

## GENERALIZED UNI-SOFT INTERIOR IDEALS IN ORDERED SEMIGROUPS

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ABSTRACT. For all  $\mathcal{M}, \mathcal{N} \in P(U)$  such that  $\mathcal{M} \subset \mathcal{N}$ , we first introduced the definitions of  $(\mathcal{M}, \mathcal{N})$ -uni-soft ideals and  $(\mathcal{M}, \mathcal{N})$ -uni-soft interior ideals of an ordered semigroup and studied them. When  $\mathcal{M} = \emptyset$  and  $\mathcal{N} = U$ , we meet the ordinary soft ones. Then we proved that in regular and in intra-regular ordered semigroups the concept of  $(\mathcal{M}, \mathcal{N})$ -uni-soft ideals and the  $(\mathcal{M}, \mathcal{N})$ -uni-soft interior ideals coincide. Finally, we introduced  $(\mathcal{M}, \mathcal{N})$ -uni-soft simple ordered semigroup and characterized the simple ordered semigroups in terms of  $(\mathcal{M}, \mathcal{N})$ -uni-soft interior ideals.

### 1. INTRODUCTION

An ideal of a semigroup is a special subsemigroup satisfying certain conditions. The best way to know an algebraic structure is to begin with a special substructure of it. There are plenty of papers on ideals. After Zadeh's introduction of fuzzy set in 1965 [20], the fuzzy sets have been used in the reconsideration of classical mathematics. For example, Meng and Guo [15] researched fuzzy ideals of BCK/BCI-algebras, Koguop [13] researched fuzzy ideals of hyperlattices, and Kehayopulu and Tsingelis [9] researched fuzzy interior ideals of ordered semigroups.

This inadequacy is removed by Molodtsov [16], by the invention of soft set theory in 1999. He introduced parameterization tools to tackle various uncertainties. Due to the beauty of parameterization tools,

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several researchers attracted towards this direction. Many papers have been published in this regard. In [14], Maji et al. studied various operations on soft sets. Some new operations on soft sets have been introduced by Ali et al. in [2]. Aktas and Cagman [1], compared soft sets to the related concepts of fuzzy sets and rough sets. Also, Feng and Li [4], considered soft product operations. Jun et al., [6], applied the concept of soft set theory to ordered semigroups. Khan et al. [10, 11, 12], characterized different classes of ordered semigroups by using soft-union quasi-ideals and soft-union ideals.

In this paper, we introduced the concept of  $(\mathcal{M}, \mathcal{N})$ -uni-soft ideals and  $(\mathcal{M}, \mathcal{N})$ -uni-soft interior ideals of an ordered semigroup and studied them. We also proved that in regular and in intra-regular ordered semigroups the concept of  $(\mathcal{M}, \mathcal{N})$ -uni-soft ideals and the  $(\mathcal{M}, \mathcal{N})$ -uni-soft interior ideals coincide. Lastly, we introduced  $(\mathcal{M}, \mathcal{N})$ -uni-soft simple ordered semigroup and characterized the simple ordered semigroups in terms of  $(\mathcal{M}, \mathcal{N})$ -uni-soft interior ideals.

## 2. BASIC DEFINITIONS AND PRELIMINARIES

An ordered semigroup  $(S, \cdot, \leq)$  is a *Poset*  $(S, \leq)$  equipped with a binary operation “ $\cdot$ ” such that

- (1)  $(S, \cdot)$  is a semigroup,
- (2) If  $x, a, b \in S$ , then  $a \leq b \implies \begin{cases} xa \leq xb \\ ax \leq bx. \end{cases}$

Let  $(S, \cdot, \leq)$  be an ordered semigroup. For subsets  $A$  and  $B$  of an ordered semigroup  $S$ , we denote

$$AB := \{ab \mid a \in A, b \in B\}.$$

If  $A$  is a subset of  $S$ , we denote by  $(A]$  the subset of  $S$  defined as follows

$$(A] := \{t \in S \mid t \leq h \text{ for some } h \in A\}.$$

For  $a \in S$ , we write  $(a]$  instead of  $(\{a\})$ . For subsets  $A$  and  $B$  of an ordered semigroup  $S$ , we have  $A \subseteq (A]$ . If  $A \subseteq B$ , then  $(A] \subseteq (B]$ ,  $(A](B] \subseteq (AB]$ ,  $((A]) = (A]$  and  $((A](B]) \subseteq (AB]$ .

Let  $(S, \cdot, \leq)$  be an ordered semigroup. A non-empty subset  $A$  of  $S$  is called a *subsemigroup* of  $S$  if  $A^2 \subseteq A$ .

A non-empty subset  $A$  of  $S$  is called a *right* (resp., *left*) *ideal* of  $S$  if:

- (1)  $AS \subseteq A$  (resp.,  $SA \subseteq A$ ) and
- (2) if  $a \in A$  and  $S \ni b \leq a$ , then  $b \in A$ .

If  $A$  is both a right and a left ideal of  $S$ , then it is called an ideal of  $S$ .

A non-empty subset  $A$  of  $S$  is called a *interior ideal* of  $S$  if:

- (1)  $SAS \subseteq A$

(2) if  $a \in A$  and  $S \ni b \leq a$ , then  $b \in A$ .

An ordered semigroup  $S$  is said to be regular if for every  $x \in S$  there exist  $a \in S$  such that  $a \leq axa$ .

An ordered semigroup  $S$  is said to be intra-regular if for all  $a \in S$  there exists  $x, y \in S$  such that  $a \leq xa^2y$ .

An ordered semigroup  $S$  is said to be left (resp., right) simple if it contains no proper left (resp., right) ideal.

An ordered semigroup  $S$  is said to be simple if it contains no proper two-sided ideal.

In the following, we assume that  $U$  is an initial universe set,  $E$  is a set of parameters,  $P(U)$  denotes the power set of  $U$  and  $A, B, C, \dots \subseteq E$ . And we will assume that  $\emptyset \subseteq \mathcal{M} \subset \mathcal{N} \subseteq U$ .

A soft set theory is introduced by Molodstov [16], and Çağman [3] provided new definitions and various results on soft set theory.

