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Solution of Inverse Kinematic Problem of a 2DOF Robot Using Decomposition Method

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Abstract

The inverse kinematics problem of a two degree of freedom (2DOF) planar robot arms is solved using Adomian's decomposition method (ADM), after converting to a system of two nonlinear algebraic equations. The advantage of the method is that it gives the solutions as functions of the desired position of the end effecter and the length of the arms. The accuracy of the solutions can be increased up to desired order. The solutions haven't any singularity. The problem must be solved once for any structure and the results can be used for any path and finally, the method is fast and simple to understand.

Keywords: *Inverse Kinematics, ADM, Robots arm, 2DOF.*

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1. Introduction

A robot is a mechanical intelligent motion agent which can perform tasks on its own, or with external guidance .In practice a robot is usually an electro-mechanical machine which guides by computer and electronic programming. A manipulator is an arm like mechanism on a robot that consists of a series of segments, usually sliding or rotating which grasp and move objects with a number of degrees of freedom, under automatic or manual control by an end-effecter connected to it.

The kinematics is a fundamental part of the multidisciplinary research area of robotics. The position kinematic model relates the joints positions and the end-effecter posture. The direct kinematics analysis is the process of calculating the end-effecter posture from the joint positions given, while the inverse kinematics analysis is the process of obtaining the joint positions necessary to establish a desired end-effecter posture [11]. The method commonly used for direct kinematic modeling, is based on the Denavit–Hartenberg convention that is a consistent and concise description of the kinematic relations between the links of a kinematic chain introduced at 1955 [6]. The inverse kinematics is an ill posed problem because of the problems such as multiple or even infinite solutions, nonlinearity, singularities and uncertainties. Scientists and engineers use artificial neural network (ANN) to solve the inverse kinematics problems [7, 8, 9]. In this approach, a network will be trained to learn a desired set of joint angles positions from a given set of end-effecter positions.

The ADM was first introduced by George Adomian at the beginning of the 1980's for solving a wide range of problems whose mathematical models yield equation or system of equations involving algebraic, differential, integral and integro-diffrential terms [2, 3, 5]. This method is simple and efficient for the solution of algebraic or system of algebraic equations that gives accurate approximate solutions. Abbaui and Cherruault [1] applied ADM to solve nonlinear algebraic equation $f(x)=0$ and proved the convergence of the series solution E. Babolian et al. applied the ADM to solve a system of nonlinear equations [4]. A. R. Vahidi, et.al. used restarted ADM to improve the solutions of systems of nonlinear algebraic equations [12].

In this paper the inverse kinematic problem of a 2DOF robot arm is solved using the well-known ADM after converting to a system of nonlinear algebraic equations. The problem is introduced in section 2. Section 3 belongs to describing the method and its application to the problem. Section 4 gives the application of results in controlling the end effecter to move on three different lines in a planar surface.

2. The problem

The structure of a planar 2DOF manipulator is given in Figure 1.

In the structure given in figure 1, l_1 and l_2 are the constant lengths of the arms and the variables θ_1 and θ_2 are joint angles measured from positive horizontal axes. The arm lengths considered to be equal to reduce the structural limitations. Direct kinematics formulation of this structure is the position of end effecter, $M(p,q)$, as functions of θ_1 and θ_2. This obtains easily by the simple geometrical relations

$$p = l cos\theta_1 + l cos\theta_2, \quad (1)$$

$$q = l sin\theta_1 + l sin\theta_2. \quad (2)$$

The inverse kinematic problem is to find the values of θ_1 and θ_2 for a known position $M(p,q)$ of end effecter. Defining$x = cos\theta_1$and$y = sin\theta_2$, (1) and (2) converts to the algebraic equations

$$p = lx + l\sqrt{1-y^2} \quad (3)$$

$$q = l\sqrt{1-x^2} + ly, \quad (4)$$

or

$$\frac{p}{l} - x = \sqrt{1-y^2} \quad (5)$$

$$\frac{q}{l} - y = \sqrt{1-x^2} \quad (6)$$

After squaring and some simple calculations (5) and (6) can be rewrites in the form

$$x = \frac{p^2-l^2}{2pl} + \frac{l}{2p}(x^2+y^2), \quad (7)$$

$$y = \frac{q^2-l^2}{2ql} + \frac{l}{2q}(x^2+y^2). \quad (8)$$

In the section 3, the system of equations (7) and (8) will be solved using the ADM.

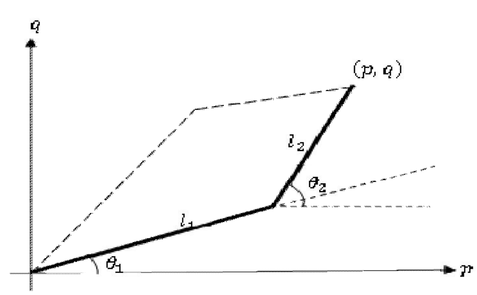

Figure 1.The structure of the planar 2DOF manipulator.

