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Electrostatics Modes in Mono-Layered Graphene

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Abstract: In this paper, we investigated the corrected plasmon dispersion relation for graphene in presence of a constant magnetic field which it includes a quantum term arising from the collective electron density wave interference effects. By using quantum hydrodynamic plasma model which incorporates the important quantum statistical pressure and electron diffraction force, the longitudinal plasmons are the electrostatic collective excitations of the solid electron gas. It shows the importance of quantum term from the collective electron density wave interference effects. By plotting the dispersion relation derived, it has been found that dispersion relation of surface modes depends significantly on Bohm's potential and statistical terms and it should be taken into account in the case of magnetized or unmagnetized plasma; we have noticed successful description of the quantum hydrodynamic model. So, the quantum corrected hydrodynamic model can effectively describe the Plasmon dispersion spectrum in degenerate plasmas, since it takes into account the full picture of collective electron-wave interference via the quantum Bohm's potential. By plotting the dispersion relation, the behavior of different wave types was predicted. It was found that one of them should not be propagated to the specific wave number. By drawing of contour curve of these modes, the areas that modes can be propagated were obtained.

Keywords: Hydrodynamic Equations, Graphene, Electrostatic Waves, Dispersion Relation.

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1. Introduction

One of the new materials in nano-structure emergence in recent few years is Graphene. Graphene has become one of the most amazing and technologically appealing fields of scientific research which has captured enormous amount of attention among researchers of diverse fields [1]. Actually, Graphene is an allotropes of carbon that is structurally as a single layer of carbon atoms connected in a honeycomb lattice in the form of a two-dimensional crystal material. The electrodynamics properties of graphene are summarized in below, the structure of energy bands in graphene net changes in the spectrum is parabolic. Furthermore, the dynamic frequency dependent conductivity of graphene with strong nonlinear characteristics predicts its very promising applications in developments of future advanced terahertz source and detector technologies (terahertz gap technology) [2]. It has been found that graphene, usually considered as a gapless semiconductor, shows a profoundly different behavior from semiconductors, regarding the plasmon excitation resonances [3]. The quantum hydrodynamics (QHD) model, since the first developments several decades ago, has become one of the most convenient and useful methods in description of collective modes in quantum plasmas. Recent development of effective hydrodynamic models incorporating the electron recoil, spin magnetization, and relativistic effects has turned the hydrodynamics approach into a direct method of evaluation of the collective modes in wide variety of plasmas [4, 5].

The most important component of a QHD, which causes different dispersion effects in quantum plasma compared to that of a classical counterpart, is the degeneracy pressure. However, the second order effects, such as the quantum electron diffraction and spin magnetization effects, has been shown to lead to observable effects on ion acoustic and magneto-sonic wave propagations and instabilities in quantum plasmas. If the background ions form a monolayer planar honeycomb lattice, the degenerate electrons fill the conical band dispersion container, the so-called Dirac cones .Such Dirac cones are described by a linear energy dispersion relation as $E = \hbar k_F v_F$ (with the characteristic Fermi energy of $E_F = \hbar k_F v_F$), quite similar to that of the massless photon gas, except that the valence free electrons in graphene possess subluminal particle velocities [6].

2. Dispersion Relations

Let’s consider a high-frequency wave which is propagated in the infinite and homogeneous plasma sheet which consists of quantum electron fluid doped in two-dimensional ion lattice. This sheet is located in $z = 0$ and confined both sides by vacuum. The external and constant magnetic field $\vec{\mathbf{B}} = B_0 \hat{\mathbf{y}}$ which is in plane is applied (in Figure 1).

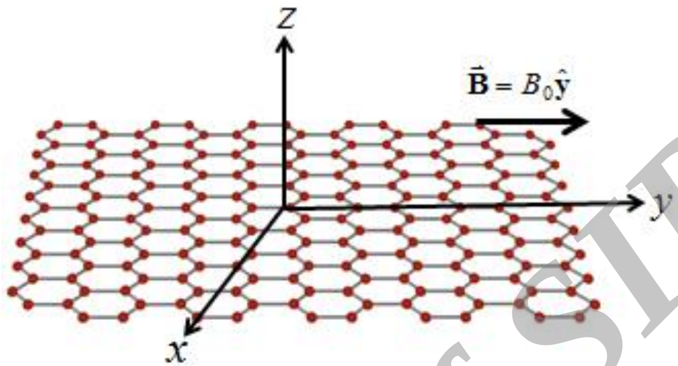

Fig. 1. Schematic of a mono layered Graphene and an in plane magnetic field.