**Definition 2.1.** [16, 3] A *soft set*  $f_A$  over  $U$  is defined to be the set of ordered pairs

$$f_A = \{(x, f_A(x)) \mid x \in E, f_A(x) \in P(U)\},$$

where  $f_A : E \rightarrow P(U)$  such that  $f_A(x) = \emptyset$  if  $x \notin A$ .

The function  $f_A$  is also called an *approximation function*.

It is clear from Definition 2.1, that a soft set is a *parameterized family* of subsets of  $U$ . Note that the set of all soft sets over  $U$  will be denoted  $S(U)$ .

Define an ordered relation " $\tilde{\subseteq}_{(\mathcal{M}, \mathcal{N})}$ " on  $P(U)$  as follows:

For any  $f_A, f_B \in S(U)$ ,  $\emptyset \subseteq \mathcal{M} \subset \mathcal{N} \subseteq U$ , we defined

$$f_A \tilde{\subseteq}_{(\mathcal{M}, \mathcal{N})} f_B \iff f_A(x) \cap \mathcal{N} \subseteq f_B(x) \cup \mathcal{M},$$

and we define a relation " $=_{(\mathcal{M}, \mathcal{N})}$ " as follows:

$$f_A =_{(\mathcal{M}, \mathcal{N})} f_B \iff f_A \tilde{\subseteq}_{(\mathcal{M}, \mathcal{N})} f_B \text{ and } f_B \tilde{\subseteq}_{(\mathcal{M}, \mathcal{N})} f_A.$$

The *soft union* of  $f_A$  and  $f_B$ , denoted by  $f_A \tilde{\cup} f_B = f_{A \cup B}$ , is defined by

$$(f_A \tilde{\cup} f_B)(x) = f_A(x) \cup f_B(x) \text{ for all } x \in E.$$

The *soft intersection* of  $f_A$  and  $f_B$ , denoted by  $f_A \tilde{\cap} f_B = f_{A \cap B}$ , is defined by

$$(f_A \tilde{\cap} f_B)(x) = f_A(x) \cap f_B(x) \text{ for all } x \in E.$$

For a soft  $f_A$  over  $U$  and  $\delta \subseteq U$ . The  $\delta$ -exclusive set of  $(f_A, S)$ , denoted by  $e_A(f_A; \delta)$ , is defined as

$$e_A(f_A; \delta) = \{x \in L \mid f_A(x) \subseteq \delta\}$$

For a non-empty subset  $A$  of  $S$ , the *characteristic soft set*  $(\chi_A, S)$  over  $U$  is a soft set defined as follows:

$$\chi_A : S \longrightarrow P(U), x \longmapsto \begin{cases} U, & \text{if } x \in A, \\ \emptyset, & \text{if } x \in S \setminus A. \end{cases}$$

For the characteristic soft set  $(\chi_A, S)$  over  $U$ , the soft set  $(\chi_A^c, S)$  over  $U$  is given as follows:

$$\chi_A^c : S \longrightarrow P(U), x \longmapsto \begin{cases} \emptyset, & \text{if } x \in A, \\ U, & \text{if } x \in S \setminus A. \end{cases}$$

### 3. $(\mathcal{M}, \mathcal{N})$ -UNI-SOFT INTERIOR IDEALS OF ORDERED SEMIGROUPS

In this section, we define  $(\mathcal{M}, \mathcal{N})$ -uni-soft interior ideals of ordered semigroups and study their properties as regards soft set operations and soft uni-product. Also, it is shown that every  $(\mathcal{M}, \mathcal{N})$ -uni-soft ideal is a  $(\mathcal{M}, \mathcal{N})$ -uni-soft interior ideal.

**Definition 3.1.** Let  $(S, \cdot, \leq)$  be an ordered semigroup. A soft set  $(f_S, S)$  over  $U$  is called  $(\mathcal{M}, \mathcal{N})$ -uni-soft left ideal over  $U$  if:

- (1)  $x \leq y \implies f_S(x) \cap \mathcal{N} \subseteq f_S(y) \cup \mathcal{M}$  for all  $x, y \in S$  and
- (2)  $f_S(xy) \cap \mathcal{N} \subseteq f_S(y) \cup \mathcal{M}$  For all  $x, y \in S$ .

A soft set  $(f_S, S)$  over  $U$  is called  $(\mathcal{M}, \mathcal{N})$ -uni-soft right ideal over  $U$  if:

- (1)  $x \leq y \implies f_S(x) \cap \mathcal{N} \subseteq f_S(y) \cup \mathcal{M}$  for all  $x, y \in S$  and
- (2)  $f_S(xy) \cap \mathcal{N} \subseteq f_S(x) \cup \mathcal{M}$  For all  $x, y \in S$ .

A soft set  $f_S$  of  $S$  over  $U$  is called a  $(\mathcal{M}, \mathcal{N})$ -uni-soft ideal of  $S$  over  $U$  if it is both a  $(\mathcal{M}, \mathcal{N})$ -uni-soft left and a  $(\mathcal{M}, \mathcal{N})$ -uni-soft right ideal of  $S$  over  $U$ .

**Example 3.2.** Let  $S = \{e, a\}$  be an ordered semigroup defined by the order relation  $e \leq a$  with the following multiplication table:

$\cdot$	e	a
e	e	a
a	a	e

Define a soft set  $(f_S, S)$  over  $U$  as follows:

$$f_S : S \longrightarrow P(U), x \longmapsto f_S(x) = \begin{cases} \gamma & \text{if } x = e, \\ \gamma & \text{if } x = a. \end{cases}$$

Where  $\mathcal{M} \subseteq \gamma \subset \mathcal{N}$ . Then  $f_S(xy) \cap \mathcal{N} = f_S(e) \cap \mathcal{N} = \gamma \cap \mathcal{N} = \gamma = \gamma \cup \mathcal{M} = f_S(e) \cup \mathcal{M} = f_S(y) \cup \mathcal{M}$ , for every  $x, y \in S$ . Therefore,  $(f_S, S)$  is a  $(\mathcal{M}, \mathcal{N})$ -uni-soft left (resp., right) ideal over  $U$ .

