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ω-NARROWNESS AND RESOLVABILITY OF TOPOLOGICAL GENERALIZED GROUPS

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Abstract. A topological group H is called *ω-narrow* if for every neighbourhood V of it's identity element there exists a countable set A such that $VA = H = AV$. A semigroup G is called a *generalized group* if for any $x \in G$ there exists a unique element $e(x) \in G$ such that $xe(x) = e(x)x = x$ and for every $x \in G$, there exists $x^{-1} \in G$ such that $x^{-1}x = xx^{-1} = e(x)$. Also, let G be a topological space and the operation and inversion mapping are continuous, then G is called a topological generalized group. If $\{e(x) \mid x \in G\}$ is countable and for any $a \in G$, $\{x \in G | e(x) = e(a)\}$ is an ω-narrow topological group, then G is called an ω-narrow topological generalized group. In this paper, ω-narrow and resolvable topological generalized groups are introduced and studied.

1. Introduction and preliminaries

Generalized groups are an interesting extension of groups. This notion was first introduced by Molaei in [8]. A *generalized group* is a non-empty set G admitting an operation called multiplication, which satisfies the following conditions:

1. $(xy)z = x(yz)$ for all $x, y, z \in G$.
2. For each $x \in G$ there exists a unique element $z \in G$ such that $zx = xz = x$ (we denote z by $e(x)$).
3. For each $x \in G$ there exists an element $y \in G$ called inverse of x such that $xy = yx = e(x)$.

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It is well known that each x in G has a unique inverse in G, and the inverse of x is denoted by x^{-1} [8]. Moreover, for a given $x \in G$, we have $e(e(x)) = e(x)$, $(x^{-1})^{-1} = x$ and $e(x^{-1}) = e(x)$.

Definition 1.1. [7] If G and H are two generalized groups, then a map $f : G \to H$ is called a homomorphism if $f(ab) = f(a)f(b)$ for all $a, b \in G$.

Theorem 1.2. [7] *Let $f : G \to H$ be a homomorphism where G and H are two generalized groups. Then*

1. $f(e(a)) = e(f(a))$,
2. $f(a^{-1}) = (f(a))^{-1}$,

for all $a \in G$.

Recall that a non-empty subset H of a generalized group G is called a *generalized subgroup* if H is a generalized group under the multiplication on G [7].

Theorem 1.3. [7] *Let H be a non-empty subset of a generalized group G. Then, H is a generalized subgroup of G if and only if $ab \in H$ and $a^{-1} \in H$ for all $a, b \in H$.*

We recall that a *paratopological generalized group* is a generalized group G endowed with a Hausdorff topology such that the multiplicative mapping $m : G \times G \to G$ defined by $(x, y) \mapsto x.y$ is continuous [12]. A paratopological generalized group with continuous inversion $I : G \to G$ defined by $x \mapsto x^{-1}$ is called a *topological generalized group* [9]. Moreover, if $a \in G$ then $G_{e(a)} = \{g \in G \mid e(g) = e(a)\}$ is closed in G [12, Theorem 3], $G_{e(a)}$ is a topological group with the operation on G, and G is the disjoint union of such topological groups, i.e., $G = \dot{\bigcup}_{a \in G} G_{e(a)}$ [10]. The first infinite ordinal is denoted by ω.

Theorem 1.4. [2] *Let G be a paratopological generalized group such that the family $\mathcal{F} = \{G_{e(a)}\}_{a \in G}$ is locally finite. Then every $G_{e(a)}$ is closed and open in G.*

Proposition 1.5. [2] *Let H be a dense generalized subgroup of a topological generalized group G such that the family $\mathcal{F} = \{G_{e(a)}\}_{a \in G}$ is locally finite. Then $H_{e(a)}$ is dense in $G_{e(a)}$ for every $a \in G$.*

2. Main results

We start our main results with the following proposition.

