Numerical Solution of Delay Fractional Optimal Control Problems using Modification of Hat Functions

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Extended Abstract

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Introduction

Optimal control problems occur in engineering, science and many other fields. An optimal control problem is a problem of optimization of an objective functional on a set of state and control variables, which is called the performance index, subject to dynamic constraints on the states and controls. In the case that the dynamic constraints include delay fractional differential equation, the problem is called a delay fractional optimal control problem. In this paper, we consider the following optimal control problem

$$\min \quad J = \int_0^{t_f} h(t, x(t), u(t))dt, \tag{1}$$

subject to:

$$\begin{aligned} &D^{\alpha}x(t) = c(t)x(t) + d(t)u(t) + e(t)x(t-\mu) + f(t)u(t-\delta) + g(t), \\ &x(t) = a(t), \quad t \in [-\mu, 0], \\ &u(t) = b(t), \quad t \in [-\delta, 0], \\ &D^{(i)}x(0) = x_0^i, \quad i = 1, 2, \ldots, n-1, \end{aligned} \tag{2}$$

where $0 \le t \le t_f$, $0 < \mu, \delta < t_f$, $n-1 < \alpha \le n$. Here, D^{α} is the Caputo fractional derivative. We suggest a numerical method based on the modification of hat functions for solving problem (1)-(2).

Material and methods

In this method, we introduce the fractional order operational matrix of integration, product operational matrix and also delay operational matrix of the modification of hat basis functions. These matrices together with the Gauss-Legendre integration formula and Lagrange multiplier method are used to reduce the considered delay fractional optimal control problem to a system of nonlinear algebraic equations which can be solved using a suitable numerical method.

Results and discussion

Some test problems are provided and solved by the new technique in order to demonstrate the applicability, high accuracy and efficiency of this method. For employing the Gauss-Legendre integration formula, we use $m = 10$. The obtained results are compared with the other

existing methods. Also, the numerical results reported in the tables indicate that the accuracy improve by increasing the number of basis functions.

Conclusion

The following results are concluded from this research:

- The operational matrices of modification of hat functions have many zeros. Therefore the method is computationally very attractive. This matter has been approved by reporting the computing time.
- The main problem is reduced to a system of nonlinear algebraic equations which can be solved using iterative methods easily. Thus, the method simplifies the problem.
- By increasing the number of basis functions, the results improve which shows that the numerical solution is convergent.

Keywords: Delay fractional optimal control problem, Modification of hat functions, Riemann-Liouville integral, Caputo derivative, Operational matrix of integration, Product operational matrix, Delay operational matrix.

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