**Definition 3.3.** Let  $(S, \cdot, \leq)$  be an ordered semigroup. A soft set  $(f_S, S)$  over  $U$  is called  $(\mathcal{M}, \mathcal{N})$ -uni-soft interior ideal over  $U$  if:

- (1)  $x \leq y \implies f_S(x) \cap \mathcal{N} \subseteq f_S(y) \cup \mathcal{M}$  for all  $x, y \in S$  and  
 (2)  $f_S(xay) \cap \mathcal{N} \subseteq f_S(a) \cup \mathcal{M}$  For all  $a, x, y \in S$ .

In Example 3.2, one can easily show that  $(f_S, S)$  is a  $(\mathcal{M}, \mathcal{N})$ -uni-soft interior ideal over  $U$ .

**Example 3.4.** [11] Let  $S = \{a, b, c, d\}$  be an ordered semigroup with the following multiplication table and the ordered relation:

$\cdot$	$a$	$b$	$c$	$d$
$a$	$a$	$a$	$a$	$a$
$b$	$a$	$a$	$a$	$a$
$c$	$a$	$a$	$b$	$a$
$d$	$a$	$a$	$b$	$b$

$$\leq := \{(a, a), (b, b), (c, c), (d, d), (a, b), (a, d)\}.$$

Let  $(f_S, S)$  be a soft set over  $U = \mathbb{Z}$  defined by

$$f_S : S \longrightarrow P(U), \quad x \longmapsto f_S(x) = \begin{cases} 6\mathbb{N} & \text{if } x = a, \\ 3\mathbb{Z} & \text{if } x \in \{b, d\}, \\ 3\mathbb{N} & \text{if } x = c. \end{cases}$$

Where  $\mathcal{M} \subseteq 6\mathbb{N} \subset 3\mathbb{N} \subset 3\mathbb{Z} \subseteq \mathcal{N}$ . Then  $(f_S, S)$  is a  $(\mathcal{M}, \mathcal{N})$ -uni-soft interior ideal over  $U$ .

**Theorem 3.5.** *Let  $(S, \cdot, \leq)$  be an ordered semigroup. Then  $(f_S, S)$  is a  $(\mathcal{M}, \mathcal{N})$ -uni-soft interior ideal of  $S$  over  $U$  if and only if the  $\delta$ -exclusive set of  $(f_S, S)$  is an interior ideal of  $S$  for all  $\delta \in P(U)$ , where  $\mathcal{M} \subset \delta \subset \mathcal{N}$ .*

*Proof.* Suppose that  $(f_S, S)$  is a  $(\mathcal{M}, \mathcal{N})$ -uni-soft interior ideal of  $S$  over  $U$  and  $\delta \in P(U)$ . First, we need to show that  $xay \in e_S(f_S; \delta)$ , for all  $a \in e_S(f_S; \delta)$ ,  $x, y \in S$ . By hypothesis, we have  $f_S(xay) \cap \mathcal{N} \subseteq f_S(a) \cup \mathcal{M} \subseteq \delta \cup \mathcal{M} = \delta$  and  $\delta \subset \mathcal{N}$ , which means that  $f_S(xay) \subseteq \delta$ , that is  $xay \in e_S(f_S; \delta)$ . Now, for all  $x \in S$  and  $y \in e_S(f_S; \delta)$  such that  $x \leq y$ , since  $(f_S, S)$  is a  $(\mathcal{M}, \mathcal{N})$ -uni-soft interior ideal of  $S$  over  $U$ , so from  $x \leq y$  we have  $f_S(x) \cap \mathcal{N} \subseteq f_S(y) \cup \mathcal{M} \subseteq \delta \cup \mathcal{M} = \delta$ , we conclude that  $f_S(y) \subseteq \delta$ , that is  $y \in e_S(f_S; \delta)$ . Hence the  $\delta$ -exclusive set of  $(f_S, S)$  is an interior ideal of  $S$  for all  $\delta \in P(U)$  where  $\mathcal{M} \subset \delta \subset \mathcal{N}$ .

Conversely, assume that the  $\delta$ -exclusive set of  $(f_S, S)$  is an interior ideal of  $S$  for all  $\delta \in P(U)$  where  $\mathcal{M} \subset \delta \subset \mathcal{N}$  and  $a, x, y \in S$ . Let  $f_S(xay) \cap \mathcal{N} \supset \delta = f_S(a) \cup \mathcal{M}$  for  $\delta \in P(U)$ . It follows that  $f_S(a) \subseteq \delta$  and  $f_S(xay) \supset \delta$ , that is  $a \in e_S(f_S; \delta)$  and  $xay \notin e_S(f_S; \delta)$ . Which is a contradiction to the fact that  $e_S(f_S; \delta)$  is an interior ideal of  $S$ . Hence  $f_S(xay) \cap \mathcal{N} \subseteq f_S(a) \cup \mathcal{M}$  for all  $a, x, y \in S$ . If there are  $x, y \in S$

such that  $x \leq y$ . Let  $f_S(x) \cap \mathcal{N} \supset \delta = f_S(y) \cup \mathcal{M}$  then  $\delta \subseteq P(U)$ , which means that  $f_S(y) \subseteq \delta$  and  $f_S(x) \supset \delta$ , that is  $y \in e_S(f_S; \delta)$  and  $x \notin e_S(f_S; \delta)$ . Which is again a contradiction to the fact that  $e_S(f_S; \delta)$  is an interior ideal of  $S$ . Hence if  $x \leq y$ , then  $f_S(x) \cap \mathcal{N} \subseteq f_S(y) \cup \mathcal{M}$ , for all  $x, y \in S$ . Therefore  $(f_S, S)$  is a  $(\mathcal{M}, \mathcal{N})$ -uni-soft interior ideal of  $S$  over  $U$ .  $\square$

**Lemma 3.6.** *Let  $(S, \cdot, \leq)$  be an ordered semigroup. A non-empty subset  $I$  of  $S$  is an ideal of  $S$  if and only if the characteristic function  $(\chi_I^c, S)$  is a  $(\mathcal{M}, \mathcal{N})$ -uni-soft ideal over  $U$ .*

*Proof.* It follows from Theorem 3.5.  $\square$

**Theorem 3.7.** *Let  $(S, \cdot, \leq)$  be an ordered semigroup. Then the soft union of two  $(\mathcal{M}, \mathcal{N})$ -uni-soft interior (resp., left, right) ideals over  $U$  is also a  $(\mathcal{M}, \mathcal{N})$ -uni-soft interior (resp., left, right) ideal over  $U$ .*

