



## Dynamics of value at risk: copula -VaR approach optimized with PSO meta-heuristic algorithm<sup>1</sup>

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### Introduction

Value at Risk (VaR) is a measure for estimating potential portfolio losses due to market risk (Yu et al., 2018). Despite the simple concept, measuring the VaR is a difficult statistical problem due to the normality assumption and time-varying conditional quantiles. The introduction of CAViaR model and its extent to the multivariate CAViaR approach (MCAViaR) have solved these deficiencies. The main drawback of this model is that it only considers linearly conditioned quantiles. Furthermore, there is some instability in the optimization technique, which does not always ensure convergence to the unique minimizer. To address these issues, copula models have been developed that provide a flexible non-linear multivariate representation among quantiles. Copula functions are easily able to extend the measurement of market risk to a multivariate state (Hotta, Lucas, Palaro, 2008). In this study, we present the VaR model that establishes a nonlinear relationship between univariate quantiles estimated by univariate CAViaR models. An important parameter of Copula functions is the degree of dependency between tail distributions, the incorrect estimation of which also leads to inaccurate interpretation. The portfolio dependency coefficient was calculated using optimized copula models and the PSO meta-heuristic algorithm.

According to the literature, this study aims to answer the following questions:

### The main question

- Is it possible to provide an optimized copula -VaR approach using the PSO meta-heuristic algorithm to investigate VaR dynamics?

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**Sub-questions**

1. How does the PSO meta-heuristic algorithm optimize the estimation of the copula model's dependence coefficient?
2. Is the VaR-copula model capable of considering nonlinear relationships in the analysis of VaR dynamics when compared to the CAViaR model, which is a model with linear relationship measurement?
3. Does the optimized VaR-copula model outperform the multivariate CAViaR model (MCAViaR) in terms of estimating VaR dynamics?

**Research Methods**

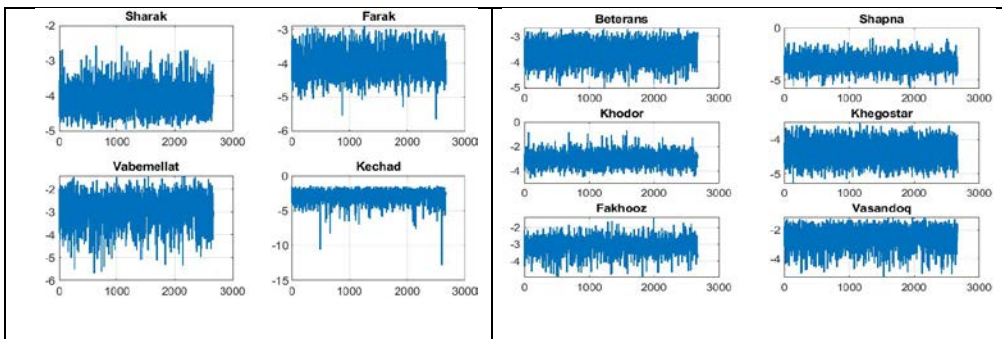
To evaluate the degree of tail dependencies between different random variables and to obtain high accuracy measurements of tail risk, the univariate CAViaR approach was extended to a multivariate CAViaR (MCAViaR) approach. Particle swarm optimization (PSO) was used to estimate the dependence parameter. Meanwhile, the MCAViaR model and the Clayton and t student-type hybrid Copula model are used to estimate VaR dynamics. The study sample consists of ten large and active stock companies, and the study period runs from April 2009 to March 2020.

The study's input variable is the daily adjusted price of ten, which was obtained from TSETMC.com. Measurement variables also include logarithmic returns, stock dependencies, beta coefficients, and time-varying quantities  $q_t(\alpha)$ .

**Research findings**

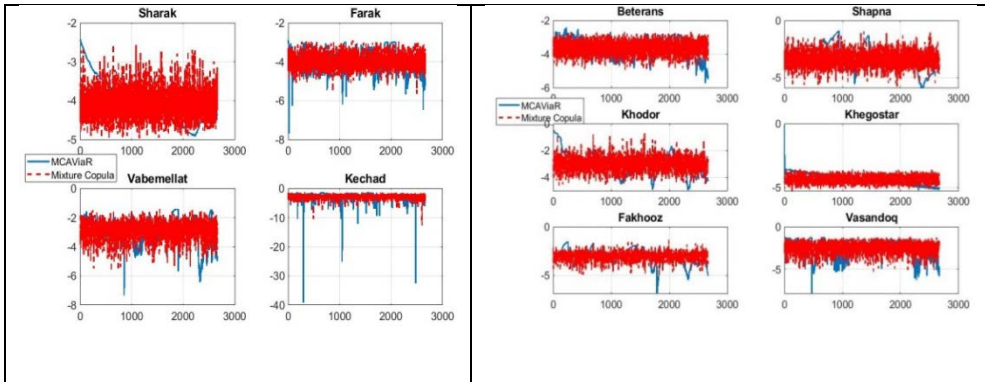
**Estimation of time-varying quantiles of combined copula model**

Figure 1 shows the time-varying quantiles of each stock using the hybrid copula model for  $\alpha=0.05$ . Despite the CAViaR model, it is clear from these diagrams that the hybrid copula method can model VaR dynamics with time-varying quantiles without making any linear assumptions. It is also clear that the dynamics of the quantiles in this model are more significant than the MCAViaR model. To compare these quantiles with the quantiles obtained from the MCAViaR model, the diagrams of both models are plotted together in Figure 2 for  $\alpha = 0.05$ . By comparing these two results, it is clear that the quantiles of the hybrid copula model have a higher frequency and thus accurately represent the dynamics.



**Figure 1.** Quantiles time series of the hybrid copula model per  $\alpha = 0.05$





**Figure 2.** Comparison of MCAViaR model quantiles and combined copulas per  $\alpha=0.05$

**Discussion and conclusion**

According to the findings of this study, meta-heuristic algorithms have a high ability to solve optimization problems. Specifically, financial optimization can be used as a powerful tool in financial studies to find globally optimal solutions. The optimal estimation of the degree of dependence between different stock pairs using the hybrid copula model and PSO algorithm and the use of the estimated parameter of each stock in estimating time-varying quantiles showed that the hybrid copula model is well able to model VaR dynamics. And this dynamism is more significant than the MCAViaR model. The results of the first sub-question showed that meta-heuristic algorithms have a high ability in optimization problems and estimate copula models with great precision through optimization. The degree of dependence of the two models of copula Clayton and t student for the pair of studied stock showed that for two different models, different values of dependence are obtained that finally the weighted combined dependence of these two models can be considered as the value of dependence between a pair of stocks. To examine the second sub-question, we first estimate the coefficients of this model. The coefficients are very small or zero, which indicates that the MCAViaR model can be separated by two independent CAViaR equations. Therefore, it seems that using the MCAViaR model to study the VaR dynamics of Iranian capital market stocks only leads to the complexity of the problem and these dynamics can be achieved by solving two CAViaR equations independently and simultaneously. Examination of time-varying quantiles of each of the studied stocks with the MCAViaR approach and hybrid copula model showed that the hybrid copula model is able to model time-varying quantiles. The results of the Kupiec post-test (Third sub-question) showed that the hybrid copula model performed better than the MCAViR model in estimating dynamic quantiles and consequently estimating VaR dynamics. This result confirms the results obtained by De Luca, et al, (2019).

**Keywords:** VaR Estimation, MCAViaR Model, Copula, Optimization, PSO Meta-Heuristic Algorithm.

## References

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