Proposition 2.1. *Let G be a compact paratopological generalized group with the locally finite family $\mathcal{F} = \{G_{e(a)}\}_{a \in G}$. Then the inverse function I from G to G is continuous, and so G is a topological generalized group.*

Proof. Let $a \in G$. Then $G_{e(a)}$ is compact, since $G_{e(a)}$ is closed. Thus, the restriction of I to $G_{e(a)}$ is continuous by [3, Proposition 2.3.3]. Since the family $\mathcal{F}$ is locally finite, the inverse function I is continuous on $G = \bigcup_{a\in G} G_{e(a)}$ [11], and so G is a topological generalized group. □

Proposition 2.2. *Suppose that G is a paratopological generalized group with locally finite family $\mathcal{F} = \{G_{e(a)}\}_{a\in G}$. Then for each compact subset F of G, the set F^{-1} is closed in G.*

Proof. If $a \in G$, then $F_{e(a)} = F \cap G_{e(a)}$ is closed and so $F_{e(a)}$ is compact. Now by [3, Lemma 2.3.5], $F^{-1}_{e(a)}$ is closed in $G_{e(a)}$, and so it is closed in G. Since the family $\mathcal{F}$ is locally finite, $F^{-1} = \bigcup_{a\in G} F^{-1}_{e(a)}$ is closed in G. □

Recall that a semitopological group G is said to be *ω-narrow* if for every open neighbourhood V of the neutral element in G there exists a countable set $A \subset G$ such that $VA = G = AV$ and if A is a finite set, then the semitopological group G is called precompact. A topological generalized group G is called *precompact* [1] if G_a is a precompact topological group for all $a \in e(G)$ and $card(e(G)) < \infty$. If we substitute G in Example 2.13 of this section with the closed unit interval $[0, 1]$ of $\mathbb{R}$, then we observe that a compact topological generalized group need not be precompact. Also, we note that every compact topological generalized group G in which the family $\{G_{e(a)}\}_{a\in G}$ is locally finite is precompact.

Proposition 2.3. *Every precompact topological generalized group G which is locally compact is compact.*

Proof. Since G is precompact, $e(G)$ is finite and G_a is a precompact topological group for all $a \in e(G)$. On the other hand, since every G_a is closed, it is locally compact too. By using [3, Theorem 3.7.22], we observe that every topological group G_a is compact and so G is compact. □

Recall that a topological space X is called *extremally disconnected* [5], if X is Hausdorff and for every open subset U the closure $\overline{U}$ is open in X.

Proposition 2.4. *Suppose that G is an extremally disconnected topological generalized group, such that the family $\mathcal{F} = \{G_{e(a)}\}_{a\in G}$ is locally finite. Then, every precompact subset of G is finite.*

Proof. Let B be a precompact subset of G and $a \in B$. Then $card(e(B)) < \infty$ and $B_{e(a)}$ is precompact. Proposition 1.4 implies that $G_{e(a)}$ is open in G and so it is extremally disconnected. By [3, Theorem 3.7.28],

$B_{e(a)}$ is finite in $G_{e(a)}$. Since $card(e(B))$ is finite, $B = \bigcup_{a\in B} B_{e(a)}$ is finite. □

Corollary 2.5. *Every precompact extremally disconnected topological generalized group is finite.*

In the following definition, we will extend the notion of ω-narrowness to topological generalized groups.

Definition 2.6. An ω-narrow topological generalized group is a topological generalized group G such that $e(G)$ is a countable set and for any $a \in e(G)$, G_a is an ω-narrow topological group.

It is clear from the above definition that every precompact topological generalized group is ω-narrow.

Proposition 2.7. *Every continuous homomorphic image H of an ω-narrow topological generalized group G is ω-narrow.*

Proof. Let $f : G \to H$ be a generalized group homomorphism which is surjective. We claim that the following conditions hold.

(i) $e(H)$ is a countable set.

(ii) $\forall h \in e(H)$, H_h is an ω-narrow topological group.

