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An integrated model for solving the multiple criteria ABC inventory classification problem

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Abstract

In this paper, we present an integrated version of the Ng model and Zhou and Fan model [W. L. Ng, A simple classifier for multiple criteria ABC analysis, European Journal of Operation Research, 177 (2007) 344-353; P. Zhou & L. Fan, A note on multi-criteria ABC inventory classification using weighted linear optimization, European Journal of Operation Research, 182 (2007) 1488-1491]. The model that Ng [1] offered, hereafter called the Ng-model, in spite of its advantages may lead to a situation in which the weights of some criteria in relation to an item would not play any role in determining overall score that item. Also, the scale transformation that he applied for transforming the measures of items under criteria into interval 0-1 is not suitable for the small-scale measures. On the other hand, for the R inventory item, the Zhou and Fan model [2], hereafter called the ZF-model, should be solved through a linear optimizer 2R times in which an inventory manager might has no any background in regard with optimizer. Furthermore, when number of items is large, the computing time would increase. Therefore, in order to remove drawbacks of both the approaches, an integrated model is presented in which objective functions are the same ZF-method but its constraints is similar to Ng model. At last, results obtained from applying the proposed model in an illustrative example are compared with Ng and ZF-models.

Keywords: *Multi-criteria ABC inventory classification*

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1. Introduction

In today's competition world, companies are attempting to gain a higher share of market. Therefore, with reduction of costs, they have to decrease the cost of goods manufactured. One of these costs is the cost of inventory items. The most usual approach for minimizing this cost is specifying the appropriate ordering policies based on the priority of each item among other items. A multi-criteria ABC inventory classification (MC-ABC) is one of these techniques where items are divided into 3 classes, namely, A (very important), B (moderately) and C (least important) and then appropriate ordering policies are selected for each item. In recent years, some papers have applied the weighted linear optimization models [1-3] for MC-ABC classification. Ng proposed one of such models in which with the scale transformation of the measures of items related to criteria into a value between 0-1 and also through a proper transformation of optimization model could classify them analytically. The scale transformation that he applied is suitable only for the large scale measures. Besides, his model might lead to a position where the weights of some criteria are taken into account zero. On the other hand, ZF offered a model in which they gained the most favorable and least favorable scores for each item and then changed them into a single score for determining the prioritization of that item. Despite advantage of this model, it is needed that for the R inventory item, via a linear optimizer, is solved $2R$ times and also processing time wdoul augment when numbers of inventory items are increased simultaneously. In order to exploit the advantages of both model and removal of weaknesses, the aim of this paper, is to offer a model in which suggested objective function would be the objective functions of ZF's method and its constraints are the same model.

2. Ng and ZF models

2.1. ZF-model

Let there are R $(r = 1,2,\dots R)$ items in warehouse which they are to be classified into classes A, B and C based on C $(c = 1,2,\dots C)$ criteria. Also, let x_{rc} and w_{rc} denote the measure and the weight of rth item against cth criterion, respectively. Furthermore, assume all the criteria are positive related to the score of the inventory items. If isn't such a case, transformations such as taking negative can be applied. The main target is that through the most favorable scores and the least favorable scores of an item and transforming those into a single score can items classify with respect to different criteria appropriately. Their model was exercised for not placing an item with high measure for an unimportant criterion as well as low measure for an important criterion as class A in Ramanathan-model [3], hereafter called the R-model. The most favorable scores for each item *i* obtain by solving iteratively the following weighted linear optimization model which it, in real, is the same R-model:

$$\begin{aligned} gI_i &= \max \sum_{c=1}^{C} w_{ic}^{\,g} x_{ic} \\ s.t \quad & \sum_{c=1}^{C} w_{ic}^{\,g} x_{rc} \le 1 \quad , \quad r = 1,\dots,R \qquad (1) \\ & w_{ic}^{\,g} \ge 0, \end{aligned}$$

Also the least favorable scores proposed by ZF-model for each item *i* are calculated as follow:

$$\begin{aligned} bI_i &= \min \sum_{c=1}^{C} w_{ic}^{\,b} x_{ic} \\ s.t \quad & \sum_{c=1}^{C} w_{ic}^{\,b} x_{rc} \ge 1 \quad , \quad r = 1,\dots,R \qquad (2) \\ & w_{ic}^{\,b} \ge 0, \end{aligned}$$

Then a single score obtain by following composite index:

$$nI_r(\lambda) = \lambda . \frac{gI_r - gI^-}{gI^* - gI^-} + (1-\lambda) . \frac{bI_r - bI^-}{bI^* - bI^-} \qquad (3)$$

where,

$gI^* = \max\{gI_r, r = 1,2,\dots R\}$,

$gI^- = \min\{gI_r, r = 1,2,\dots R\}$,

$bI^* = \max\{bI_r, r = 1,2,\dots R\}$,

$bI^- = \min\{bI_r, i = 1,2,..,R\}$, λ is a control parameter that it is equal to 0.5 in (3).

