

A CHARACTERIZATION FOR METRIC TWO-DIMENSIONAL GRAPHS AND THEIR ENUMERATION

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ABSTRACT. The *metric dimension* of a connected graph G is the minimum number of vertices in a subset B of G such that all other vertices are uniquely determined by their distances to the vertices in B . In this case, B is called a *metric basis* for G . The *basic distance* of a metric two-dimensional graph G is the distance between the elements of B . Giving a characterization for those graphs whose metric dimensions are two, we enumerate the number of n vertex metric two-dimensional graphs with the basic distance 1.

1. INTRODUCTION

Let $G = (V, E)$ be a connected simple graph. For two vertices u and v of G , the distance $d_G(x, y)$ or $d(x, y)$ of x and y is the length of a minimum path connecting x to y . For a subset $R = \{r_1, \dots, r_k\}$ of V and a vertex v , the representation of v with respect to R is the k -tuple $\langle v|R \rangle = (d(v, r_1), \dots, d(v, r_k))$. The subset R is called a resolving set for G if any vertex has a unique representation with respect to R . A resolving set B of V is called a metric basis for G if it has the minimum possible number of elements for a resolving set. The metric dimension G , denoted by $\dim_M(G)$ is then equal to this minimum number. For a study about these notions, we refer the reader to [4] and [8].

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As a simple known fact, $\dim_M(G) = 1$ if and only if G is a path. The metric dimension of an n vertex graph G is $n - 1$ if and only if G is the complete graph K_n ; see [3].

The concept of a resolving set has various applications in different areas including network discovery and verification [1], problems of pattern recognition and image processing [6], robot navigation [5], mastermind game [2], and combinatorial search and optimization [7].

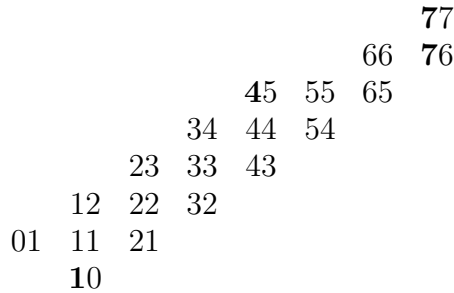
2. A CHARACTERIZATION FOR $\dim_M(G) = 2$

In this section, we aim to characterize all two metric dimensional graphs, but prior to this we need to extend the notion of a path.

Definition 2.1. Let x and d be two positive integers with $x \geq d$ and let y be a nonnegative integer. An *extended path* $\mathcal{P}(x, y, d)$ of the length x , width y , and height $2d + 1$ is a simple graph with the following properties:

- i. $V(\mathcal{P}) = \cup_{i=0}^x P_i$, where $P_i = \{v_{i,|i-d|}, v_{i,|i-d|+1}, \dots, v_{i,i+d}\}$ for $0 \leq i \leq y$ and $P_i = \{v_{i,|i-d|}, v_{i,|i-d|+1}, \dots, v_{i,i+d-1}\}$ for $y + 1 \leq i \leq x$;
- ii. neighbors of $v_{i,j}$ are $v_{k,\ell}$ with $|i - k| \leq 1$ and $|j - \ell| \leq 1$.

Example 2.2. As an example, the generalized path $\mathcal{P}(7, 4, 1)$ has vertices of the form



and there is an edge between any two vertices, which are horizontally, vertically, or diagonally adjacent. Whence any horizontal, vertical, or diagonal line is a path. Here, P_i 's are vertical lines numbered from left to right by P_0, P_1, \dots, P_7 . The length of the first diagonal path from top is $y = 4$, the left coordinate of any vertex in the last path is $x = 7$, and the left coordinate of the only vertex in the first horizontal path from down is $d = 1$.

As another example, the generalized path $\mathcal{P}(6, 3, 4)$ has vertices of the form

						69
						58
						68
						37
					47	57
					67	
					26	36
					46	56
					66	
					15	25
					35	45
					55	65
					04	14
					24	34
					44	54
					64	
					13	23
					33	43
					53	63
					22	32
					42	52
					62	
					31	41
					51	
					40	

and there is an edge between any two vertices which are horizontally, vertically, or diagonally adjacent.

Definition 2.3. Let G be a metric two-dimensional graph with the metric basis $B = \{a, b\}$. Then $d(a, b)$ is called the *basic distance* of G with respect to B and is denoted by $BD_B(G)$.

Proposition 2.4. Let x and d be two positive integers with $x \geq d$ and let y be a nonnegative integer. If $(x, y, d) \neq (1, 0, 1)$, then the generalized path $\mathcal{P}(x, y, d)$ is a metric two-dimensional graph with the metric basis $B = \{v_{0,d}, v_{d,0}\}$ and the basic distance d with respect to B . Moreover, $\langle v_{i,j} | B \rangle = (i, j)$ for each $v_{i,j} \in \mathcal{P}$.

Proof. At first we note that if $(x, y, d) \neq (1, 0, 1)$, then $\mathcal{P}(x, y, d)$ is not a path. We can therefore deduce that $\dim_M(\mathcal{P}(x, y, d)) \geq 2$. We show that $B = \{a := v_{0,d}, b := v_{d,0}\}$ is a metric basis for $\mathcal{P}(x, y, d)$. In fact, we use induction on $i + j$ to show that $\langle v_{i,j} | B \rangle = (i, j)$ for each $v_{i,j} \in \mathcal{P}$.