$H = f(G) = \bigcup_{a\in G} f(G_{e(a)})$ and by Theorem 1.2, $f(e(a)) = e(f(a))$. Thus, $f(G_a) \subset H_{f(a)}$, and so $card(e(H)) \leq card(e(G))$ since f is onto. Therefore (i) holds.

To prove (ii), let U be an open neighbourhood of $f(x) = h \in e(H)$ in H_h. Since $h \in e(H)$, $e(h) = h$ and so $e(x) \in f^{-1}(h)$. Therefore, $f^{-1}(U)$ is an open neighbourhood of $e(x)$ in G and it follows that, $f^{-1}(U) \cap G_{e(x)}$ is an open neighbourhood of $e(x)$ in the ω-narrow topological group $G_{e(x)}$. So, there exists a countable set $A_{e(x)} \subset G_{e(x)}$ such that $A_{e(x)}(G_{e(x)} \cap f^{-1}(U)) = G_{e(x)} = (G_{e(x)} \cap f^{-1}(U))A_{e(x)}$. Since $x \in f^{-1}(h)$ is arbitrary, we have

$$\begin{aligned} H_h = \bigcup_{x\in f^{-1}(h)} f(G_{e(x)}) &= \bigcup_{x\in f^{-1}(h)} f((f^{-1}(U) \cap G_{e(x)})A_{e(x)}) \\ &\subseteq \bigcup_{x\in f^{-1}(h)} (U \cap f(G_{e(x)}))f(A_{e(x)}) \\ &\subseteq \bigcup_{x\in f^{-1}(h)} (U \cap H_h)f(A_{e(x)}) \\ &= \bigcup_{x\in f^{-1}(h)} Uf(A_{e(x)}) \\ &= U \bigcup_{x\in f^{-1}(h)} f(A_{e(x)}). \end{aligned}$$

Since $f^{-1}(h)\cap e(G)$ is countable, $\bigcup_{x\in f^{-1}(h)} f(A_{e(x)})$ is a countable subset of H_h. Now, we define $A = \bigcup_{x\in f^{-1}(h)} f(A_{e(x)})$ that is a countable set in H_h. Therefore, $H_h = UA$ and by asimilar argument we have $H_h = AU$. Thus, H_h is an ω-narrow topological group and this completes the proof. □

Proposition 2.8. *The topological product of a finite family of ω-narrow topological generalized groups is an ω-narrow topological generalized group.*

Proof. Let $\mathbb{F}$ be a finite set and $\{G^i\}_{i\in\mathbb{F}}$ be a family of ω-narrow topological generalized groups. Since $G^i = \bigcup_{a\in e(G^i)} G^i_a$ for every $i \in \mathbb{F}$, we have

$$G = \prod_i [G^i = \bigcup_{a\in e(G^i)} G^i_a] = \bigcup_{a\in e(G^i)} (\prod_i G^i_a).$$

Every $\prod_{i\in\mathbb{F}} G^i_{a_i}$ is an ω-narrow topological group by [3, Proposition 3.4.3], and so G is the disjoint union of ω-narrow topological groups. Moreover, since $e(G^i)$ is countable for all $i \in \mathbb{F}$, $e(G) = \prod_{i\in\mathbb{F}} e(G^i)$ is countable and this completes the proof. □

Proposition 2.9. *Every generalized subgroup H of an ω-narrow topological generalized group G is ω-narrow.*

Proof. Since $card(e(H)) \leq card(e(G))$, our hypothesis implies that $card(e(H))$ is countable. Let $h \in e(H)$, then G_h is an ω-narrow group and H_h is it's subgroup. Thus, H_h is an ω-narrow topological group by [3, Theorem 3.4.4]. Therefore, H is an ω-narrow topological generalized group. □

Proposition 2.10. *Let G be an ω-narrow topological generalized group. Then G is first-countable if and only if G is second-countable.*

Proof. Let G be a first-countable ω-narrow topological generalized group. So, for every a in the countable set $e(G)$, G_a is a first-countable ω-narrow topological group. From [3, Proposition 3.4.5] it follows that G_a has a countable base. From $G = \dot{\bigcup}_{a\in e(G)} G_a$ we infer that G has a countable base. Thus, G is second-countable. The converse is obvious. □

Since every second countable space is separable and Lindelöf, we have the following result.