*Proof.* Let  $(f_S, S)$  and  $(g_S, S)$  be  $(\mathcal{M}, \mathcal{N})$ -uni-soft interior ideals over  $U$ . For any  $x, a, y \in S$ , we have

$$\begin{aligned} (f_S \tilde{\cup} g_S)(xay) \cap \mathcal{N} &= (f_S(xay) \cup g_S(xay)) \cap \mathcal{N} \\ &= (f_S(xay) \cap \mathcal{N}) \cup (g_S(xay) \cap \mathcal{N}) \\ &\subseteq (f_S(a) \cup \mathcal{M}) \cup (g_S(a) \cup \mathcal{M}) \\ &= (f_S(a) \cup g_S(a)) \cup \mathcal{M} \\ &= (f_S \tilde{\cup} g_S)(a) \cup \mathcal{M}. \end{aligned}$$

Furthermore, let  $x, y \in S$  such that  $x \leq y$ . Since  $(f_S, S)$  and  $(g_S, S)$  are  $(\mathcal{M}, \mathcal{N})$ -uni-soft interior ideals over  $U$ , we have

$$\begin{aligned} (f_S \tilde{\cup} g_S)(x) \cap \mathcal{N} &= (f_S(x) \cup g_S(x)) \cap \mathcal{N} \\ &= (f_S(y) \cap \mathcal{N}) \cup (g_S(y) \cap \mathcal{N}) \\ &\subseteq (f_S(y) \cup \mathcal{M}) \cup (g_S(y) \cup \mathcal{M}) \\ &= (f_S(y) \cup g_S(y)) \cup \mathcal{M} \\ &= (f_S \tilde{\cup} g_S)(y) \cup \mathcal{M}. \end{aligned}$$

Therefore  $(f_S \tilde{\cup} g_S, S)$  is a  $(\mathcal{M}, \mathcal{N})$ -uni-soft interior ideal over  $U$ . In a similar way,  $(f_S \tilde{\cup} g_S, S)$  is a  $(\mathcal{M}, \mathcal{N})$ -uni-soft left (resp., right) ideal over  $U$ .  $\square$

Let  $(S, \cdot, \leq)$  and  $(T, \cdot, \leq)$  be two ordered semigroups. Under the coordinatewise multiplication, i.e.,

$$(x, a)(y, b) = (xy, ab)$$

where  $(x, a), (y, b) \in S \times T$ , the Cartesian product

$$S \times T = \{(x, a) \mid x \in S, a \in T\}$$

is a semigroup. Define a partial order  $\leq$  on  $S \times T$  by

$$(x, a) \leq (y, b) \text{ if and only if } x \leq y \text{ and } a \leq b,$$

where  $(x, a), (y, b) \in S \times T$ . Then,  $(S \times T, \cdot, \leq)$  is an ordered semigroup.

For uni-soft sets  $(f_S, S)$  and  $(f_T, T)$  over  $U$ , we consider a uni-soft set  $(f_{S \vee T}, S \times T)$  over  $U$  in which  $f_{S \vee T}$  is defined as follows:

$$f_{S \vee T} : S \times T \longrightarrow P(U), \quad (x, a) \longmapsto f_S(x) \cup f_T(a).$$

**Theorem 3.8.** *Let  $(S, \cdot, \leq)$  be an ordered semigroup. If  $(f_S, S)$  and  $(f_T, T)$  are  $(\mathcal{M}, \mathcal{N})$ -uni-soft interior (resp., left, right) ideals over  $U$ , then  $(f_{S \vee T}, S \times T)$  is a  $(\mathcal{M}, \mathcal{N})$ -uni-soft interior (resp., left, right) ideal over  $U$ .*

*Proof.* Let  $(x, a), (y, b), (z, c) \in S \times T$ . Then

$$\begin{aligned} f_{S \vee T}((x, a), (y, b), (z, c)) \cap \mathcal{N} &= f_{S \vee T}(xyz, abc) \cap \mathcal{N} \\ &= (f_S(xyz) \cup f_T(abc)) \cap \mathcal{N} \\ &= (f_S(xyz) \cap \mathcal{N}) \cup (f_T(abc) \cap \mathcal{N}) \quad (\#). \end{aligned}$$

Since  $(f_S, S)$  and  $(f_T, T)$  are  $(\mathcal{M}, \mathcal{N})$ -uni-soft interior ideal over  $U$ , we have  $f_S(xyz) \cap \mathcal{N} \subseteq f_S(y) \cup \mathcal{M}$  and  $f_T(abc) \cap \mathcal{N} \subseteq f_T(b) \cup \mathcal{M}$ . Hence from equation (#) we have

$$\begin{aligned} (f_S(xyz) \cap \mathcal{N}) \cup (f_T(abc) \cap \mathcal{N}) &\subseteq (f_S(y) \cup \mathcal{M}) \cup (f_T(b) \cup \mathcal{M}) \\ &= (f_S(y) \cup f_T(b)) \cup \mathcal{M} \\ &= f_{S \vee T}(y, b) \cup \mathcal{M}. \end{aligned}$$

Furthermore, let  $(x, a), (y, b) \in S \times T$  be such that  $(x, a) \leq (y, b)$ . Then

$$\begin{aligned} f_{S \vee T}(x, a) \cap \mathcal{N} &= (f_S(x) \cup f_T(a)) \cap \mathcal{N} \\ &= (f_S(x) \cap \mathcal{N}) \cup (f_T(a) \cap \mathcal{N}) \\ &\subseteq (f_S(y) \cup \mathcal{M}) \cup (f_T(b) \cup \mathcal{M}) \\ &= (f_S(y) \cup f_T(b)) \cup \mathcal{M} \\ &= f_{S \vee T}(y, b) \cup \mathcal{M}. \end{aligned}$$

