

Locally Socle of $C(X)$

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Abstract

Let $C_F(X)$ be the socle of $C(X)$ and $LC_F(X) = \{f \in C(X) : \overline{S_f} = X\}$, where S_f is the union of all open subsets U in X such that $|U \setminus Z(f)| < \infty$, $LC_F(X)$ is called the locally socle of $C(X)$ and it is a z -ideal of $C(X)$ containing $C_F(X)$. We characterize spaces X for which the equality in the relation $C_F(X) \subseteq LC_F(X) \subseteq C(X)$ is hold. In fact, we show that X is an almost discrete space if and only if $LC_F(X) = C(X)$. We note that if X is an infinite space, then $C_F(X) \subsetneq C(X)$. We also observe that $|I(X)| < \infty$ if and only if $C_F(X) = LC_F(X)$. Moreover, it is shown that if $|I(X)| < \infty$, then $LC_F(X)$ is never essential in any subring of $C(X)$, while $LC_F(X)$ is an intersection of essential ideals of $C(X)$. We determine the conditions such that $LC_F(X)$ is not prime in any subring of $C(X)$ which contains the idempotent of X . We investigate the primness of $LC_F(X)$ in some subrings of $C(X)$.

Keywords: Socle, z -ideal, Almost discrete space, Locally socle, Essential ideal.

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