Then by sorting the composite scores $nI_r(\lambda)$'s in descending order, the items are classified based on ABC classification analysis.

2.1. Ng-model

In this model, Ng first transformed the measures of each item in respect to the all criteria via the scale transformation $\frac{x_{rc} - \min_{r=1,2,\dots,R}\{x_{rc}\}}{\max_{r=1,2,\dots,R}\{x_{rc}\} - \min_{r=1,2,\dots,R}\{x_{rc}\}}$ into a value within closed interval [0-1].

In the mentioned scale transformation, the larger measure of an item is closer to value

1. Also, he assumed the criteria are ranked in a descending order as $w_{rc} - w_{r(c+1)} \ge 0, c = 1,2,\dots(C-1)$.
2. Then he offered the below weighted linear optimization model to assign the scores of items:

$$\max \boldsymbol{S_r} = \sum_{c=1}^{C} \boldsymbol{w_{rc}}\ \boldsymbol{x_{rc}}$$
$$\boldsymbol{s.t} \quad \sum_{c=1}^{C} \boldsymbol{w_{rc}} = 1 \qquad (4)$$
$$\boldsymbol{w_{rc}} - \boldsymbol{w_{r(c+1)}} \geq 0, \quad \boldsymbol{c} = 1,2,\dots(\boldsymbol{C}-1)$$
$$\boldsymbol{w_{rc}} \geq 0, \qquad \boldsymbol{c} = 1,2,\dots,\boldsymbol{C}$$

Where first constraint is a normalization constraint and second constraint show the ranking of criteria. Ng in his paper, with a simple transformation from above problem and without it had need to be solved by a linear optimizer could gain the scores of items via 5 stages.

1. Transform the measures x_{rc} using transformation $\frac{x_{rc} - \min_{r=1,2,\dots,R}\{x_{rc}\}}{\max_{r=1,2,\dots,R}\{x_{rc}\} - \min_{r=1,2,\dots,R}\{x_{rc}\}}$ into a value in 0-1.

2. Calculate all partial averages $\frac{1}{c}\sum_{u=1}^{c} x_{ru}$, $c = 1,\dots, C$ for each item *r*.

3. Select the maximum value of partial averages as the score S_r *r*th item.

4. Sort the scores S_r 's in the descending order.

5. Group the items based on ABC analysis.

3. The proposed model

The Ng-model is simple and easy to understand. It is also very flexible so that it can easily integrate additional information from inventory managers for inventory classification. Despite its many advantages, Ng-model leads to a situation where the score of each item is independent of the weights obtained from the model.

That is, the weights do not have any role for determining the total score of each item. This may lead to a situation where an item is inappropriately classified and not reflect the real position of this inventory item.

To show how the Ng-model leaving out the weights, consider the following example.

Item	First criterion	Second criterion	Third criterion
Item 1	2	2	2

Here we have one item with three criteria. It is clear that based on the Ng-model the score of this item is $S_i = \max\{2, \frac{1}{2}(2+2), \frac{1}{3}(2+2+2)\} = 2$. On the other hand, if we solve the Ng-model for this item we have

$$\begin{aligned} \max \quad & S_i = 2w_{11} + 2w_{12} + 2w_{13} \\ s.t \quad & w_{11} + w_{12} + w_{13} = 1, \\ & w_{11} \geq w_{12} \geq 2w_{13} \geq 0. \end{aligned}$$

The optimal solution of the above model is: $w_{11}^* = 1, w_{12}^* = w_{13}^* = 0$. As we see, the weight of the second and third criterion is 0, which means that these two criteria do not have any meaning. In actual applications, to become a zero the weight of an item against a certain criterion means that we throw away the corresponding part of the obtained data.