Therefore  $(f_{S \vee T}, S \times T)$  is a  $(\mathcal{M}, \mathcal{N})$ -uni-soft interior ideal over  $U$ . Similarly, we show that If  $(f_S, S)$  and  $(f_T, T)$  are  $(\mathcal{M}, \mathcal{N})$ -uni-soft left (resp., right) ideals over  $U$ , then  $(f_{S \vee T}, S \times T)$  is a  $(\mathcal{M}, \mathcal{N})$ -uni-soft left (resp., right) ideal over  $U$ .  $\square$

**Theorem 3.9.** *Let  $\varphi : S \longrightarrow T$  be a homomorphism of an ordered semigroup. If  $(f_S, S)$  is a uni-soft interior (resp., left, right) ideal over*

$U$ , the pre image  $(\varphi^{-1}(f_S), S)$  of  $(f_S, T)$  under  $\varphi$  is a uni-soft interior (resp., left, right) ideal over  $U$ , where  $\varphi^{-1}(f_S)$  is given as follows:

$$\varphi^{-1}(f_S) : S \longrightarrow P(U), \quad x \longmapsto f_S(\varphi(x)).$$

*Proof.* Let  $(S, \cdot, \leq)$  be an ordered semigroup and  $\varphi : S \longrightarrow T$  be a homomorphism. Let  $x, y \in S$  and  $x \leq y$ . Since  $\varphi$  is a homomorphism of ordered semigroups from  $S$  to  $T$ , we have  $\varphi(x) \leq \varphi(y)$ . Since  $(f_S, S)$  is a  $(\mathcal{M}, \mathcal{N})$ -uni-soft interior ideal over  $U$ , we have  $f_S(\varphi(x)) \cap \mathcal{N} \subseteq f_S(\varphi(y)) \cup \mathcal{M}$ . Hence

$$\varphi^{-1}(f_S)(x) \cap \mathcal{N} = f_S(\varphi(x)) \cap \mathcal{N} \subseteq f_S(\varphi(y)) \cup \mathcal{M} = \varphi^{-1}(f_S)(y) \cup \mathcal{M}.$$

Furthermore, for any  $x, y, z \in S$ , we have

$$\begin{aligned} \varphi^{-1}(f_S)(xyz) \cap \mathcal{N} &= f_S(\varphi(xyz)) \cap \mathcal{N} \\ &= f_S(\varphi(x)\varphi(y)\varphi(z)) \cap \mathcal{N} \\ &\subseteq f_S(\varphi(y)) \cup \mathcal{M} \\ &= \varphi^{-1}(f_S)(y) \cup \mathcal{M}. \end{aligned}$$

Therefore  $(\varphi^{-1}(f_S), S)$  is a  $(\mathcal{M}, \mathcal{N})$ -uni-soft interior ideal over  $U$ . Similarly, we can show that  $(\varphi^{-1}(f_S), S)$  is a  $(\mathcal{M}, \mathcal{N})$ -uni-soft left (resp., right) ideal over  $U$ .  $\square$

**Lemma 3.10.** *Let  $(S, \cdot, \leq)$  be an ordered semigroup. Then every  $(\mathcal{M}, \mathcal{N})$ -uni-soft two-sided ideal over  $U$  is a  $(\mathcal{M}, \mathcal{N})$ -uni-soft interior ideal over  $U$ .*

*Proof.* Let  $x, y, a \in S$ . Since  $(f_S, S)$  is a  $(\mathcal{M}, \mathcal{N})$ -uni-soft right ideal over  $U$ , we have,

$$f_S(xay) \cap \mathcal{N} = f_S((xa)y) \cap \mathcal{N} \subseteq f_S(xa) \cup \mathcal{M}, \quad (1)$$

and since  $(f_S, S)$  is a  $(\mathcal{M}, \mathcal{N})$ -uni-soft right ideal over  $U$ , we have

$$f_S(xay) \cap \mathcal{N} = f_S(x(ay)) \cap \mathcal{N} \subseteq f_S(ay) \cup \mathcal{M}. \quad (2)$$

From (1) and (2), we get  $f_S(xay) \cap \mathcal{N} = f_S(x(ay) \cap \mathcal{N}) \cap \mathcal{N} \subseteq (f_S(ay) \cup \mathcal{M}) \cap \mathcal{N} = (f_S(ay) \cap \mathcal{N}) \cup (\mathcal{M} \cap \mathcal{N}) \subseteq (f_S(a) \cup \mathcal{M}) \cup \mathcal{M} = f_S(a) \cup \mathcal{M}$ .

Let  $x, y \in S$  such that  $x \leq y$ . Then  $f_S(x) \cap \mathcal{N} \subseteq f_S(y) \cup \mathcal{M}$ , because  $(f_S, S)$  is a  $(\mathcal{M}, \mathcal{N})$ -uni-soft two-sided ideal over  $U$ . Thus  $(f_S, S)$  is a  $(\mathcal{M}, \mathcal{N})$ -uni-soft interior ideal over  $U$ .  $\square$

The following example shows that the converse of the Lemma 3.10, is not true in general.



**Example 3.11.** In Example 3.4, soft set  $(f_S, S)$  is a  $(\mathcal{M}, \mathcal{N})$ -uni-soft interior ideal over  $U$ . But it is not a uni-soft left ideal over  $U$ , since  $f_S(dc) \cap \mathcal{N} = f_S(b) \cap \mathcal{N} = 3\mathbb{Z} \cap \mathcal{N} = 3\mathbb{Z} \not\subseteq 3\mathbb{N} = 3\mathbb{N} \cup \mathcal{M} = f_S(c) \cup \mathcal{M}$ , for every  $c, d \in S$ , and hence it is not a uni-soft two-sided ideal over  $U$ .

#### 4. $(\mathcal{M}, \mathcal{N})$ -UNI-SOFT INTERIOR IDEALS OF REGULAR/INTRA-REGULAR ORDERED SEMIGROUPS

In this section, we prove that in regular and in intra-regular ordered semigroups, the concepts of  $(\mathcal{M}, \mathcal{N})$ -uni-soft ideals and  $(\mathcal{M}, \mathcal{N})$ -uni-soft interior ideals coincide.