Corollary 2.11. *Every first-countable ω-narrow topological generalized group is separable and Lindelöf.*

Proposition 2.12. *Let G be a Lindelöf topological generalized group, such that the family $\mathcal{F} = \{G_a\}_{a\in e(G)}$ is locally finite. Then G is ω-narrow.*

Proof. Let b be an arbitrary element of $e(G)$, then by Theorem 1.4 G_b is open and closed in G and so G_b is Lindelöf. Thus, G_b is an ω-narrow topological group by [3, Proposition 3.4.6]. Since $G = \dot{\bigcup}_{a\in e(G)} G_a$ and every G_a is open, $e(G)$ must be countable. Thus, G is ω-narrow. □

Being locally finite is necessary in Proposition 2.12 as it is illustrated in the following example.

Example 2.13. Let $G = \mathbb{R} \setminus \{0\}$ be a subspace of the real line. Then G with the multiplication $x.y = x$ is a Lindelöf topological generalized group such that for every $a \in G$, $e(a) = a^{-1} = a$. Since $G_{e(a)} = \{a\}$ for every $a \in G$, $\{G_{e(a)}\}_{a\in G}$ is not locally finite. Moreover, Since $e(G) = G = \mathbb{R}\setminus\{0\}$, the set $e(G)$ is not countable set, and so G is not ω-narrow.

The smallest cardinal number c such that every family of pairwise disjoint non-empty open subsets of X has cardinility less than or equal to c, is called *Souslin number* [5], or *cellularity* of the space X and it is denoted by $c(X)$. If $c(X)$ is countable, then we say that X has the *Souslin property.*

Proposition 2.14. *Let G be a topological generalized group that has the Souslin property and the family $\mathcal{F} = \{G_a\}_{a\in e(G)}$ is locally finite. Then G is ω-narrow.*

Proof. Let $a \in e(G)$. Since the family $\mathcal{F}$ is locally finite, G_a is open in G by Proposition 1.4. Thus, $c(G_a) \leq c(G)$, and so G_a has the Souslin property. Now [3, Theorem 3.4.7] implies that G_a is ω-narrow. Moreover, Since $\mathcal{F}$ is the family of pairwise disjoint non-empty open subsets of G, we have $card(e(G)) \leq c(G)$. Therefore, $card(e(G))$ is countable and this completes the proof. □

Clearly, every separable space has the Souslin property. Thus, we have the following result.

Corollary 2.15. *Let G be a separable topological generalized group, such that the family $\{G_a\}_{a\in e(G)}$ is locally finite. Then G is ω-narrow.*

Proposition 2.16. *If a topological generalized group G contains an ω-narrow dense generalized subgroup, such that the family $\mathcal{F} = \{G_a\}_{a\in e(G)}$ is locally finite, then G is ω-narrow.*

Proof. This follows from [3, Theorem 3.4.9] and Proposition 1.5.

□

Recall that ω is the first infinite ordinal. The *invariance number* $inv(G_a)$ [3] of a topological group G_a is countable, i.e., $inv(G_a) \leq \omega$, if for each open neighbourhood U of the neutral element $e(a)$ in G_a, there exists a countable family γ of open neighbourhoods of $e(a)$ such that for each $x \in G_a$, there exists $V \in \gamma$ satisfying $xVx^{-1} \subset U$.

Definition 2.17. Let G be a topological generalized group. Then $inv(G) = max\{inv(G_a) \mid a \in e(G)\}$ is called *the invariance number of* G and if $inv(G)$ is countable, then G is called ω*-balanced*.

Clearly, every generalized subgroup of an ω-balanced topological generalized group is ω-balanced.