Also the approach which Ng applied for the small scale measures is not suitable. Because of this, we use scale transformation $\frac{\max_{r=1,2....,R}\{x_{rc}\} - x_{rc}}{\max_{r=1,2....,R}\{x_{rc}\} - \min_{r=1,2....,R}\{x_{rc}\}}$ for these group of measures. Contrary to the Ng-model, in mentioned transformation, the scale of a larger measure of an item related to a certain criterion is closer to zero. Therefore, objective function at model 4, for the large scale measure would be maximizing and for the small scale measure is exchanged as minimizing. On the other hand, the model that ZF presented is needed to be solved 2R times by linear optimizer. Furthermore, although the model that has been presented by obtaining the most favourable and least favourable scores is trying to prevent the unsuitable earmarking of an item as class A. But generally, has done this without ranking of criteria's importance. Whereas, in real world, the inventory managers consider some criteria more important than the others. In proposed approach an attempt has been performed such that using ZF-approach, we can remove the points of weakness the Ng-model. In mentioned model, the objective function are the same ZF with this difference that the most favorable score's objective function for the large scale measures is maximization and for the measures of small scale is minimization and conversely, objective function of the least favorable scores for the measures of large scale is minimization and for the measures of small scale is maximization, but the constraints of both model is the same as Ng model. Therefore, MC-ABC classification problem is transformed into a multiple objective decision making (MODM) problem. In this paper, we use a new approach to solve this problem, by introducing variables w_l and w_s for the measures of large scale and the measures of small scale, respectively, which are selected as satisfaction of decision maker in relation to small and large scale objectives. We select value $w = 0.5$ for both those, by reason of similarity of their affect on objectives. Also, obviously, each maximizing and minimizing objective function can reduce to minimizing and maximizing, respectively, by multiplying that in -1. Suppose that $c = 1,\ldots, c_1$ is set the large scale criteria and $c = c_1 + 1,\ldots, C$ is set the small scale criteria. Then, objective functions of the measures of large scale and the measures of small scale related to the most favorable problem for each item r are as, respectively:

$$g'I_r = \max \sum_{c=1}^{c_1} w(w_{rc}{}^{g}(x_{rc})),$$

And

$$g''I_r = \min \sum_{c=c_1+1}^{C} w(w_{rc}{}^{g}(x_{rc})) = \max \sum_{c=c_1+1}^{C} w(w_{rc}{}^{g}(-x_{rc}))$$

By letting w_l and w_s equal to 0.5, above two-objective problem are transformed into following one objective linear optimization problem:

$$\begin{aligned} & \boldsymbol{gI_r} = 0.5\max\Big(\sum_{c=1}^{c_1} \boldsymbol{w_{rc}}^{g}\boldsymbol{x_{rc}} + \sum_{c=c_1+1}^{C} \boldsymbol{w_{rc}}^{g}(-\boldsymbol{x_{rc}})\Big), \\ & \boldsymbol{s.t} \quad \sum_{c=1}^{C} \boldsymbol{w_{rc}}^{g} = 1, \\ & \quad \boldsymbol{w_{rc}}^{g} - \boldsymbol{w_{r(c+1)}}^{g} \geq 0, \quad \boldsymbol{c} = 1,2,\ldots,(\boldsymbol{C}-1), \\ & \quad \boldsymbol{w_{rc}}^{g} \geq 0, \quad \boldsymbol{c} = 1,2,\ldots,\boldsymbol{C}, \end{aligned} \tag{5}$$

And the least favorable scores are calculated as follow:

$$\begin{aligned} & \boldsymbol{bI_r} = 0.5\ \min\Big(\sum_{c=1}^{c_1} \boldsymbol{w_{rc}}^{b}\boldsymbol{x_{rc}} + \sum_{c=c_1+1}^{C} \boldsymbol{w_{rc}}^{b}(-\boldsymbol{x_{rc}})\Big), \\ & \boldsymbol{s.t} \quad \sum_{c=1}^{C} \boldsymbol{w_{rc}}^{b} = 1, \\ & \quad \boldsymbol{w_{rc}}^{b} - \boldsymbol{w_{r(c+1)}}^{b} \geq 0, \quad \boldsymbol{c} = 1,2,\ldots,(\boldsymbol{C}-1), \\ & \quad \boldsymbol{w_{rc}}^{b} \geq 0, \quad \boldsymbol{c} = 1,2,\ldots,\boldsymbol{C}, \end{aligned} \tag{6}$$

Since achieved scores from above mentioned weighted linear optimization models place within a 0-1 scale for all items. Therefore, these are transformed into a single score using the following composite index of small scale:

$$\boldsymbol{nI_r}(\lambda) = \lambda.\frac{\boldsymbol{gI}^* - \boldsymbol{gI_r}}{\boldsymbol{gI}^* - \boldsymbol{gI}^-} + (1-\lambda).\frac{\boldsymbol{bI}^* - \boldsymbol{bI_r}}{\boldsymbol{bI}^* - \boldsymbol{bI}^-} \tag{7}$$

The above acquired score for an item with a high priority is close to zero and for an item with low priority approach to one. Therefore in our model, we should be sorted our items in the ascending order.