**Theorem 4.1.** *Let  $(S, \cdot, \leq)$  be a regular ordered semigroup. Then every  $(\mathcal{M}, \mathcal{N})$ -uni-soft interior ideal over  $U$  is a  $(\mathcal{M}, \mathcal{N})$ -uni-soft two-sided ideal over  $U$ .*

*Proof.* Let  $(f_S, S)$  is a  $(\mathcal{M}, \mathcal{N})$ -uni-soft interior ideal over  $U$  and let  $x, y \in S$ . Since  $S$  is a regular, then there exist  $a, b \in S$  such that  $x \leq xax$  and  $y \leq yby$ . We have

$$\begin{aligned} f_S(xy) \cap \mathcal{N} &\subseteq (f_S((xax)y) \cup \mathcal{M}) \cap \mathcal{N} \\ &= (f_S((xa)xy) \cap \mathcal{N}) \cup (\mathcal{M} \cap \mathcal{N}) \\ &\subseteq (f_S(x) \cup \mathcal{M}) \cup \mathcal{M} \\ &= f_S(x) \cup \mathcal{M}, \end{aligned}$$

and

$$\begin{aligned} f_S(xy) \cap \mathcal{N} &\subseteq (f_S(x(yby)) \cup \mathcal{M}) \cap \mathcal{N} \\ &= (f_S(xy(by)) \cap \mathcal{N}) \cup (\mathcal{M} \cap \mathcal{N}) \\ &\subseteq (f_S(y) \cup \mathcal{M}) \cup \mathcal{M} \\ &= f_S(y) \cup \mathcal{M}. \end{aligned}$$

Now, let  $x, y \in S$  such that  $x \leq y$ . Then  $f_S(x) \cap \mathcal{N} \subseteq f_S(y) \cup \mathcal{M}$ , because  $(f_S, S)$  is a  $(\mathcal{M}, \mathcal{N})$ -uni-soft interior ideal of  $S$  over  $U$ . Therefore  $(f_S, S)$  is a  $(\mathcal{M}, \mathcal{N})$ -uni-soft two-sided ideal over  $U$ .  $\square$

By Lemma 3.10 and Theorem 4.1, we have the following:

*Remark 4.2.* In regular ordered semigroups the concepts of  $(\mathcal{M}, \mathcal{N})$ -uni-soft two-sided ideals and  $(\mathcal{M}, \mathcal{N})$ -uni-soft interior ideals coincide.

**Theorem 4.3.** *Let  $(S, \cdot, \leq)$  be an intra-regular ordered semigroup and  $(f_S, S)$  is a  $(\mathcal{M}, \mathcal{N})$ -uni-soft interior ideal over  $U$ . Then  $(f_S, S)$  is a  $(\mathcal{M}, \mathcal{N})$ -uni-soft two-sided ideal over  $U$ .*

*Proof.* Let  $(f_S, S)$  be a  $(\mathcal{M}, \mathcal{N})$ -uni-soft interior ideal over  $U$ . Let  $x, y \in S$ , since  $S$  is a intra-regular then there exists  $a, b \in S$  such that  $x \leq ax^2a$  and  $y \leq by^2b$ . Since  $(f_S, S)$  is an uni-soft interior ideal of  $S$ , we have

$$\begin{aligned} f_S(xy) \cap \mathcal{N} &\subseteq (f_S((ax^2a)y) \cup \mathcal{M}) \cap \mathcal{N} \\ &= (f_S((ax)x(ay)) \cap \mathcal{N}) \cup (\mathcal{M} \cap \mathcal{N}) \\ &\subseteq (f_S(x) \cup \mathcal{M}) \cup \mathcal{M} \\ &= f_S(x) \cup \mathcal{M}, \end{aligned}$$

and

$$\begin{aligned} f_S(xy) \cap \mathcal{N} &\subseteq (f_S(x(by^2b)) \cup \mathcal{M}) \cap \mathcal{N} \\ &= (f_S((xb)y(yb)) \cap \mathcal{N}) \cup (\mathcal{M} \cap \mathcal{N}) \\ &\subseteq (f_S(y) \cup \mathcal{M}) \cup \mathcal{M} \\ &= f_S(y) \cup \mathcal{M}. \end{aligned}$$

Let  $x, y \in S$  be such that  $x \leq y$ . Then  $f_S(x) \cap \mathcal{N} \subseteq f_S(y) \cup \mathcal{M}$ , because  $(f_S, S)$  is a  $(\mathcal{M}, \mathcal{N})$ -uni-soft interior ideal of  $S$  over  $U$ . Thus  $(f_S, S)$  is a  $(\mathcal{M}, \mathcal{N})$ -uni-soft two-sided ideal of  $S$  over  $U$ .  $\square$

By Lemma 3.10 and Theorem 4.3, we have the following:

*Remark 4.4.* In intra regular ordered semigroups, the concepts of  $(\mathcal{M}, \mathcal{N})$ -uni-soft two-sided ideals and  $(\mathcal{M}, \mathcal{N})$ -uni-soft interior ideals coincide.

**Theorem 4.5.** *If  $S$  is a monoid with identity  $e$ . Then every  $(\mathcal{M}, \mathcal{N})$ -uni-soft interior ideal over  $U$  is a  $(\mathcal{M}, \mathcal{N})$ -uni-soft two sided ideal over  $U$ .*

*Proof.* Let  $(f_S, S)$  is a  $(\mathcal{M}, \mathcal{N})$ -uni-soft interior ideal over  $U$  and  $x, y \in S$ . Then  $f_S(xy) \cap \mathcal{N} = f_S(xye) \cap \mathcal{N} \subseteq f_S(y) \cup \mathcal{M}$  and  $f_S(xy) \cap \mathcal{N} = f_S(exy) \cap \mathcal{N} \subseteq f_S(x) \cup \mathcal{M}$ . Furthermore, let  $x, y \in S$  be such that  $x \leq y$ . Then  $f_S(x) \cap \mathcal{N} \subseteq f_S(y) \cup \mathcal{M}$ , because  $(f_S, S)$  is a  $(\mathcal{M}, \mathcal{N})$ -uni-soft interior ideal of  $S$  over  $U$ . Therefore  $(f_S, S)$  is a  $(\mathcal{M}, \mathcal{N})$ -uni-soft two-sided ideal over  $U$ .  $\square$

## 5. $(\mathcal{M}, \mathcal{N})$ -UNI-SOFT SIMPLE ORDERED SEMIGROUPS

In this section, we define  $(\mathcal{M}, \mathcal{N})$ -uni-soft simple ordered semigroups and characterize simple ordered semigroups in terms of  $(\mathcal{M}, \mathcal{N})$ -uni-soft interior ideals.

