ABSTRACT

Updated Lagrangian Description of Large Deformation Analysis of Unsaturated Soils under Dynamic and Static Loadings

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Extensive areas of the earth are covered by unsaturated soils. Besides, construction of fills, embankments and earth dams are related to compacted soils, which are a very widespread class of unsaturated soils, and there are many geotechnical problems with these soils. The unsaturated soil is assumed to be a three-phase porous media with a solid phase and two fluid phases, water and air. Therefore, considerable attempts have been made to explain or to predict shear strength and volume change behaviour of unsaturated soils in terms of effective stress. Bishop [1] modified classical expression of effective stress for saturated soils to the unsaturated soils in the following form:

$$\sigma' = (\sigma - p_a) + \chi(p_a - p_w)$$

in which σ' is effective stress, p_a and p_w are pore air and pore water pressure respectively, $(p_a - p_w)$ is suction and χ is a parameter that mainly depends on degree of saturation. Many researchers showed that the use of a unique effective stress relation was not fully satisfactory to describe the various aspects of unsaturated soil behavior. The more realistic behavior of unsaturated soils could be explained using state variables and two independent tensorial variables $(p_a - p_w)$ and $(\sigma - p_a)$ i.e. the total stress and suction, respectively) and state surface of void ratio (e) and degree of saturation (S_r) that provide the variation of them with applied total stress and suction. For a given soil, if the fabric does not change significantly, void ratio and degree of saturation are main factors controlling permeability. There are also some empirical relationships based on suction. Thus the permeability of water and air are functions of void ratio and degree of saturation.

To obtain the governing equations for the mechanics of unsaturated porous media, the continuity relations of water and air and the dissolved air in water (Henry's law) are used.

The motion of water and air can be described by generalized Darcy's law. The total mechanical equilibrium is derived by summing of linear momentum balance of three phases and neglecting relative acceleration of fluids.

The basic approach in the incremental step-by-step solution is to assume that the solution for time t is known and that the solution for the time t+dt is required. In large deformation analysis, special attention must be given to the fact that the configuration of the body is changing continuously. In the updated - lagrangian (UL) formulation, all static and kinematic variables are referred to the configuration at time t, and the second Piola-Kirchhoff stress and lagrangian strain tensors can be employed effectively at time t+dt.

For linearizing, we use the Jaumann stress increments instead of second Piola-kirchhohh stress increments. By using the appropriate stress-strain relation for unsaturated soils and the weighted residual methods, the integral equations are obtained. Utilizing the appropriate shape functions, the basic finite element equations are obtained. In the solution of the nonlinear response, the governing equation must be satisfied at the complete time by using a step-by-step incremental analysis and an iteration procedure like modified Newton-Rophson method is used. For a quantitative solution, the resultant equations are discretized in time by Newmark's solution and the final finite element equations are obtained.

Finally, by the UDAMN finite element code, which developed by authors, two problems, i.e. the settlement of unsaturated ground under loading and step construction of an unsaturated embankment, have been solved by this method.

Keywords: Unsaturated Soil; Large Deformation Analysis; Updated Lagrangian Description; Dynamic Analysis; Finite Element; State Variable; State Surface