4. Problem transformation

The linear optimization models (5) and (6) can be solved with a linear optimizer, but by reason of that an inventory's manager might did not had any familiarity with optimization issue of MODM models, thus as similar as Ng, via a suitable transformation, an attempt has been performed so that we can calculate the scores of all item without any need to optimizer. In reminder this section we describe the transformation process. Whereas the constraints of our proposed model is the same constraints of Ng-model, therefore those are, as described in his paper, as follow:

$$\sum_{c=1}^{C} cu_{rc} = 1, \qquad r = 1,\ldots, R, \tag{8}$$

And

$$u_{rc} \geq 0, \qquad r = 1,\ldots,R \text{ and } c = 1,\ldots,C, \tag{9}$$

Where

$$u_{rc} = w_{rc} - w_{r(c+1)}, \; r = 1,\ldots, R \quad \text{And} \quad c = 1,\ldots,(C-1), \tag{10}$$

$$u_{rC} = w_{rC}, \qquad r = 1,\ldots, R \tag{11}$$

But since the objective functions in our model has been decomposed based on the criteria of small and large scales, therefore, it is needed to be corrected as below, let:

$$l_{rc} = \sum_{m=1}^{c} z_{rm}, \qquad r = 1,\ldots, R \tag{12}$$

Where z_{rm} is given as follow:

$$\begin{cases} z_{rm} = x_{rm}, & \text{if} \quad 1 \leq m \leq c_1, \\ z_{rm} = -x_{rm}, & \text{if} \quad c_1 + 1 \leq m \leq C, \end{cases} \tag{13}$$

Using Eq. (13), the score S_r of the rth item of both problem (5) and (6) is equivalent to:

$$\begin{aligned} S_r &= 0.5\left(\sum_{c=1}^{c_1} w_{rc}\, x_{rc} + \sum_{c=c_1+1}^{C} w_{rc}\,(-x_{rc})\right) \\ &= 0.5\sum_{c=1}^{C} u_{rc} l_{rc}, \quad r = 1,\ldots,R \end{aligned} \tag{14}$$

We can verify Eq. (14) by substituting u_{rc} and l_{rc} from Eqs. (10), (11), (12) and (13) into (14):

$$\begin{aligned} S_r &= 0.5\sum_{c=1}^{C} u_{rc} l_{rc} \\ &= 0.5\left(\sum_{c=1}^{C-1}\left((w_{rc} - w_{r(c+1)})\sum_{m=1}^{c} z_{rm}\right) + u_{rC}\sum_{m=1}^{C}(z_{rm})\right) \\ &= 0.5[((w_{r1} - w_{r2})x_{r1}) + ((w_{r2} - w_{r3})(x_{r1} + x_{r2})) \\ &+ ((w_{r3} - w_{r4})(x_{r1} + x_{r2} + x_{r3})) + \cdots \\ &+ ((w_{rc_1-1} - w_{rc_1})(x_{r1} + \cdots + x_{rc_1})) + \\ &((w_{rc_1} - w_{rc_1+1})(x_{r1} + \cdots + x_{rc_1} - x_{rc_1+1})) + \cdots \\ &+ ((w_{r(C-1)} - w_{rC})(x_{r1} + \cdots + x_{rc_1} - x_{rc_1+1} - x_{rc_1+2} - \cdots - x_{r(C-1)}) \\ &+ w_{rC}(x_{r1} + \cdots + x_{rc_1} - x_{rc_1+1} - x_{rc_1+2} - \cdots - x_{rC})] \end{aligned}$$

$$=0.5[(w_{r1}x_{r1}-w_{r2}x_{r1})$$
$$+(w_{r2}x_{r1}+w_{r2}x_{r2}-w_{r3}x_{r1}-w_{r3}x_{r2})$$
$$+(w_{r3}x_{r1}+w_{r3}x_{r2}+w_{r3}x_{r3}$$
$$-w_{r4}x_{r1}-w_{r4}x_{r2}-w_{r4}x_{r3})+\cdots$$
$$+(w_{rc_1-1}x_{r1}+\cdots+w_{rc_1-1}x_{rc_1}$$
$$-w_{rc_1}x_{r1}-\cdots-w_{rc_1}x_{rc_1})$$
$$+(w_{rc_1}x_{r1}+\cdots+w_{rc_1}x_{rc_1}$$
$$-w_{rc_1}x_{rc_1+1}-w_{rc_1+1}x_{r1}-\cdots-w_{rc_1+1}x_{rc_1}$$
$$+w_{rc_1+1}x_{rc_1+1})+\cdots$$
$$+(w_{r(C-1)}x_{r1}+\cdots+w_{r(C-1)}x_{rc_1}$$
$$-w_{r(C-1)}x_{rc_1+1}-w_{r(C-1)}x_{rc_1+2}-\cdots-w_{r(C-1)}x_{r(C-1)}$$
$$-w_{rC}x_{r1}-\cdots-w_{rC}x_{rc_1}$$
$$+w_{rC}x_{rc_1+1}+w_{rC}x_{rc_1+2}+\cdots+w_{rC}x_{r(C-1)})$$
$$+(w_{rC}x_{r1}+\cdots+w_{rC}x_{rc_1}-w_{rC}x_{rc_1+1}$$
$$-w_{rC}x_{rc_1+2}-\cdots-w_{rC}x_{rC})]$$
$$=0.5[w_{r1}x_{r1}+\cdots+w_{rc_1}x_{rc_1}$$
$$-w_{rc_1+1}x_{rc_1+1}-\cdots-w_{rC}x_{rC}]$$
$$=0.5(\sum_{c=1}^{c_1}w_{rc}x_{rc}+\sum_{c=c_1+1}^{C}w_{rc}(-x_{rc}))$$
$$=0.5\sum_{c=1}^{C}u_{rc}l_{rc}$$