**Definition 5.1.** An ordered semigroup  $S$  is called  $(\mathcal{M}, \mathcal{N})$ -uni-soft simple if for any  $(\mathcal{M}, \mathcal{N})$ -uni-soft ideal of  $S$ , we have  $f_S(x) \cap \mathcal{N} \subseteq f_S(y) \cup \mathcal{M}$ , for all  $x, y \in S$ .

**Theorem 5.2.** *Let  $(S, \cdot, \leq)$  be an ordered semigroup. Then  $S$  is  $(\mathcal{M}, \mathcal{N})$ -uni-soft simple if and only if for any  $(\mathcal{M}, \mathcal{N})$ -uni-soft ideal  $(f_S, S)$  of  $S$ , if  $e_S(f_S, S) \neq \emptyset$ , then  $e_S(f_S, \delta) = S$ , for all  $\delta \in P(U)$  where  $\mathcal{M} \subset \delta \subset \mathcal{N}$ .*

*Proof.* Let  $(f_S, S)$  be a  $(\mathcal{M}, \mathcal{N})$ -uni-soft ideal of  $S$  over  $U$ , and  $e_S(f_S, \delta) \neq \emptyset$ . We need to show that  $x \in e_S(f_S, \delta)$  for all  $x \in S$ , where  $\mathcal{M} \subset \delta \subset \mathcal{N}$ . Since  $e_S(f_S, \delta) \neq \emptyset$ , then there exists  $y \in e_S(f_S, \delta)$ , that is  $f_S(y) \subseteq \delta$ . By hypothesis we have,

$$f_S(x) \cap \mathcal{N} \subseteq f_S(y) \cup \mathcal{M} \subseteq \delta \cup \mathcal{M} = \delta.$$

Notice that  $\delta \subset \mathcal{N}$ , which means that  $f_S(x) \subseteq \delta$ , that is  $x \in e_S(f_S, \delta)$ .

Conversely, suppose that for any  $(\mathcal{M}, \mathcal{N})$ -uni-soft ideal  $(f_S, S)$  over  $U$ , we have  $e_S(f_S, \delta) = S$  for all  $\mathcal{M} \subset \delta \subset \mathcal{N}$ . We need to show that  $f_S(x) \cap \mathcal{N} \subseteq f_S(y) \cup \mathcal{M}$  for all  $x, y \in S$ . Let if

$$f_S(x) \cap \mathcal{N} \supset \delta = f_S(y) \cup \mathcal{M}.$$

Then  $f_S(y) \subseteq \delta$  and  $f_S(x) \supset \delta$ , which means that  $x \notin e_S(f_S, \delta) = S$ , a contradiction. So  $f_S(x) \cap \mathcal{N} \subseteq f_S(y) \cup \mathcal{M}$  for all  $x, y \in S$ .  $\square$

**Theorem 5.3.** *Let  $(S, \cdot, \leq)$  be an ordered semigroup and soft set  $(f_S, S)$  a  $(\mathcal{M}, \mathcal{N})$ -uni-soft right (resp., left) ideal over  $U$ . Then  $I_x = \{y \in S \mid f_S(y) \cap \mathcal{N} \subseteq f_S(x) \cup \mathcal{M}\}$  is a right (resp., left) ideal of  $S$ , for all  $x \in S$ .*

*Proof.* Let  $x \in S$ . Then  $I_x \neq \emptyset$  since  $x \in I_x$ . Suppose that  $y \in I_x$  and  $s \in S$ , Then  $ys \in I_x$ . Since  $(f_S, S)$  is a  $(\mathcal{M}, \mathcal{N})$ -uni-soft ideal over  $U$  and  $y, s \in S$ , we have

$$\begin{aligned} f_S(ys) \cap \mathcal{N} &= (f_S(ys) \cap \mathcal{N}) \cap \mathcal{N} \\ &\subseteq (f_S(y) \cup \mathcal{M}) \cap \mathcal{N} \\ &= (f_S(y) \cap \mathcal{N}) \cup (\mathcal{M} \cap \mathcal{N}) \\ &\subseteq (f_S(x) \cup \mathcal{M}) \cup \mathcal{M} && \text{since } y \in I_x \\ &= f_S(x) \cup \mathcal{M}. \end{aligned}$$

Hence  $ys \in I_x$ .

Furthermore, let  $s, y \in S$  be such that  $s \leq y$ . If  $y \in I_x$ , Then  $s \in I_x$ . Since  $(f_S, S)$  is a  $(\mathcal{M}, \mathcal{N})$ -uni-soft ideal over  $U$  and  $s \leq y$ , we have

$$\begin{aligned} f_S(s) \cap \mathcal{N} &= (f_S(s) \cap \mathcal{N}) \cap \mathcal{N} \\ &\subseteq (f_S(y) \cup \mathcal{M}) \cap \mathcal{N} \\ &= (f_S(y) \cap \mathcal{N}) \cup (\mathcal{M} \cap \mathcal{N}) \\ &\subseteq (f_S(x) \cup \mathcal{M}) \cup \mathcal{M} && \text{since } y \in I_x \\ &= f_S(x) \cup \mathcal{M}. \end{aligned}$$

So  $s \in I_x$ . Therefore,  $I_x$  is a right ideal of  $S$ , for all  $x \in S$ .  $\square$

By left right dual of Theorem 5.3, we have the following result.

**Theorem 5.4.** *Let  $(S, \cdot, \leq)$  be an ordered semigroup and soft set  $(f_S, S)$  a  $(\mathcal{M}, \mathcal{N})$ -uni-soft ideal over  $U$ . Then for all  $x \in S$ , the set*

$$I_x = \{y \in S \mid f_S(y) \cap \mathcal{N} \subseteq f_S(x) \cup \mathcal{M}\}$$

*is a right ideal of  $S$ .*

**Theorem 5.5.** *An ordered semigroup  $S$  is simple if and only if it is  $(\mathcal{M}, \mathcal{N})$ -uni-soft simple.*

*Proof.* Assume that  $S$  is simple. Let  $(f_S, S)$  be a  $(\mathcal{M}, \mathcal{N})$ -uni-soft ideal over  $U$  and  $x, y \in S$ . By Theorem 5.4, the set  $I_x$  is an ideal of  $S$ . Since  $S$  is simple, we have  $I_x = S$ . Then  $b \in I_x$ , from which we have that  $f_S(y) \cap \mathcal{N} \subseteq f_S(x) \cup \mathcal{M}$ . Thus  $S$  is  $(\mathcal{M}, \mathcal{N})$ -uni-soft simple.