Our closed hydrodynamic set of equations consists of the continuity, momentum and Poisson's equations, written as:

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} = -\frac{e}{m_c}\left[-\nabla \Phi_{ind} + \mathbf{v} \times \mathbf{B}\right] - \frac{\nabla P_{2D}}{m_c\, n} + \frac{\hbar^2}{2\, m_c} \nabla \left(\frac{\nabla^2 \sqrt{n}}{\sqrt{n}} \right) \tag{1}$$

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\, \mathbf{v}) = 0 \tag{2}$$

$$\nabla \cdot \vec{\mathbf{E}} = -\frac{e}{\varepsilon_0}(n - n_0) \tag{3}$$

Where, n , P_{2D} and E are the number of electron density, electrostatic potential induced and pressure fluid quantum and the total electric field. In order to calculate the electron fluid pressure fermion quantum degenerate, for example, it is assumed that the plasma is in a state of complete degenerate. In two-dimensional, Dirac pressure for the quantum fluid plasma is defined $P_{2D} = \sqrt{2\pi} / 3\hbar v_F n^{3/2}$ which $\sigma(r') = -e\, n(r')$ is the surface charge density.

Selecting changes of perturbed parameters like $\varphi = \varphi(z) \exp[i\ (k_x x - \omega t)]$, we can express the set of equations mentioned below, regarding the linear perturbation. By substituting relationships in equation (1) and with regard to linear disturbances can obtain the following equations.

$$-i\,\omega \vec{\mathbf{v}} = -\frac{e}{m_c} \nabla \left(\frac{e n}{8\pi \varepsilon_0 k_x}\right) - \frac{e}{m_c} \vec{\mathbf{v}} \times \vec{\mathbf{B}}_0 - \frac{E_F}{2\, m_c n_0} \nabla n + \frac{\hbar^2}{4\, m_c^2 n_0} \nabla \left(\nabla^2 n\right), \tag{4}$$

$$-i\omega n + n_0 \nabla \cdot \vec{\mathbf{v}} = 0, \tag{5}$$

Also the inducted electrostatic potential is defined as $\Phi_{ind} = -e\,n/(2\varepsilon_0 k_x)$ and $k_F = \sqrt{2\pi n_0}$ are used [7]. The Eq. (4) and the z-component of the curl of the Eq. (4), one can derive a relation between $\vec{\mathbf{v}}$ and n :

$$\left(1-\frac{e^2B_0^2}{m_c^2\omega^2}\right)\nabla.\vec{\mathbf{v}} = -\frac{i e^2}{8\pi\varepsilon_0\omega k_y}\nabla^2 n - \frac{i E_F}{2m_c\omega n_0}\nabla^2 n + \frac{i\hbar^2}{4m_c^2\omega n_0}\nabla^4 n \tag{6}$$

By substituting 5 on 6 and $\nabla^2 = \frac{\partial^2}{\partial z^2} - k_y^2$. We calculate the following equation.

$$\left(\frac{e^2n_0}{8\pi\varepsilon_0 m_c k_y} + \frac{E_F}{2m_c} + \frac{\hbar^2k_y^2}{4m_c^2}\right)\frac{\partial^2 n}{\partial z^2} - \left(\frac{e^2B_0^2}{m_c^2} + \frac{e^2n_0}{8\pi\varepsilon_0 m_c}k_y + \frac{E_F}{2m_c}k_y^2 + \frac{\hbar^2}{4m_c^2} - \omega^2\right)n = 0 \tag{7}$$

To obtain equation (7), very slow nonlocal variations are neglected i.e. $k_y^{-2}(\partial^4/\partial z^4) << \partial^2/\partial z^2 << k_y^2$. The following solution is proposed for the Eq. (7)

$$n(z) = \begin{cases} 0 & z \neq 0 \\ C\exp\left(-k_z\,|z|\right) & z = 0 \end{cases}, \tag{8}$$