Now, using Eq. (14), the problems (5) and (6) change as follow, respectively:

$$\begin{aligned} gI_r &= 0.5\max \sum_{c=1}^{C} u_{rc}l_{rc}, \\ s.t \quad & \sum_{c=1}^{C} cu_{rc} = 1 \\ & u_{rc} \geq 0, \quad c = 1,\dots,C, \end{aligned} \tag{15}$$

and

$$\begin{aligned} bI_r &= 0.5\min \sum_{c=1}^{C} u_{rc}l_{rc}, \\ s.t \quad & \sum_{c=1}^{C} cu_{rc} = 1 \\ & u_{rc} \geq 0, \quad c = 1,\dots,C, \end{aligned} \tag{16}$$

Since problem (15) and (16) have only one equality constraint, the optimum solution is one of decision variables $u_{rc}\geq 0$, $c=1,\dots,C$ and should be equal to $\frac{1}{c}$

Hence, the optimal solutions of objective functions (15) and (16) for rth item , $r=1,\dots,R$, is $0.5(\max_{c=1,\dots,c_1,\dots,C}(\frac{1}{c}\sum_{m=1}^{c}z_{rm}))$ and $0.5(\min_{c=1,\dots,c_1,\dots,C}(\frac{1}{c}\sum_{m=1}^{c}z_{rm}))$, respectively, where z_{rm} for $m=1,\dots,c_1,\dots,C$ is determined according to Eq. (13). In general, in the proposed model, the score each inventory item r obtains by following steps.

1. Transform the large scale measures for a certain criterion via transformation $\frac{x_{rc}-\min_{r=1,2,\dots,R}\{x_{rc}\}}{\max_{r=1,2,\dots,R}\{x_{rc}\}-\min_{r=1,2,\dots,R}\{x_{rc}\}}$ and the large scale measures using transformation $\frac{\max_{r=1,2,\dots,R}\{x_{rc}\}-x_{rc}}{\max_{r=1,2,\dots,R}\{x_{rc}\}-\min_{r=1,2,\dots,R}\{x_{rc}\}}$ into a value within 0-1.

2. Calculate all partial averages $0.5\,(\frac{1}{c}\sum_{m=1}^{c} z_{rm}),\ c=1,\dots,C$ for each item *r*, by considering Eq. 13.
3. Select the maximum value between these partial averages as the most favourable score and minimum value between these partial averages as the least favourable score for each item *r*.
4. Transform these s cores into a single score using Eq. 7. Consider this score as final score S_r for item *r*. The score S_r for an item with the higher priority is closer to zero.
5. Sort the scores S_r's in the ascending order
6. Group the items based on ABC analysis.

5. Illustrative example

In order to compare the proposed model with results of ZF and Ng models, we apply the data in [1-4]. All 47 inventory items under three criteria the annual dollar usage, average unit cost and lead time are shown in Table 1. We assume the descending order of criteria is as described by Ng. Also the converted measures into interval 0-1 and partial averages have been presented in Table 1. The most favourable scores, the least favourable scores for each of items and their composite scores generated by our model have been presented in Table 2. The classification results using our model, ZF and Ng models have been compared together with in this Table as well. To this end, we remain the number of items in classes A, B and C according to the same number of items in traditional ABC (TABC) method, i.e. 10 items for class A, 14 items for class B and 23 items for class C.

When comparison with the TABC, only 32 items of the suggested model remain in the same classes. In other words, by implementing the suggested model, 8 out of 10 of class A in the TABC classification reclassify in the same class, and other two items are classified in class B. Out of 14 in class B, 7 remain in the same class B, 1 is transferred to class A and 6 to class C. Moreover, out of 23 items of class C in the TABC classification, 17 remain in the same class C, 1 are transferred to class A and 5 to class B.