Conversely, suppose  $S$  contains proper ideals and let  $I$  be such ideal of  $S$ . By Lemma 3.6, we know that  $(\chi_I^c, S)$  is a  $(\mathcal{M}, \mathcal{N})$ -uni-soft ideal of  $S$ . We have that  $S \subseteq I$ . Indeed, let  $x \in S$ . Since  $S$  is  $(\mathcal{M}, \mathcal{N})$ -uni-soft simple,  $\chi_I^c(x) \cap \mathcal{N} \subseteq \chi_I^c(y) \cup \mathcal{M}$  for all  $y \in S$ . Now, let  $z \in I$ . Then we have

$$\chi_I^c(x) \cap \mathcal{N} \subseteq \chi_I^c(z) \cup \mathcal{M} = U \cup \mathcal{M} = U.$$

Notice that  $\mathcal{M} \subset \mathcal{N}$ , we conclude that  $\chi_I^c(x) \subseteq U$ , which implies that  $\chi_I^c(x) = U$ , that is  $x \in I$ . Thus we have that  $S \subseteq I$ , and so  $S = I$ . We get a contradiction to hypothesis that  $S$  contains proper ideals.  $\square$

**Lemma 5.6.** [8, 9] *An ordered semigroup  $S$  is simple if and only if for every  $a \in S$ , we have  $S = (SaS)$ .*

**Theorem 5.7.** *Let  $(S, \cdot, \leq)$  be an ordered semigroup. Then  $S$  is simple if and only if for every  $(\mathcal{M}, \mathcal{N})$ -uni-soft interior ideal  $(f_S, S)$  of  $S$ , we have  $f_S(x) \cap \mathcal{N} \subseteq f_S(y) \cup \mathcal{M}$ , for all  $x, y \in S$ .*

*Proof.* Suppose  $S$  is simple. Let  $(f_S, S)$  be a  $(\mathcal{M}, \mathcal{N})$ -uni-soft interior ideal over  $U$  and  $x, y \in S$ . Since  $S$  is simple and  $y \in S$ , by Lemma 5.6, we have  $S = (SyS)$ . Since  $x \in S$ , we have  $x \in (SyS)$ , then  $x \leq ayb$  for

some  $a, b \in S$ . Since  $(f_S, S)$  is a  $(\mathcal{M}, \mathcal{N})$ -uni-soft interior ideal over  $U$ , We have

$$\begin{aligned} f_S(x) \cap \mathcal{N} &= (f_S(x) \cap \mathcal{N}) \cap \mathcal{N} \\ &\subseteq (f_S(ayb) \cup \mathcal{M}) \cap \mathcal{N} \\ &= (f_S(ayb) \cap \mathcal{N}) \cup (\mathcal{M} \cap \mathcal{N}) \\ &\subseteq (f_S(y) \cup \mathcal{M}) \cup \mathcal{M} \\ &= f_S(y) \cup \mathcal{M}. \end{aligned}$$

Conversely, suppose that for every  $(\mathcal{M}, \mathcal{N})$ -uni-soft interior ideal  $(f_S, S)$  over  $U$ , we have  $f_S(x) \cap \mathcal{N} \subseteq f_S(y) \cup \mathcal{M}$ , for all  $x, y \in S$ . Now let  $(f_S, S)$  be any  $(\mathcal{M}, \mathcal{N})$ -uni-soft ideal of  $S$  over  $U$ , then it is a  $(\mathcal{M}, \mathcal{N})$ -uni-soft interior ideal of  $S$  over  $U$ . So we have  $f_S(x) \cap \mathcal{N} \subseteq f_S(y) \cup \mathcal{M}$ , for all  $x, y \in S$ . Thus  $S$  is  $(\mathcal{M}, \mathcal{N})$ -uni-soft simple, then by Theorem 5.5, we conclude that  $S$  is simple.  $\square$

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GENERALIZED UNI-SOFT INTERIOR IDEALS IN ORDERED SEMIGROUPS

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ایده‌آل‌های داخلی اجتماع-نرم تعمیم‌یافته در نیم‌گروه‌های مرتب

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برای هر  $M, N \in P(U)$  به گونه‌ای که  $M \subset N$ ، ابتدا مفاهیم ایده‌آل‌های  $(M, N)$ -اجتماع-نرم و ایده‌آل‌های داخلی  $(M, N)$ -اجتماع-نرم برای نیم‌گروه‌های مرتب را معرفی کرده و آن‌ها را مورد مطالعه قرار می‌دهیم. در حالت خاص که  $M = \emptyset$  و  $N = U$ ، مفاهیم فوق با مفاهیم موجود قبلی ایده‌آل نرم و ایده‌آل داخلی نرم، سازگار می‌باشند. سپس ثابت می‌کنیم که در نیم‌گروه‌های مرتب منظم و نیم‌گروه‌های مرتب درون-منظم، مفاهیم تعریف شده فوق برهم منطبق می‌باشند. در انتها، ضمن معرفی مفهوم نیم‌گروه مرتب ساده  $(M, N)$ -اجتماع-نرم، به رده‌بندی نیم‌گروه‌های مرتب ساده بر حسب ایده‌آل‌های داخلی  $(M, N)$ -اجتماع-نرم می‌پردازیم.

کلمات کلیدی: مجموعه نرم، ایده‌آل‌های  $(M, N)$ -اجتماع-نرم، نیم‌گروه‌های مرتب درون-منظم، ایده‌آل‌های داخلی  $(M, N)$ -اجتماع-نرم، نیم‌گروه ساده  $(M, N)$ -اجتماع-نرم، منظم، نیم‌گروه مرتب ساده.