Proposition 2.18. *Let G be an ω-narrow topological generalized group, then G is ω-balanced.*

Proof. Let a be an arbitrary element of $e(G)$. Since G is ω-narrow, G_a is an ω-narrow group. By [3, Proposition 3.4.10], the invariance number of G_a is countable and so G is ω-balanced. □

The converse of Proposition 2.18 need not be true. Indeed, a topological generalized group G with multiplication $a * b = a$ and discrete topology is ω-balanced, while it is ω-narrow if and only if $e(G) = G$ is countable.

Proposition 2.19. *The invariance number of a first-countable topological generalized group G is countable.*

Proof. Let a be an arbitrary element of $e(G)$. Then, G_a is a first-countable topological group. By [3, Theorem 3.4.11] we have $inv(G_a) \leq \omega$. Thus, the invariance number of G is countable. □

3. **Resolvability of topological generalized groups**

A topological space X is called *irresolvable* if each pair of dense subsets of X has non-empty intersection; otherwise, X is called *resolvable* [6]. X is called *hereditarily irresolvable* if every non-empty subspace of X is irresolvable [6].

Hewitt studied resolvable and irresolvable spaces in [6]. The following theorem is needed in the sequel.

Theorem 3.1. [6] *Every topological space X can be represented as a disjoint union $X = F \cup E$, where F is closed and resolvable and E is open and hereditarily irresolvable.*

It is easily seen that the representation of X in Theorem 3.1 is unique. It will henceforth be called "Hewitt representation" of X. The next proposition is an immediate consequence of [4, Lemma 3.1] and Theorem 1.4.

Proposition 3.2. *Suppose that G is a topological generalized group such that the family $\mathcal{F} = \{G_a\}_{a\in e(G)}$ is locally finite. Then G is resolvable if and only if G_a is resolvable for every $a \in e(G)$.*

The assumption that the family $\mathcal{F} = \{G_a\}_{a\in e(G)}$ is locally finite is essential in the proof of Proposition 3.2. For example, the real line $\mathbb{R}$ is resolvable and $\mathbb{R}$ with the multiplication $x.y = x$ is a topological generalized group such that the family $\{\mathbb{R}_a\}_{a\in e(\mathbb{R})}$ is not locally finite and if $a \in e(\mathbb{R})$, then $\mathbb{R}_a = \{a\}$ is irresolvable.

Proposition 3.3. *Let G be a topological generalized group and let the family $\mathcal{F} = \{G_a\}_{a\in e(G)}$ be locally finite. If for every $a \in e(G)$, E_a is a hereditarily irresolvable subspace of G_a, then $\bigcup_{a\in e(G)} E_a$ is hereditarily irresolvable subspace of G.*

Proof. Suppose to the contrary that $\bigcup_{a\in e(G)} E_a$ is not hereditarily irresolvable. So there is a resolvable subspace A in $\bigcup_{a\in e(G)} E_a$. Now it follows that for some $a \in e(G)$, $A_a = A \cap G_a$ is a non-empty open subspace of A and so, it is resolvable. Therefore, A_a is a resolvable subspace of E_a, which is a contradiction. □

Proposition 3.4. *Let G be a topological generalized group such that the family $\mathcal{F} = \{G_a\}_{a\in e(G)}$ is locally finite. Then, $F \cup E$ is the Hewitt representation of G if and only if for any $a \in e(G)$, $F_a \cup E_a$ is the Hewitt representation of G_a, where $F_a = F \cap G_a$ and $E_a = E \cap G_a$.*

Proof. Let $F_a \cup E_a$ be the Hewitt representation of G_a, where $a \in e(G)$. We claim that $(\cup_{a\in e(G)} F_a) \cup (\cup_{a\in e(G)} E_a)$ is the Hewitt representation of G. $\cup_{a\in e(G)} F_a$ is resolvable and it is closed since the family $\{G_a\}_{a\in e(G)}$ is locally finite. On the other hand, $\cup_{a\in e(G)} E_a$ is an open subspace of G which is hereditarily irresolvable by the above proposition. It is clear that $(\cup_{a\in e(G)} F_a) \cap (\cup_{a\in e(G)} E_a) = \emptyset$. Thus, our claim is proved.