3. The method and solution

In this section, at first the ADM will be described for the general form of a system of nonlinear algebraic equations then it will be used to solve the underlying inverse kinematic problem.

3.1 The ADM for a system of nonlinear equations

Consider the following system of nonlinear equations

$f_i(x_1, x_2, \ldots, x_n) = 0, \quad i = 1,2, \ldots, n,$ (9)

where $f_i : R^n \to R$. Equation (9) can be written in the canonical form

$x_i = c_i(x_1, \ldots, x_n) + N_i(x_1, \ldots, x_n),$
$i = 1,2, \ldots, n,$ (10)

where c_i's are constants and N_i's are nonlinear functions of their arguments generally. The standard ADM [9] uses the solution x_i in terms of the series

$x_i = \sum_{j=0}^{\infty} x_{i,j} \qquad i = 1,2, \ldots, n.$ (11)

The nonlinear functions, N_i's in (10) are expressed in terms of an infinite series called Adomian polynomials[12] as

$N_i(x_1, \ldots, x_n) = \sum_{j=0}^{\infty} A_{i,j} \quad i = 1,2, \ldots, n$ (12)

$A_{i,j}$'s given in (12) depend upon

$x_{1,0}, x_{1,1}, \ldots, x_{1,j}, x_{2,1}, \ldots, x_{2,j}, \ldots, x_{n,1}, \ldots, x_{n,j}$.

In view of the equation (11) and (12),

$N_i\left(\sum_{j=0}^{\infty} x_{1,j}\lambda^j, \ldots, \sum_{j=0}^{\infty} x_{n,j}\lambda^j\right) =$
$\sum_{j=0}^{\infty} A_{i,j}, \qquad i = 1,2, \ldots, n,$ (13)

from which we obtain

$$A_{i,j} = \frac{1}{j!}\left[\frac{d^j}{d\lambda^j} N_i\left(\sum_{j=0}^{\infty} x_{1,j}\lambda^j, \ldots, \sum_{j=0}^{\infty} x_{n,j}\lambda^j\right)\right]_{\lambda=0},$$
$i = 1,2, \ldots, n.$ (14)

Where λ is the parameter introduced for convenience. Hence the equation (10) can be written as

$\sum_{j=0}^{\infty} x_{i,j} = c_i + \sum_{j=0}^{\infty} A_{i,j}, \ i = 1,2, \ldots, n.$ (15)

The ADM defines the components $x_{i,j}$, $j \geq 0$, by the following recursive relations

$x_{i,o} = c_i,$ (16)

$x_{i,j+1} = A_{i,j}, \ \ i = 1,2, \ldots, n, \ \ j = 0,1,2, \ldots.$ (17)

Finally the approximate solution x_i can be approximated by the truncated series

$\varphi_{i,k} = \sum_{j=0}^{k} x_{i,j},$ (18)

that

$\lim_{k\to\infty} \varphi_{i,k} = x_i, \qquad i = 1,2, \ldots, n,$ (19

gives a converge solution which the convergence is proved[13].

3.2 Application of ADM to inverse kinematics problem

In order to solve the system of nonlinear algebraic equations(7) and (8) using ADM, let $x = \sum_{i=0}^{\infty} x_i$, $y = \sum_{i=0}^{\infty} y_i$, $x^2 + y^2 = \sum_{i=0}^{\infty} A_i$ to obtain

$$\sum_{i=0}^{\infty} x_i = \frac{p^2 - l^2}{2pl} - \sum_{i=0}^{\infty} A_i, \quad (20)$$

$$\sum_{i=0}^{\infty} y_i = \frac{q^2 - l^2}{2ql} - \sum_{i=0}^{\infty} A_i. \quad (21)$$

The equations (20) and (21) give the recursive relations

$$x_0 = \frac{p^2 - l^2}{2pl}, \quad (22)$$

$$y_0 = \frac{q^2 - l^2}{2ql}, \quad (23)$$

$$x_{i+1} = A_i, \quad (24)$$

$$y_{i+1} = A_i. \quad (25)$$

Here x, y are used rather than x_1, x_2 adopted in section 2. Choosing $k = 1$ in (18) the approximate solution $x \approx \varphi_{1,1}$ and $y \approx \varphi_{2,1}$ for $l = 1$ will be obtained as

$$x \approx \varphi_{1,1} = \frac{-q^4 + p^6 q^2(-1 + 12q^2) - p^2 q^2 (2 - 8q^2 + q^4) + p^4(-1 + 8q^2 - 26q^4 + 4q^6)}{16 p^5 q^4} \quad (26)$$

$$y \approx \varphi_{2,1} = \frac{-q^4 + p^6 q^2(-1 + 4q^2) - p^2 q^2 (2 - 8q^2 + q^4) + p^4(-1 + 8q^2 - 26q^4 + 12q^6)}{16 p^4 q^5} \quad (27)$$

