the fire station j.

Note 1. In Table 1, $pr_{.,.}$, $w_.$, and $\beta_.$ are in $[0,1]$.

Note that p_{kj} is obtained previously. In the other words, we can consider the frequency of the event k in region of the fire station j for example for a period of ten years. But, pr_{kj} is given by managers, and it shows the possibility of occurring the event k in the region of fire station j. To more clarify, we give an example. Consider, a school is recently founded in the region of the fire station j, and suppose there was not any school in this region. Therefore, some events have rarely occurred there. But, the possibility of rare occurring or never occurring events is now raised. Thus pr_{kj} and p_{kj} can be completely different. When all the situations are the same, β_j imposes the manager's decision to assign the resource to the fire station j. As an example, consider there are two fire stations with the same conditions, need a type of equipment. Also, consider one of them is in the center of the city, and the other is on the outskirts. Suppose that there is one equipment. In this situation, it is usually preferred to assign this equipment to the fire station that is in the center of the city. Therefore, the manager can consider the higher weight to the fire station that is in the center of the city to impose her/his preferences in the model.

To compute the associated penalty, we must do some necessary computations. We consider the relative frequency for the event k in the region of the fire station j as follows:

$$u_{kj} = \frac{p_{kj}}{\sum_{j=1}^{n} p_{kj}}. \tag{1}$$

Remark 1. Note that one can consider u_{kj} introduced in (1) as an estimation of the probability of occurring the event k in the region of the fire station j.

Definition 1. Let

$$v_{kj} = \max\{u_{kj}, pr_{kj}\}, \tag{2}$$

where pr_{kj} is given by the manager and u_{kj} is introduced in (1). Now, we define $v_{kj}w_k$ as the impact of the event k on the station j, for $j = 1, \ldots, n$ and $k = 1, \ldots, q$.

Note that we could not find any definition for v_{kj} in the literature since this model was proposed for a real problem that occurred in Mashhad. We did not have no more information in this real-world situation. Therefore, using experts in Fire Department of Mashhad, we proposed this formula. In this formula, we consider historical data p_{kj} and experts consideration pr_{kj}. Thus, $w_k v_{kj}$ shows how important the event k in the region of the station j.

Definition 2. We denote the possible events in the station j by Q_j defined as follows:

$$Q_j = \{k | f_k > 0, v_{kj} > 0\}. \tag{3}$$

where v_{kj} is defined in (2) and f_k is defined in Table 1.
We now introduce the decision variables in Table 2.

Table 2: Decision variables

x_j:	shows the number of the resources that are assigned to station j.
y_{jk}:	shows the shortage in the station j to service the event k.
h_{jk}:	shows the shortage in the station j to service the event k with considering the neighboring services.
d_{jk}:	if station j can not service the event k, takes 1, otherwise takes 0.

2.1 The objective function

The objective of the problem is to minimize the sum of the associated penalties. We here consider three penalties for servicing the event k in the station j based on the lack of ability for providing responsible service, the number of shortages and the number of shortages with considering neighboring services. Therefore, we consider the following penalty for the station j:

$$\sum_{k \in Q_j} (d_{jk} + y_{jk} + h_{jk})\, w_k v_{kj}, \tag{4}$$

where Q_j and v_{kj} are defined in (3) and (2), respectively, d_{jk}, y_{jk}, and h_{jk} are defined in Table 2, and w_k is defined in Table 1. To explain (4), we give two illustrative examples.

Illustrative Example 1. Consider there are five fire stations and only one resource exists. Also, the structure of the neighboring fire stations is given in Figure 1. Suppose that each of the stations 1, 3, 4, and 5 need one unit

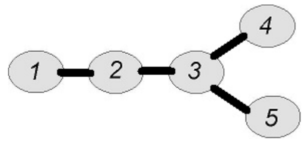

Figure 1: the graph of the neighborhoods in the illustrative Example 1.

of the resource and all situation are in the same. From Figure 1, the station 3 is the best candidate, because, the station 3 while servicing the occurred events in its own region, can participate to service the occurred events in the stations 4 and 5's regions. One can easily obtain the same result by using the sum of the penalties defined in (4) for each fire station.