Where

$$k_z = \left(\frac{e^2B_0^2}{m_c^2} + \frac{e^2n_0}{8\pi\varepsilon_0 m_c}k_y + \frac{E_F}{2m_c}k_y^2 + \frac{\hbar^2}{4m_c^2} - \omega^2\right)\Big/\left(\frac{e^2n_0}{8\pi\varepsilon_0 m_c k_y} + \frac{E_F}{2m_c} + \frac{\hbar^2k_y^2}{4m_c^2}\right) \tag{9}$$

and,

$$-i\,\omega J_{1x} = \frac{e}{m_c}J_{1z}B_0 \tag{10}$$

$$-i\,\omega J_{1y} = -\frac{e^2n_0}{2m_c k_x}\left(k_z^2 - k_y^2\right)E_{1x} + i\,\frac{e\,E_F}{2m_c}k_y n_1 - i\,\frac{e\,\hbar^2}{4m_c^2}k_y\left(k_z^2 - k_y^2\right)n_1 \tag{11}$$

$$-i\,\omega J_{1z} = -\frac{e^2n_0}{2m_c k_y}\left(k_z^2 - k_y^2\right)E_{1z} - \frac{eE_F}{2m_c}k_z n_1 + \frac{e\hbar^2}{4m_c^2}\left(k_z^2 - k_y^2\right)n_1 \tag{12}$$

By definition of $A = \frac{E_F}{2m_c} + \frac{\hbar^2 k_y^2}{4m_c^2}$ and using some algebraic mathematical, one would easy the set mentioned equations.

$$a_{11}\ J_{1y} + a_{12}\ J_{1z} = b_1 E_{1y} \tag{13}$$

$$a_{21}\ J_{1y} + a_{22} J_{1z} = b_2 E_{1z} \tag{14}$$

Where the definition of coefficients are like the following

$$a_{11} = i\left(\omega^2 - \omega_c^2 + Ak_y^2\right) \qquad a_{21} = k_y k_z A$$

$$a_{12} = k_y k_z A \qquad a_{22} = i\left(\omega^2 - Ak_z^2\right)$$

$$b_1 = \frac{e^2 n_0 \omega}{2m_c k_y}\left(k_z^2 - k_y^2\right) \qquad b_2 = \frac{e^2 n_0 \omega}{2m_c k_y}\left(k_z^2 - k_y^2\right)$$

Using the relation between $\vec{\mathbf{E}}$ and $\vec{\mathbf{J}}$ conductivity tensor as σ can be written as follows.

$$\sigma = \begin{pmatrix} \sigma_{yy} & \sigma_{yz} \\ \sigma_{zy} & \sigma_{zz} \end{pmatrix} \tag{15}$$

Using the Ampère's law can be written the dielectric tensor as follows.

$$\mathrm{K} = 1 + \frac{i}{\varepsilon_0 \omega}\sigma \tag{16}$$

By calculating the determination of Eq. (18), one can drive the dispersion relation for propagation of electrostatic surface waves on the single-layer graphene.

$$\alpha k_F \left(\frac{k_z^2 - k_y^2}{k_y}\right)^2 \left(\omega^2 - \Omega_c^2 + Ak_z^2\right) + \alpha k_F \left(\frac{k_z^2 - k_y^2}{k_y}\right)^2 \left(\omega^2 - Ak_y^2\right) + \alpha \left(\frac{k_z^2 - k_y^2}{k_y}\right)^2 + \left|I\right| = 0 \tag{17}$$

Where,

$$\left|I\right| = \omega^4 - \omega^2\left(\Omega_c^2 - Ak_z^2 + Ak_y^2\right) - Ak_y^2 \Omega_c^2$$

3. Discussion

In this section, the numerical and analytical discussion is presented about the relationship dispersion i.e. Eq. (17). For two-dimensional single-layer graphene the numerical density of electrons can be between $n_0 \approx 10^{12} cm^{-2}$ and

