## Locally Socle of C(X)

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## **Abstract**

Let  $C_F(X)$  be the socle of C(X) and  $LC_F(X) = \left\{f \in C(X) : \overline{S_f} = X\right\}$ , where  $S_f$  is the union of all open subsets U in X such that  $|U \setminus Z(f)| < \infty$ ,  $LC_F(X)$  is called the locally socle of C(X) and it is a z-ideal of C(X) containing  $C_F(X)$ . We characterize spaces X for which the equality in the relation  $C_F(X) \subseteq LC_F(X) \subseteq C(X)$  is hold. In fact, we show that X is an almost discrete space if and only if  $LC_F(X) = C(X)$ . We note that if X is an infinite space, then  $C_F(X) \subsetneq C(X)$ . We also observe that  $|I(X)| < \infty$  if and only if  $C_F(X) = LC_F(X)$ . Moreover, it is shown that if  $|I(X)| < \infty$ , then  $LC_F(X)$  is never essential in any subring of C(X), while  $LC_F(X)$  is an intersection of essential ideals of C(X). We determine the conditions such that  $LC_F(X)$  is not prime in any subring of C(X) which contains the idempotent of X. We investigate the primness of  $LC_F(X)$  in some subrings of C(X).

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