Illustrative Example 2. Consider there are two fire stations and three possible events. Also, each fire station is without a neighbor (i.e. $N(1) = N(2) = \emptyset$). The other data are given as follows (see tables 1–2 and equations (2) and (3)):

$$T = 8, \quad Q_1 = \{2, 3\}, \quad Q_2 = \{1, 2, 3\},$$
$$f = [4,\ 6,\ 2], \quad w = \frac{1}{0.7}[0.2,\ 0.3,\ 0.7],$$
$$v = \begin{bmatrix} 0.2 \; 0.9 \; 0.7 \\ 0.4 \; 0.9 \; 0.2 \end{bmatrix}$$

Now, we give the motivation for considering d_{kj} in our proposed model. If d_{kj} is removed from (4), we have the following penalty for the station j:

$$\sum_{k \in Q_j} (y_{jk} + h_{jk})\, w_k v_{kj}. \tag{5}$$

Since $N(1) = N(2) = \emptyset$, (5) can be considered as follows:

$$\sum_{k \in Q_j} (y_{jk})\, w_k v_{kj}. \tag{6}$$

Now, consider two feasible solutions, **(a):** $x_1 = x_2 = 4$, **(b):** $x_1 = 2$ and $x_2 = 6$. For **(a)**, we have

$$y_{12} = 2,\ y_{13} = 0,\ y_{21} = 0,\ y_{22} = 2,\ y_{23} = 0,$$

and based on (6), the sum of penalties is $\dfrac{2 \times 0.9 \times 0.3 + 2 \times 0.9 \times 0.3}{0.7} = \dfrac{1.08}{0.7}$. For **(b)**, we have

$$y_{12} = 4,\ y_{13} = 0,\ y_{21} = 0,\ y_{22} = 0,\ y_{23} = 0,$$

and based on (6), the sum of penalties is $\frac{4 \times 0.9 \times 0.3}{0.7} = \frac{1.08}{0.7}$. Thus, when we do not consider d_{kj} in the model, there is no difference between **(a)** and **(b)**. But, in **(b)**, only the event 2 in the station 1 is not covered. Whereas in **(a)**, the event 2 in stations 1 and 2 is not covered. Now, if we consider (4) as the associated penalty, then **(b)** is preferred to **(a)**.

2.2 The model

Now, the proposed integer program is given as follows.

$$\min\ z = \sum_{j=1}^{n} \beta_j \left\{ \sum_{k \in Q_j} (d_{jk} + y_{jk} + h_{jk})\, w_k v_{kj} \right\} \tag{7}$$

s.t.

$$\sum_{j=1}^{n} x_j \leq T. \tag{8}$$

$$y_{jk} \geq f_k - x_j \qquad \forall j \in \{1, \ldots, n\}\ k \in Q_j. \tag{9}$$

$$h_{jk} \geq f_k - x_j - \sum_{\bar{j} \in N(j)} x_{\bar{j}} \qquad \forall j \in \{1, \ldots, n\}\ k \in Q_j. \tag{10}$$

$$d_{jk} \leq y_{jk} \qquad \forall j \in \{1, \ldots, n\}\ k \in Q_j. \tag{11}$$

$$d_{jk} \geq \frac{y_{jk}}{M_f}, \qquad \forall j \in \{1, \ldots, n\}\ k \in Q_j, \text{where } M_f = \max_{k=1,\ldots,q} f_k. \tag{12}$$

$$y_{jk},\ h_{jk} \in \mathbb{N} \cup \{0\}, \text{and } d_{jk} \in \{0, 1\} \qquad \forall j \in \{1, \ldots, n\}\ k \in Q_j.$$

$$x_j \in \mathbb{N} \cup \{0\} \qquad \forall j \in \{1, \ldots, n\}.$$

The constraint (8) shows that the number of assigned resources must be less than or equal to T. From Table 2, y_{kj} shows the shortage for servicing the event k in the station j. Thus, if we assign x_j unit of the resource to station j, then y_{kj} is $\max\{0, f_k - x_j\}$. Based on constraints (9), if $x_j \geq f_k$, then the corresponding constraint is redundant. Since the objective function is minimization, then y_{kj} takes zero. In the other case, if $x_j < f_k$, then the corresponding constraint ensures that $y_j \geq f_k - x_j > 0$. Similar to above, since the objective function is minimization, then y_{kj} takes $f_k - x_j$. Therefore, constraints (9) ensure that y_{kj} is $\max\{0, f_k - x_j\}$. Similar to constraints (9), constraints (10) ensure that $h_{jk} = \max\{0, f_k - x_j - \sum_{\bar{j} \in N(j)} x_{\bar{j}}\}$. Together constraints (11) and (12) guarantee, if y_{kj} is zero, then d_{kj} is zero and if $y_{kj} > 0$, then d_{kj} is 1.