Table 1. The measures of items, transformed values and partial averages against criteria

Item number	Annual dollar usage ($)	Average unit cost ($)	Lead time (day)	Annual dollar usage (Transformed)	Average unit cost (Transformed)	Lead time (Transformed)	Partial average		
							1	2	3
1	5840.64	49.92	2	1	0.2187	-0.8333	1	0.6953	0.1284
2	5670	210	5	0.9707	1	-0.3333	0.9707	0.9853	0.5458
3	5037.12	23.76	4	0.8619	0.0909	-0.5000	0.8619	0.4764	0.1539
4	4769.56	27.73	1	0.8159	0.1104	-1.0000	0.8159	0.4631	-0.0245
5	3478.8	57.98	3	0.5939	0.2581	-0.6666	0.5939	0.4260	0.0618
6	2936.67	31.24	3	0.5007	0.1275	-0.6666	0.5007	0.3141	-0.0128
7	2820	28.2	3	0.4806	0.1127	-0.6666	0.4806	0.2966	-0.0244
8	2640	55	4	0.4497	0.2437	-0.5000	0.4497	0.3467	0.0644
9	2423.52	73.44	6	0.4124	0.3335	-0.1666	0.4124	0.3729	0.1931
10	2407.5	160.5	4	0.4097	0.7584	-0.5000	0.4097	0.5840	0.2227
11	1075.2	5.12	2	0.1806	0.0000	-0.8333	0.1806	0.0903	-0.2175
12	1043.5	20.87	5	0.1751	0.0769	-0.3333	0.1751	0.1260	-0.0271
13	1038	86.5	7	0.1742	0.3973	0.0000	0.1742	0.2857	0.1905
14	883.2	110.4	5	0.1476	0.5139	-0.3333	0.1476	0.3307	0.1094
15	854.4	71.2	3	0.1426	0.3226	-0.6666	0.1426	0.2326	-0.0671
16	810	45	3	0.1350	0.1947	-0.6666	0.1350	0.1648	-0.1123
17	703.68	14.66	4	0.1167	0.0466	-0.5000	0.1167	0.0816	-0.1122
18	594	49.5	6	0.0978	0.2167	-0.1666	0.0978	0.1572	0.0493
19	570	47.5	5	0. 0937	0.2069	-0.3333	0. 0937	0.1503	-0.0109
20	467.6	58.45	4	0.0761	0.2603	-0.5000	0.0761	0.1682	-0.0545
21	463.6	24.4	4	0.0754	0.0942	-0.5000	0.0754	0.0848	-0.1101
22	455	65	4	0.0739	0.2913	-0.5000	0.0739	0.1831	-0.0446
23	432.5	86.5	4	0.0701	0.3973	-0.5000	0.0701	0.2337	-0.0108
24	398.4	33.2	3	0.0642	0.1371	-0.6666	0.0642	0.1006	-0.1551
25	370.5	37.05	1	0.0594	0.1559	-1.0000	0.0594	0.1076	-0.2615
26	338.4	33.84	3	0.0539	0.1402	-0.6666	0.0539	0.0970	-0.1575
27	336.12	84.03	1	0.0535	0.3852	-1.0000	0.0535	0.2193	-0.1871
28	313.6	78.4	6	0.0496	0.3577	-0.1666	0.0496	0.2036	0.0802
29	268.68	134.34	7	0.0419	0.6308	0.0000	0.0419	0.3363	0.2242
30	224	56	1	0.0342	0.2484	-1.0000	0.0342	0.1413	-0.2391
31	216	72	5	0.0328	0.3265	-0.3333	0.0328	0.1792	0.0086
32	212.08	53.02	2	0.0322	0.2338	-0.8333	0.0322	0.1330	-0.1893
33	197.92	49.48	5	0.0297	0.2166	-0.3333	0.0297	0.1231	-0.0290
34	190.89	7.07	7	0.0285	0.0096	0.0000	0.0285	0.0190	0.0127
35	181.8	60.6	3	0.0269	0.2708	-0.6666	0.0269	0.1488	-0.1229
36	163.28	40.82	3	0.0238	0.1743	-0.6666	0.0238	0.0990	-0.1561
37	150	30	5	0.0215	0.1215	-0.3333	0.0215	0.0715	-0.0634
38	134.8	67.4	3	0.0189	0.3040	-0.6666	0.0189	0.1614	-0.1145
39	119.2	59.6	5	0.0162	0.2660	-0.3333	0.0162	0.1411	-0.0170
40	103.36	51.68	6	0.0135	0.2273	-0.1666	0.0135	0.1204	0.0247
41	79.2	19.8	2	0.0093	0.0717	-0.8333	0.0093	0.0405	-0.2507
42	75.4	37.7	2	0.0087	0.1591	-0.8333	0.0087	0.0839	-0.2218
43	59.78	29.89	5	0.0060	0.1209	-0.3333	0.0060	0.0634	-0.0688
44	48.3	48.3	3	0.0040	0.2108	-0.6666	0.0040	0.1074	-0.1506
45	34.4	34. 4	7	0.0016	0.1430	0.0000	0.0016	0.0723	0.0482
46	28.8	28.8	3	0.0006	0.1156	-0.6666	0.0006	0.0581	-0.1834
47	25.38	8.46	5	0.0000	0.0164	-0.3333	0.0000	0.0082	-0.1056
Min	25.38	5.12	1						
Max	5840.64	210	7						

As we see, Comparing the ZF's model with the suggested model, only 33 out of 47 remain in the same classes.

