Extended Operational Matrix For Solving Fractional Population Growth Model

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Extended Abstract

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In this paper, we apply the extended triangular operational matrices of fractional order to solve the fractional Voltrra ' s model for population growth of a species in a closed system. The fractional derivative is considered in the Caputo sense. This technique is based on generalized operational matrix of triangular functions. The introduced method reduces the proposed problem to solving a system of algebraic equations. Illustrative examples are included to demonstrate the validity and the applicability of the proposed method.

Introduction

In recent years, fractional calculus and differential equations have found enormous applications in mathematics, physics, chemistry, and engineering [1-3]. Some authors have presented the numerical methods for some differential and integral equation problems involved fractional derivatives [4-6]. In this paper, we consider the fractional population growth model (FPGM) of a species in a closed system [7,8]. The model is characterized by the nonlinear fractional Volterra integro-differential equation:

$$\frac{d^{\alpha} y(t)}{dt^{\alpha}} = y(t)(a - by(t) - c\int_0^t y(s)ds),\ 0 \le t, \tag{1}$$

subject to the initial condition

$$y(0) = y_0, \tag{2}$$

where $y(t)$ is the population of identical individuals at time t, y_0 is a constant describing the order of the time fractional derivative, $a > 0$ is the birth rate coefficient, $b > 0$ is the crowding coefficient, and $c > 0$ is the toxicity coefficient [13,14].

Solving FPGM

In this sectionan effective direct method for solving FPGM is presented. Using the definitions of the fractional derivatives and integrals, it is suitable to rewrite Eq.(1) in the following form

$$y(t) = y_0 + aI^{\alpha} y(t) - bI^{\alpha} y^2(t) - cI^{\alpha} y(t)\int_0^t y(s)ds. \tag{3}$$

Approximations of $y(t), y^2(t)$ and y_0 with respect to TFs may be written as

$$y_0 \simeq Y_0^T T(t) = T^T(t)Y_0, \tag{4}$$

$$y(t) \simeq Y^T T(t) = T^T(t)Y, \tag{5}$$

$$[y(t)]^2 \simeq Y_2^T T(t) = T^T(t)Y_2, \tag{6}$$

where 2m-vectors Y_0, Y and Y_2 are TF coefficients. Also elements of Y_2 are 2nh power of the entries of vector Y. From Eqs.(5), last term in Eq.(3) can be approximated as

$$I^{\alpha}(y(t)\int_0^t y(s)ds)\simeq I^{\alpha}(T^T(t)Y\int_0^t Y^T T(s)ds)) =$$

$$I^{\alpha}(T^T(t)YY^T PT(t))\simeq \hat{B}^T P_{\alpha}T(t) \tag{7}$$

also

$$I^{\alpha}(y^2(t))\simeq I^{\alpha}(Y_2^T T(t)) = Y_2^T P_{\alpha}T(t). \tag{8}$$

where $\hat{B}$ is 2m-vector with elements equal to the diagonal entries of matrix $YY^T P$. By substituting Eqs.(4,5,6), and (8) in Eq.(3), we get

$$Y^T T(t) = Y_0^T T(t) + aY^T P_{\alpha}T(t) - bY_2^T P_{\alpha}T(t) - c\hat{B}^T P_{\alpha}T(t), \tag{9}$$

$$Y - P_{\alpha}^T(aY - bY_2 - c\hat{B}) = Y_0. \tag{10}$$

Eq.(10) is a nonlinear system of algebraic equations. Components of unknown vector Y can be obtained by solving this system using an iterative method. Hence, the approximate solution $y(t)\simeq y_m(t) = Y^T T(t)$ can be computed without using any projection method.

Conclusion

In this paper, we present a computational method based on TFs for solving nonlinear fractional Volterra's population model by means of triangular operational matrix. Generalized operational matrix of TF is derived and used to reduce FPGM to a system of algebraic equations. The advantage of this method is low cost of setting up the equations without applying any projection method. Also results show good accuracy in comparison with other methods.

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