Algorithm 1 Greedy phase

Step 1 {Initialization} Let $x^* = (0, \dots, 0)^T_{n\times 1}$, and let $z^* = z(x^*)$.

Step 2 {Main loop}
For $t = 1$ to T do:

- Let $z_* = +\infty$.
  For $j = 1$ to n do:
  - Let $\bar{x} = x^*$, $\bar{x_j} = \bar{x_j} + 1$ and $\bar{z} = z(\bar{x})$.
  - **If** $\bar{z} < z_*$, **then** let $z_* = \bar{z}$ and $\bar{j} = j$.

  endfor
- Let $x^*_{\bar{j}} = x^*_{\bar{j}} + 1$, and let $z^* = z_*$. **If** $z^* = 0$, then stop, the optimal solution is x^*.

endfor

Step 3 Return x^* as an initial solution.

3 A heuristic algorithm

In this section, we propose a heuristic algorithm to solve the problem. Our proposed algorithm has two phases. In the first phase, we propose a greedy procedure to construct a feasible solution (the greedy phase). Then, in the second phase, we apply a neighborhood search on the constructed solution (the improvement phase).

3.1 Greedy phase

In this phase, we construct a feasible solution. At first, we let $x_1 = \dots = x_n = 0$. Then, in each iteration, we compute the corresponding difference of the objective function for each fire station, when the number of the assigned resources in a fire station is increased one unit. Then, we select the station j to increase x_j, when the corresponding difference of the objective function is the best.
To evaluate $x = (x_1, \dots, x_n)^T$, let $y_{kj} = \max\{0, f_k - x_j\}$, and let $h_{jk} = \max\{0, f_k - x_j - \sum_{\bar{j}\in N(j)} x_{\bar{j}}\}$. Also, if y_{kj} is zero, then let $d_{kj} = 0$ and otherwise let $d_{kj} = 1$. Now, the corresponding objective value of x can be computed by (7). We denote the corresponding objective function value of x by $z(x)$. The greedy phase is given in Algorithm 1.

Note 2. In Algorithm 1, if we can find a solution x^* with $z(x^*) = 0$, then we have found a solution without any penalty. Therefore, we terminate the algorithm, and we do not use the phase 2.

3.2 Improvement phase

Here, we describe the neighborhood search procedure. We use a simple local search to improve the quality of the solution constructed by Algorithm 1. We first give some definitions.

Definition 3. We denote the maximum number of requirements in the station j by b_j and define it as follows:

$$b_j = \max_{k \in Q_j} \{f_k\}, \qquad j = 1, \ldots, n, \tag{13}$$

where Q_j is defined by (3).

It is evident that if $x_j \geq b_j$, then we do not have any shortage in the station j.

Definition 4. Let

$$\Delta_j = \begin{cases} \min\{f_k - x_j | k \in Q_j, f_k > x_j\}, & x_j < b_j, \\ 0, & x_j \geq b_j, \end{cases} \tag{14}$$

where Q_j is defined by (3). Δ_j is the minimum number of the resources that if assigned to station j, an uncovered event (event that does not receive a response service) will be covered.

To explain our proposed local search, we need to define a move in the search space (see [10]). For the station j with $x_j < b_j$, we define

$$M_j = \left\{\bar{j} | x_{\bar{j}} \geq \Delta_j, \bar{j} \neq j\right\}, \tag{15}$$

where Δ_j is defined by (14). Note that, M_j is the set of stations that their current availability is more than the minimum requirement of the station j. Now, for the station j with $x_j < b_j$, a feasible solution can be obtained from the current feasible solution by $x_j = x_j + \Delta_j$ and $x_{\bar{j}} = x_{\bar{j}} - \Delta_j$, where $\bar{j} \in M_j$. We denote this move by $Mov(j, \bar{j}, \Delta_j)$.