Using (26) and (27) in (3) and (4), the approximate solutions of the direct problem (p_c, q_c) obtains as follows.

$$p_c(p,q) \approx l\varphi_{1,1} + l\sqrt{1 - \varphi_{2,1}{}^2} \quad (28)$$

$$q_c(p,q) \approx l\sqrt{1 - \varphi_{1,1}{}^2} + l\varphi_{2,1} \quad (29)$$

Obviously, the point (p_c, q_c) includes the approximate horizontal and vertical coordinates of the end effecter as functions of desired coordinates (p, q). The procedure can be repeated to obtain more accurate approximations up to desired accuracy.

4. Results and discussions

The end effecter coordinates obtained as the functions of desired coordinates in the section 3 using the ADM. The results for two terms of the approximate solution (18) are illustrated as (28) and (29). Here we examine these functions to obtain some desired paths of the robot end effecter given in the figure 1. Three paths are considered. At first we choose the horizontal path $q = 1$ while q is in [0, 1.8]. Some points of this line, by the increment 0.1, are calculated using (28) and (29) and plotted in figure 2.

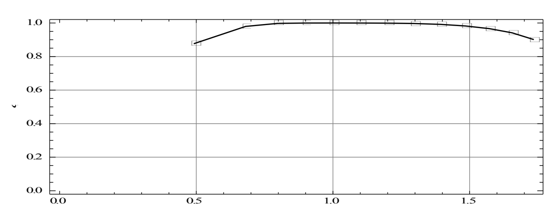

Figure 2.The calculated path of robot end effecter for desired horizontal line by two terms of (18).

There are some important points in the figure 1 that must be considered.

- Due to the structure of robot arms, as shown in the figure 1, for the line $q = 1$there is an upper limit for the value of horizontal coordinate, i.e. $p_{max} = \sqrt{3}$.
- Due to the square root in the (28) and (29), there aren't real solutions for all points in the surface. We just show the points with real p_c andq_c.

As the second path, we choose the vertical line $p = 1$. The figure 3 shows some points of this line.

It can be seen in the figure 3 that as in horizontal line given in the figure 2, there is a lower limit due to the approximate calculations and an upper limit arising from the structure of the robot arm given in the figure 1.

As the last example we calculate the path of an inclined line with the slope of one, i.e. $p = q$. Some points of this line are given in figure 4.

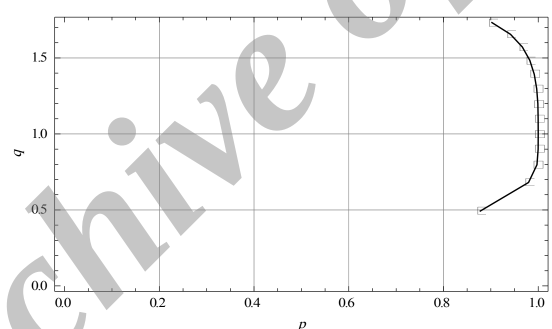

Figure 3.The calculated path of robot end effecter for desired vertical line by two terms of (18).

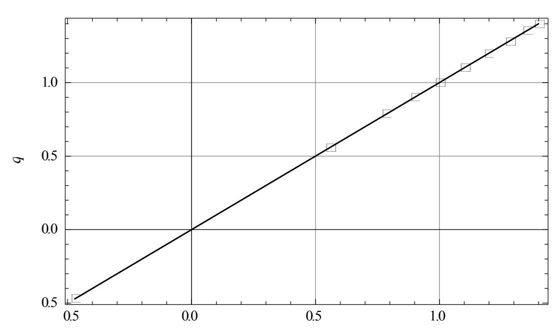

Figure 4.The calculated path of robot end effecter for inclined line by two terms of (18).

The structure of the robot arm for the given line, $p = q$, imposes the upper limit of $\sqrt{2}$ on both p and q. This limit can be seen in the figure 4.

More accurate solutions can be obtained by calculating more terms in ADM. The figures 5-7 are plotted with the approximate solutions p_c and q_c considering $k = 6$ in the series solution (18) for three predefined paths.

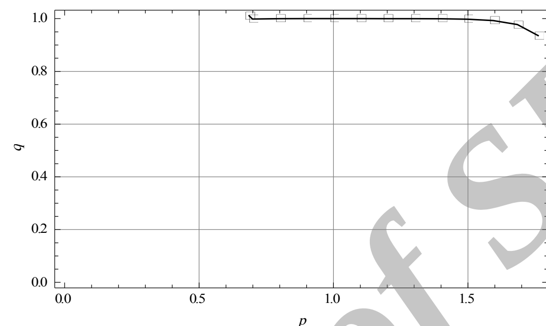

Figure 5.The calculated path of robot end effecter for horizontal line by seven terms of (18).