By implementing the suggested model, 6 out of 10 items of class A in ZF's model are reclassified in the same class while the reminder 4 item are grouped into class B.

Out of 14 in class B, 8 reclassify in the same class, 2 are grouped to class A and 4 to class C. also, out of 23 of class C in the classification of ZF's model, 19 remain in the same class C, 2 are transferred to class A and 2 to class B.

In fact, the difference of these recent two approaches is due to ranking of criteria in the suggested model and different schemes of scoring the items.

But while comparison with Ng-approach, by reason of similarity the sequence of ranking of criteria, 39 items remain in the same classes. 9 out of 10 class A items in the Ng-model reclassify in the same class, and other one items is reclassified in class B. Out of the 14 items in class B, 10 remain in the class B, 1 is transferred into class A and 3 into class C. in addition, out of the 23 items of class C in the Ng-model, 20 remain in the same class C, 3 are transferred into class B.

This slight difference is due to usage of applied scale transformation for the criterion lead time and also acquiring the most favourable scores and the least favourable scores and then converting these into a single score by Eq. 7.

Table 2. The most favourable scores, the least favourable scores, composite scores and comparison results with Ng and ZF models

Item number	Annual dollar usage($)	Average unit cost($)	Lead time (day)	gI_i	bI_i	nI_i	Proposed - model	Ng-model	ZF – model	TABC
2	5670	210	5	0.4926	0.2729	0.0074	A	A	A	A
1	5840.64	49.92	2	0.5000	0.0642	0.2585	A	A	A	A
3	5037.12	23.76	4	0.4309	0.0769	0.3123	A	A	A	A
10	2407.5	160.5	4	0.2920	0.1113	0.4098	A	A	A	A
4	4769.56	27.73	1	0.4079	-0.0122	0.4460	A	A	C	A
5	3478.8	57.98	3	0.2965	0.0309	0.5044	A	A	B	A
9	2423.52	73.44	6	0.2062	0.0965	0.5146	A	A	A	A
8	2640	55	4	0.2248	0.0322	0.5755	A	B	B	A
13	1038	86.5	7	0.1428	0.0871	0.5902	A	A	A	B
6	2936.67	31.24	3	0.2503	-0.0064	0.5976	A	A	C	C
14	883.2	110.4	5	0.1658	0.0547	0.6072	B	B	A	B
7	2820	28.2	3	0.2403	-0.0122	0.6149	B	B	C	A
29	268.68	134.34	7	0.1681	0.0209	0.6466	B	A	A	A
28	313.6	78.4	6	0.1018	0.0248	0.7087	B	B	A	B
23	432.5	86.5	4	0.1168	-0.0054	0.7310	B	B	B	C
18	594	49.5	6	0.0786	0.0246	0.7323	B	B	A	B
31	216	72	5	0.0896	0.0043	0.7455	B	B	B	C
15	854.4	71.2	3	0.1163	-0.0335	0.7664	B	C	C	B
12	1043.5	20.87	5	0.0875	-0.0135	0.7706	B	B	B	C
40	103.36	51.68	6	0.0602	0.0067	0.7730	B	B	B	B
19	570	47.5	5	0.0751	-0.0054	0.7731	B	B	B	C
22	455	65	4	0.0915	-0.0223	0.7774	B	C	B	B
39	119.2	59.6	5	0.0705	-0.0085	0.7815	B	B	B	C
20	467.6	58.45	4	0.0841	-0.0272	0.7910	B	C	B	B
33	197.92	49.48	5	0.0615	-0.0145	0.7980	C	B	B	C
45	34.4	34. 4	7	0.0361	0.0008	0.8046	C	B	B	B
34	190.89	7.07	7	0.0142	0.0063	0.8199	C	B	B	C
16	810	45	3	0.0824	-0.0561	0.8286	C	C	C	C
38	134.8	67.4	3	0.0807	-0.0572	0.8317	C	C	C	C
35	181.8	60.6	3	0.0744	-0.0614	0.8432	C	C	C	C
37	150	30	5	0.0357	-0.0317	0.8453	C	C	B	B
27	336.12	84.03	1	0.1096	-0.0935	0.8474	C	C	C	C
43	59.78	29.89	5	0.0317	-0.0344	0.8528	C	C	C	B
17	703.68	14.66	4	0.0583	-0.0561	0.8528	C	C	C	B
21	463.6	24.4	4	0.0424	-0.0550	0.8675	C	C	C	B
44	48.3	48.3	3	0.0537	-0.0753	0.8812	C	C	C	C
11	1075.2	5.12	2	0.0903	-0.1087	0.8857	C	C	C	C
24	398.4	33.2	3	0.0503	-0.0775	0.8875	C	C	C	C
36	163.28	40.82	3	0.0495	-0.0780	0.8889	C	C	C	C
26	338.4	33.84	3	0.0485	-0.0787	0.8907	C	C	C	C
32	212.08	53.02	2	0.0665	-0.0946	0.8923	C	C	C	C
47	25.38	8.46	5	0.0041	-0.0528	0.9034	C	C	C	B
30	224	56	1	0.0706	-0.1195	0.9189	C	C	C	C
46	28.8	28.8	3	0.0290	-0.0917	0.9264	C	C	C	C
42	75.4	37.7	2	0.0419	-0.1109	0.9372	C	C	C	C
25	370.5	37.05	1	0.0538	-0.1307	0.9498	C	C	C	C
41	79.2	19.8	2	0.0202	-0.1253	0.9770	C	C	C	C
gI^*				0.5000						
gI^-				0.0041						
bI^*					0.2729					
bI^-					-0.1307					