Note 3. When a move is done, an effective procedure to compute the difference of the objective function value, can decrease the computational time. Consider x is a feasible solution and $\bar{x}$ is the obtained feasible solution by $Mov(j, \bar{j}, \Delta_j)$. Also, consider $z(x)$ and $z(\bar{x})$ are the objective function values of x and $\bar{x}$, respectively. We now define

Algorithm 2 Local search (the improvement phase).

Step 1 Let x^* is the solution constructed by Algorithm 1, and let $z^* = z(x^*)$.

Step 2 Let $S = \{j | x_j^* < b_j\}$.

Step 3 **For** $j = 1, \dots, |S|$ do:

- **3-1** Compute Δ_j by (14).
- **3-2** **For** $\bar{j} \in M_j$, defined by (15), **if** $\Delta^z_{j,\bar{j},\Delta_j} < 0$, defined by (16), then $Mov(j, \bar{j}, \Delta_j)$, update x^* and z^*, and **goto Step 2**.

Step 4 **Return** x^* as an approximate solution.

$$\Delta^z_{j,\bar{j},\Delta_j} = z(\bar{x}) - z(x), \tag{16}$$

where $z(.)$ is defined by (4). It is obvious that $\Delta^z_{j,\bar{j},\Delta_j}$ can be computed by subtracting the sum of increased penalties of station $\bar{j}$ and its neighbors from the sum of the decreased penalties of station j and its neighboring stations. Therefore, the other stations don not play any role in the computation of $\Delta^z_{j,\bar{j},\Delta_j}$; thus in the implementation, the value of $\Delta^z_{j,\bar{j},\Delta_j}$ can be computed effectively. Also, One can use an approximation of $\Delta^z_{j,\bar{j},\Delta_j}$ in the implementation (similar to [4]).

We now give our proposed local search (as the improvement phase) in Algorithm 2.

Proposition 1. *Our proposed heuristic gives a feasible solution.*

Proof. In the greedy phase (Algorithm 1), our proposed heuristic begins from zero ($x = 0$), and then in each iteration of Step 2, only one unit is added to one entry of x. Thus, the return solution is a non-negative integer vector. The second phase starts by the feasible solution given by phase 1. In each iteration of Step 3 in Algorithm 2, each move $Mov(j, \bar{j}, \Delta_j)$ is done when $x_{\bar{j}} \geq \Delta_j$ (see (15)) and $\Delta_j > 0$ defined in (14) is an integer number. Thus, after each move the solution remains feasible. □

Note 4. There are some problems such that our proposed heuristic does not give the optimal solution (e.g., MASH2 in table 3). Also, we are not able to prove that our proposed heuristic is an ε-approximation algorithm [10]. However, the numerical results show that our proposed heuristic gives very good solution in an acceptable time for real-world problems (see Table 3).

4 Numerical experiments

To show the efficiency of our proposed algorithm, we compared the running time and the quality of the arrived solution of the proposed algorithm with ILOG OPL 6.3 (CPLEX 12.1) on two groups of test problems. In the first group, we used problems given by Fire Department of Masshad (MASH problems in Table 3). In the second group, we generated some test problems randomly (RAND problems in Table 3)[1]. We implemented our algorithm in Visual Studio C++ 2010. All implementations were done on a Notebook i5 2.5GHz with 4GB of RAM when only one core was used for our proposed algorithm and all cores were used for OPL. All results are given in Table 3.

Note 5. In some test problems, OPL terminated before convergence because the memory was not enough. In these situations, we reported the truncated results of OPL.

In Table 3, Gap is computed as follows:

$$Gap = \frac{z - \overline{z}}{\overline{z}},$$

where z is the objective value obtained from our algorithm and $\overline{z}$ is either the objective value of the best integer solution found by OPL or the objective value of the best node reported by OPL.