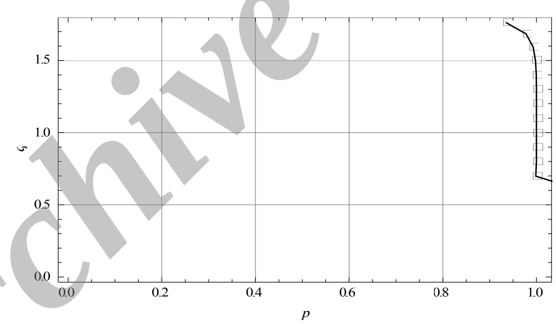

Figure 6.The calculated path of robot end effecter for vertical line by seven terms of (18).

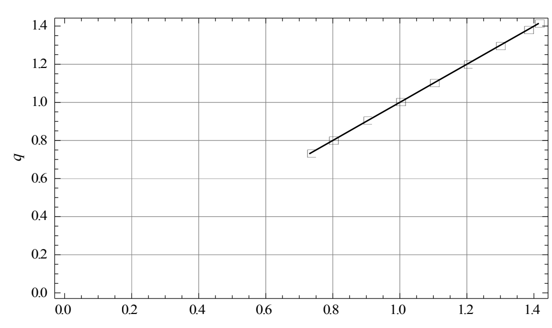

Figure 7.The calculated path of robot end effecter for inclined line by seven terms of (18).

As expected, the lines($q = 1$, $p = 1$ & $p = q$)will be obtained approximately. Comparing to the figures 2-4, it can be seen that increasing the number of calculated terms of (18), increases the accuracy of the results.

5. Conclusions

The studies show that the ADM can be used to solve the inverse kinematics problem of a planar 2DOF robot manipulator which can be used to control the robot end effecter on desired line. Increasing the number of calculated terms of the approximate solutions obtained by the ADM, gives more accurate paths. The solutions exactly show the farthest possible position of the end effecter limited by the lengths of the arms. The solutions have and lower limit for any desired line, due to the existence of square root in the direct kinematic equation. The study revealed some advantages of the ADM to the inverse kinematic problems.(a) The solutions can be obtained as functions of the desired position of the end effecter and the length of the arms. (b) The accuracy of the solutions can be increased up to desired order. (c) The solutions haven't any singularity. (d) The problem must be solved once for any structure and the results can be used for any path at any time. (e) The method is fast and simple to understand. We propose more studies in this field using the ADM, especially for more complicated structures with higher degrees of freedom.

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حل مساله‌ی سینماتیک معکوس بازوی یک روبات با دو درجه‌ی آزادی با استفاده از روش تجزیه‌ی آدومین

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چکیده

مقدمه: مساله‌ی سینماتیک معکوس بازوی مسطح یک روبات با دو درجه‌ی آزادی پس از تبدیل به دستگاهی متشکل از دو معادله‌ی جبری غیرخطی با استفاده از روش تجزیه‌ی آدومین حل شده است. برتری روش به‌کاررفته این است که پاسخ را به‌صورت تابعی از موقعیت دلخواه دست روبات و طول بازوهای آن به‌دست می‌دهد. دقت روش به‌کاررفته تا مرتبه‌ی دلخواه قابل افزایش است. روش حل منجر به هیچ تکینگی نمی‌شود. مساله یک باربرای هر ساختار حل می‌شود و سپس برای هر مسیر دلخواه قابل استفاده است. روش مورد بحث سریع و درک آن آسان است.

هدف: در این مطالعه هدف استفاده از روش تجزیه‌ی آدومین برای حل مساله‌ی سینماتیک معکوس بازوی روبات با دو درجه آزادی است.

روش بررسی: تبدیل مساله‌ی سینماتیک معکوس بازوی روبات با دو درجه آزادی به یک دستگاه معادله‌ی جبری و سپس حل آن با روش تجزیه‌ی آدومین.

نتایج: نشان داده شد که روش به کار رفته نتایج دقیق برای کنترل دست روبات روی مسیر دلخواه می دهد.

نتیجه گیری: این مقاله با استفاده از روش تجزیه‌ی آدومین به حل مساله‌ی سینماتیک معکوس بازوی روبات با دو درجه آزادی پرداخته است. نتایج نشان می‌دهند که این روش برای مساله‌ی روبات، روشی قابل کنترل و سریع است.

کلید واژه: سینماتیک معکوس، بازوی روبات، دو درجه‌ی آزادی، روش تجزیه‌ی آدومین.

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