5. Conclusion

In this paper, we presented a MODM model for MC-ABC inventory classification in which the aim was utilizing from the advantages of ZF and Ng models by removing their drawbacks. First, since the measures of items with respect to criterion lead time were small scale, we used another for converting into a 0-1 scale. With this scale transformation, the objective functions of Ng-model were decomposed into two sections of maximization and minimization so that through another problem transformation, the total scores of items obtained. Furthermore, for relieving to become a zero the weight of an item against an unimportant criterion in Ng-model, we used ZF model with Ng constraints such that the least favourable scores would not only remove recent weakness but also the effect of criteria with the small scale measures included in final score. The results showed that by applying the proposed model, the 8 items were classified in a class different from results of the Ng-model. In order to show that the results of our model is more reasonable, consider items 20 and 33 that in Ng-model had been classified in classes C and B respectively and vice versa in our model in classes B and C. Although item 20 as compared to item 33 has lager measure in relation to annual dollar usage and Average unit cost but in view of the fact that this item only is delivered to warehouse one day earlier than item 33 with respect to criterion lead time that in ranking of criteria by Ng has the shortest priority between other, it had been classified in class C. This point is due to the misuse from scale transformation by Ng for measures of lead time that was corrected by our approach. By utilizing this model for classification of inventory items in another warehouse which it has more criteria in relation to small scale measures and then comparing the results with Ng-model, will be apparent more logicality the proposed model.

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یک مدل بهبود یافته برای حل مسئله طبقه‌بندی موجودی ABC چند معیاره

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چکیده

در این مقاله یک رویکری تلفیقی از مدل های ان جی (۲۰۰۷) و ژو و فن (۲۰۰۷) را برای مسئله طبقه‌بندی ABC چند معیاره ارائه می دهیم. مدلی که ان جی (۲۰۰۷) پیشنهاد کرد، از این پس مدل ان جی، علیرغم مزایایش ممکن است به وضعیتی منجر گردد که در آن اوزان برخی معیارها در رابطه با یک کالا هیچ نقشی را در تعیین امتیاز کلی آن بازی نکنند. همچنین، تبدیل مقیاسی که او جهت تبدیل مقادیر اقلام در رابطه با معیارهای متفاوت به بازه ۱-۰ بکار برد برای مقادیر کوچک مقیاس مناسب نمی باشد. از سوی دیگر، برای R کالای موجودی، مدل ژو و فن (۲۰۰۷)، از این پس مدل ژو و فن، بایست $2R$ مرتبه از طریق یک بهینه کننده حل شود که در آن یک مدیر موجودی ممکن است هیچ پیش زمینه ای در خصوص بهینه کننده خطی نداشته باشد. بعلاوه، هنگامی که تعداد اقلام زیاد شود، زمان محاسباتی افزایش خواهد یافت. از اینرو، به منظور برطرف نمودن نقایص مدل های اخیر، مدلی تلفیقی را ارایه می دهیم که در آن توابع هدف همان توابع هدف مدل ژو و فن است اما قیود آن قیود مدل ان جی است. به منظور مقایسه مدل پیشنهادی با مدل ان جی و ژو و فن یک مطالعه موردی نیز ارائه می شود.

کلید واژه: مسئله طبقه‌بندی ABC چند معیاره، برنامه‌ریزی خطی، کنترل موجودی

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