The mean of gaps of our proposed algorithm and the best integer solution found by OPL is only 3.7e-4. Despite OPL used all cores to perform its computations and the implementation of our algorithm used only one core, the running time of our proposed algorithm is very less than OPL's running time. From table 3, it is clear that, in 63% of problems, the given solution is equal to the best integer found by CPLEX. In 17% of problems, the given solution is better than CPLEX's solution. In these problems, CPLEX was terminated because the memory was not enough. In 20% of problems, the given solution is worse than CPLEX's solution. But in these problems the maximum gap from feasible solution given by CPLEX is only $9.6e-3$ and the running time of our proposed algorithm was 20 times better than CPLEX's running time, approximately. In some problems, CPLEX is not able to find the optimal solution. In these situations, the lower bound given by CPLEX is suitable for comparing the result. From Table 3, the maximum gap from the lower bound is $1.2e-2$. Thus we can say that the maximum error of our heuristic on problems is less than $2e-2$ when the running time is almost 10 times better than CPLEX's running time. Therefore, from Table 3, we can conclude that our proposed algorithm is applicable for solving the medium scale problems in a real time.

[1] For acquiring hold of the test problems, please contact the corresponding author (rghanbari@um.ac.ir) or download at http://profsite.um.ac.ir/∼rghanbari/IJNAO.rar

Table 3: Numerical Results

			Our Algorithm		OPL			Gap	
Problem	n	q	Best	Time (sec)	Best(integer)	Best(node)	Time(sec)	Integer	Node
MASH1	34	120	1883.5649	0.16	1883.5649	1883.5649	65.52	0	0
MASH2	34	120	3100.1002	1.03	3098.9425	3074.4241	1729.29	4e-4	8.4e-4
MASH3	34	120	1731.2355	0.67	1731.2355	1731.2355	267.40	0	0
MASH4	34	120	6538.2014	2.87	6536.7751	6509.6999	1012.93	2e-4	4.4e-4
MASH5	34	120	591.8589	0.83	591.8589	591.8589	141.85	0	0
MASH6	34	120	1630.4577	0.05	1614.9958	1614.9958	5.10	9.6e-3	9.6e-3
MASH7	34	120	0	0.00	0	0	0.08	0	0
MASH8	34	120	2164.0976	1.28	2164.0976	2147.2527	1285.28	0	7.8e-4
MASH9	34	120	530.2508	0.41	530.2508	530.2508	1.53	0	0
MASH10	34	120	1841.3675	2.87	1841.3675	1791.8676	873.59	0	2.76e-2
MASH11	34	120	222.7586	1.20	222.7586	222.7586	26.21	0	0
MASH12	34	120	7034.8948	3.62	7034.8948	6976.9721	689.24	0	8.3e-3
MASH13	34	120	929.6792	0.27	929.6792	929.6792	1.23	0	0
MASH14	34	120	1815.4193	1.57	1816.1045	1756.3611	1189.90	-4e-4	3.36e-2
RAND1	5	100	89.1726	0.00	89.1726	89.1726	0.03	0	0
RAND2	30	100	4766.9702	0.08	4766.9702	4766.9702	1.89	0	0
RAND3	60	100	9055.7206	11.79	9055.7206	9042.4482	698.93	0	1.5e-3
RAND4	100	100	7181.3464	87.25	7181.5357	7123.6819	895.88	-3e-4	8.1e-3
RAND5	5	150	2492.4775	0.00	2492.4775	2492.4775	4.35	0	0
RAND6	30	150	5116.5434	1.28	5118.1958	5075.0990	850.25	-1e-4	8.2e-3
RAND7	60	150	3993.0682	16.35	3993.0682	3948.5215	784.03	0	1.1e-2
RAND8	100	150	9308.8906	66.46	9309.9365	9047.4370	45.04	-1e-4	2.9e-2
RAND9	5	200	168.5741	0.00	168.5741	168.5741	0.11	0	0
RAND10	30	200	933.2573	0.66	933.2573	933.2573	154.83	0	0
RAND11	60	200	3437.8746	15.98	3439.9514	3389.9545	706.86	-6e-4	1.4e-2
RAND12	100	200	3186.5378	131.57	3185.4674	3159.9767	762.33	3e-4	8.4e-3
RAND13	5	250	22.2134	0.00	22.2134	22.2134	0.2	0	0
RAND14	30	250	93.9136	1.36	93.9136	93.9136	9.44	0	0
RAND15	60	250	1076.3503	12.33	1074.8568	1063.2535	890.00	14e-4	1.2e-2
RAND16	100	250	2707.3912	123.80	2706.2883	2667.7602	860.47	4e-4	1.5e-2

^{t}: truncated (Out of memory).