$n_0 \approx 10^{14} cm^{-2}$ [8]. We have used for our calculation of the amount $n_0 \approx 10^{13} cm^{-2}$. First, consider the case where there is no external field (i. e. $\Omega_c = 0$). Therefore, the equation (17) to be reduced the as following

$$\omega^4 - \left(\frac{k_z^2 - k_y^2}{k_y}\right)\left(A\,k_y - 3/2\alpha\right)\omega^2 - \alpha\left(\frac{k_z^2 - k_y^2}{k_y}\right)\left(A\,k_y - \alpha\right) = 0 \tag{18}$$

By drawing the dispersion relation, one would find that there are two branches (lower and higher). Although both of them are starting from zero, by increasing the wave number they have different behavior. By increased gradually of the wave number k_y / k_F, lower-branch reaches a certain amount (cutoff frequency), in spite of the fact that the higher-branch increases.

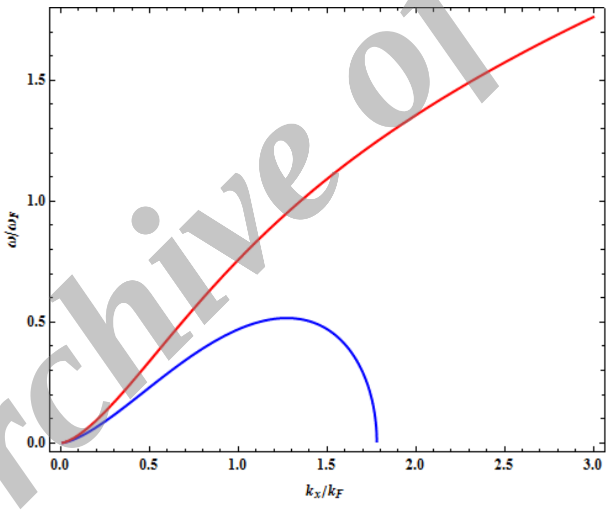

Fig. 2. Schematic of a normalized dispersion relation ω / ω_F in terms of k_y / k_F in the case of no magnetic field.

Figure 3 illustrates the contour curves for higher- branches of quantum electrostatic wave, the areas in which for different values of the magnetic field and the wave number, wave can be stable or unstable. Stable regions are shown in blue color.

Figure 3 clearly shows that the unstable area is very small. To stimulate and propagate lower-branch, one would choose the exact wave number and magnetic fields of the blue waves.

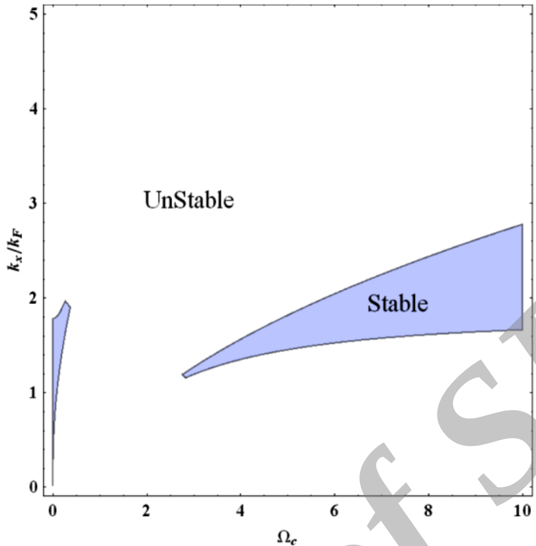

Fig. 3. Schematic of stable and unstable areas for different values of the magnetic field and the wave number for the lower branch

4. Conclusion

In summarize, quantum hydrodynamic model is an effective way to study the waves in various media. In this paper, it has tried to use the quantum hydrodynamic model for studying of propagation of the electrostatic surface wave in single layer graphene, in the presence of an external and uniform magnetic field. The direction of magnetic field was selected in plane of graphene sheet. Considering the set of the quantum hydrodynamic equations, for fluid Dirac, the dispersion relation was obtained. Numerical values were used to analyze the dispersion relation. By plotting the normalized dispersion relation, the behavior of two different wave types (i.e. the lower- and higher- branches) was predicted.

It was found that the lower-branch should not be propagated to the specific wave number (cut-off frequency). By drawing of the contour curve of the higher-branches modes, the areas that modes can be propagated were obtained.

5. References

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