Conversely, let $F \cup E$ be the Hewitt representation of G. By Theorem 1.4, $F_a = F \cap G_a$ is an open subset of F and so it is resolvable. It is also clear that F_a is a closed subset of G_a. On the other hand, since every subspace of a hereditarily irresolvable space is hereditarily irresolvable, then $E_a = E \cap G_a$ is an open and hereditarily irresolvable subspace of G_a. Now we can see $F_a \cap E_a = \emptyset$ and so, $G_a = F_a \cup E_a$ is the Hewitt representation of G_a. □

Proposition 3.5. *Let G be a topological generalized group and let H be a dense generalized subgroup of G. If the family $\mathcal{F} = \{G_a\}_{a\in e(G)}$ is* *locally finite and $H \neq G$, then G_a is a resolvable topological group for some $a \in e(G)$.*

Proof. By hypothesis H is a proper dense generalized subgroup of $G = \dot{\bigcup}_{a\in e(G)} G_a$. Thus, there exists $a \in e(G)$ such that $H_a = H \cap G_a$ is a proper subgroup of G_a. On the other hand, by Proposition 1.5 H_a is dense in G_a. Therefore, H_a is a proper dense subgroup of G_a and so by [4, Lemma 3.3], G_a is resolvable. □

Proposition 3.6. *Let G be a resolvable topological generalized group and $a \in e(G)$. If $int(G_a) \neq \emptyset$, then G_a is resolvable.*

Proof. Since G is resolvable, $int(G_a)$ is resolvable and the topological group G_a is a homogeneous space containing $int(G_a)$. Thus, G_a is resolvable. □

Note that Proposition 3.6 implies that if for some $a \in e(G)$, $int(G_a) \neq \emptyset$ and G_a is irresolvable, then G is irresolvable.

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ω-باریک و حل‌پذیر بودن گروه‌های تعمیم‌یافته‌ی توپولوژیک

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گروه توپولوژیک H را ω-باریک گویند هرگاه برای هر همسایگی V از عضو همانی آن، مجموعه‌ی شمارای A وجود داشته باشد به‌طوری‌که $VA = H = AV$. نیم‌گروه G گروهی تعمیم‌یافته نامیده می‌شود اگر برای هر $x \in G$، عضو یکتای $e(x) \in G$ وجود داشته باشد به‌طوری‌که، $xe(x) = x = e(x)x$ و برای هر $x \in G$، عضو $x^{-1} \in G$ وجود داشته باشد به‌طوری‌که، $xx^{-1} = e(x) = x^{-1}x$. هم‌چنین فرض کنید G فضای توپولوژیک نیز باشد به‌طوری‌که نگاشت عمل دوتایی و نگاشت معکوس روی آن پیوسته باشند، در این‌صورت G گروه تعمیم‌یافته‌ی توپولوژیک نامیده می‌شود. اگر $\{e(x) \mid x \in G\}$ شمارا باشد و برای هر $a \in G$ مجموعه‌ی $\{x \in G \mid e(x) = e(a)\}$ گروه توپولوژیک ω-باریک باشد، آنگاه G گروه تعمیم‌یافته‌ی توپولوژیک ω-باریک نامیده می‌شود. در این مقاله، گروه‌های تعمیم‌یافته‌ی توپولوژیک ω-باریک و حل‌پذیر را معرفی می‌کنیم و مورد مطالعه قرار می‌دهیم.

کلمات کلیدی: گروه تعمیم‌یافته‌ی توپولوژیک ω-باریک، گروه تعمیم‌یافته‌ی توپولوژیک حل‌پذیر، عدد پایداری، گروه تعمیم‌یافته‌ی توپولوژیک پیش‌فشرده.