*: optimal.

5 Conclusions and further works

Assigning available resources to fire stations is a main task of fire department's administrator in a city. The importance of this problem increases when the number of available resources are inadequate. In this situation, the goal is to assign the limited available resources to fire stations such that the associated penalties of the shortages are minimized. Here, we first gave a mathematical approach to consider some penalties for the shortage. Next, we gave an integer program to minimize the sum of associated penalties. The proposed model can be used in many other problems arisen from health services, emergency management, and so on. We also proposed a heuristic to efficiently solve the problem in a reasonable time. Our proposed heuristic had two phases. In the first phase, using a greedy approach, our proposed heuristic constructed a proper feasible solution. Next, in the second phase, we proposed a local search to improve the quality of the solution constructed in the first phase. To show the efficiency of our proposed heuristic, we compared our proposed heuristic with CPLEX based on the running time and the quality of obtained solutions on two groups of problems (real-word prob-

lems and randomly generated problems). The numerical results showed that on the 80% of benchmark problems, the obtained solution was the same as CPLEX's solutions. Also, the running time of our proposed algorithm was almost 10 times better than CPLEX's running time, in average.
Finally, areas of further research within the framework of our proposed model and heuristic include

1. The second phase of our heuristic was a local search; so we expect that if we can propose a advanced search method such as tabu search or variable neighborhood search and so on, then the quality of the obtained solution will be increased.
2. In the proposed model, we assumed that some parameters were given in a numerical form (e.g., p_{kj} and pr_{kj}). We guess that the reliability of model is increased when the value of these parameters are given by intervals or even by linguistic variable. It is evident that in this situation, some advanced robust optimization method must be used to solve the model.
3. The proposed model is suitable when we have the shortage in our existing resources, and this shortage is very far from the minimum required resource to cover an event. When the level of shortage is not high or there is not a shortage in each resource, we can give a model to cover the events when they can be occurred in parallel.

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مساله تخصیص منابع در حالت کمبود به پایگاههای آتش نشانی

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چکیده : تخصیص منابع موجود به پایگاه های آتش نشانی یکی از مهمترین وظایف سازمان اتش نشانی در یک شهر است. اهمیت این مساله هنگامی که در تامین منابع دچار کمبود هستیم، افزایش می یابد. در این وضعیت، هدف تخصیص منابع دارای کمبود به پایگاه هایی است که جریمه کمبود در کل کمینه شود. در این جا، با استفاده از یک روش تحلیلی مقادیر جریمه را برای کمبود محاسبه می کنیم. سپس، یک مدل برنامه ریزی عدد صحیح برای کمینه کردن جریمه ها ارایه می کنیم. این مدل می تواند برای بسیاری از مسایل دیگر مانند سرویس های سلامت و یا مدیریت اورژانس و غیره به کار آید. افزون براین، در این جا یک روش ابتکاری برای حل مساله در یک زمان معقول ارایه خواهیم داد. الگوریتم ابتکاری پیشنهادی دو مرحله دارد. در مرحله اول، با استفاده از یک ایده حریصانه یک جواب مناسب شدنی می سازیم. سپس در مرحله دوم، با استفاده از یک الگوریتم جستجو محلی کیفیت جواب مرحله یک را بهبود می بخشیم. برای نشان دادن کارایی الگوریتم پیشنهادی، رفتار عددی آن را با نتایج CPLEX از منظر سرعت و کیفیت جواب به رو ی دو دسته از مسایل (نمونه های واقعی و نمونه های تصادفی) مقایسه می کنیم. نتایج عددی نشان می دهد که روی ۸۰٪ مساله های آزمون جواب به دست آمده مشابه جواب CPLEX است درحالیکه به طور متوسط سرعت الگوریتم پیشنهادی ۱۰ برابر CPLEX است.

کلمات کلیدی : مساله تخصیص منابع؛ پایگاه‌های آتش نشانی؛ کمبود؛ برنامه‌ریزی عدد صحیح؛ الگوریتم ابتکاری.