The minimum possible value for $i + j$ is d . There are $d + 1$ vertices

$$v_{i,j} = v_{0,d}, v_{1,d-1}, \dots, v_{d-1,1}, v_{d,0}$$

with $i + j = d$. Consider the shortest path

$$a = v_{0,d}, v_{1,d-1}, \dots, v_{d-1,1}, v_{d,0} = b$$

to see that $\langle v_{i,j} | B \rangle = (i, j)$ for these vertices. In particular note that $d(a, b) = d$. Thus $BD_B(\mathcal{P}(x, y, d)) = d$.

Now let $\langle v_{i,j} | B \rangle = (i, j)$ for each vertex $v_{i,j}$ with $i + j < N$. Let $v_{k,\ell}$ be a vertex with $k + \ell = N$. Any path from $v_{i,j}$ to a should pass from one of the vertices $v_{i-1,j-1}, v_{i-1,j}, v_{i-1,j+1}$. The distance between each of these vertices to a is $i - 1$, by the induction hypothesis. Thus $d(v_{i,j}, a) = i$. A similar argument shows that $d(v_{i,j}, b) = j$. \square

Lemma 2.5. Let x and d be two positive integers with $x \geq d$ and let y be a nonnegative integer. Then $\mathcal{P}(x, y, d) = \mathcal{P}(x, y, 1) \cup \mathcal{P}(x, x, d - 1)$ and $\mathcal{P}(x, y, 1) \cap \mathcal{P}(x, x, d - 1)$ is a path.

Proof. Let $V(\mathcal{P}) = \cup_{i=0}^x P_i$, where $P_i = \{v_{i,|i-d|}, v_{i,|i-d|+1}, \dots, v_{i,i+d}\}$ for $0 \leq i \leq y$ and $P_i = \{v_{i,|i-d|}, v_{i,|i-d|+1}, \dots, v_{i,i+d-1}\}$ for $y+1 \leq i \leq x$.

Put

$$\begin{aligned} P'_i &= \{v'_{i,j-(d-1)} : v_{i,j} \in P_i \text{ and } j \geq i+d-2\}, \\ P''_i &= \{v''_{i-1,j} : v_{i,j} \in P_i \text{ and } j \leq i+d-2\}. \end{aligned}$$

Now if \mathcal{P}' is the subgraph of \mathcal{P} induced by $\cup_{i=0}^x P'_i$ and \mathcal{P}'' is the subgraph of \mathcal{P} induced by $\cup_{i=1}^x P''_i$, then $\mathcal{P}' = \mathcal{P}(x, y, 1)$, $\mathcal{P}'' = \mathcal{P}(x, x, d-1)$, $\mathcal{P}(x, y, d) = \mathcal{P}' \cup \mathcal{P}''$ and $\mathcal{P}' \cap \mathcal{P}''$ is the path $\{v_{1,d-1}, v_{2,d}, \dots, v_{x,x+d-2}\}$. □

Theorem 2.6. *A simple graph G is a metric two-dimensional graph with the basic distance d if and only if it is a subgraph of a generalized path $\mathcal{P}(x, y, d)$ with $(x, y, d) \neq (1, 0, 1)$ satisfying the following properties:*

- i. $v_{0,d}, v_{d,0} \in G$;
- ii. $N(v_{i,j}) \cap \{v_{i-1,j-1}, v_{i-1,j}, v_{i-1,j+1}\} \neq \emptyset$ and $N(v_{i,j}) \cap \{v_{i-1,j-1}, v_{i,j-1}, v_{i+1,j-1}\} \neq \emptyset$ for each $v_{ij} \in G$.

Proof. An inductive argument proves that any subgraph of $\mathcal{P}(x, y, d)$ with $(x, y, d) \neq (1, 0, 1)$ possessing the properties (i) and (ii) is a metric two-dimensional graph with the basis $B = \{a := v_{0,d}, b := v_{d,0}\}$ and the basic distance d .

Conversely, suppose that G is a metric two-dimensional graph with the basis $B = \{a, b\}$ and the basic distance d . Let

$$x := \max\{d(v, a) : v \in G\},$$

and

$$y := \max\{i : (i, i+d) = \langle v|B \rangle, \text{ for some } v \in G\}.$$

Define $\varphi : G \rightarrow \mathcal{P}(x, y, d)$ by $\varphi(v) = v_{i,j}$, where $(i, j) = \langle v|B \rangle$. We show that

$$\begin{aligned} |i-d| \leq j \leq i+d, & \quad \text{for } i = 0, \dots, y, \\ |i-d| \leq j \leq i+d-1, & \quad \text{for } i = y+1, \dots, x. \end{aligned}$$

We have $d(v, a) = i$ and $d(v, b) = j$, since $(i, j) = \langle v|B \rangle$. The triangle inequality implies that $d = d(a, b) \leq d(a, v) + d(v, b) = i + j$. Moreover, $j = d(v, b) \leq d(v, a) + d(a, b) = i + d$ and $i = d(v, a) \leq d(v, b) + d(b, a) = j + d$. Thus $|i-d| \leq j \leq i+d$ for each $0 \leq i \leq x$.

If $i \geq y+1$, then j cannot be $i+d$, since otherwise we should have $(i, i+d) = (i, j) = \langle v|B \rangle$ which contradicts the definition of y . Hence $j \leq i+d-1$ for $i \geq y+1$.

We therefore have $\varphi(V(G)) \subseteq V(\mathcal{P}(x, y, d))$. Now let $e = uv$ be an edge in $V(G)$. If $\varphi(u) = v_{i,j}$ and $\varphi(v) = v_{k\ell}$, then

$$i = d(u, a) \leq d(u, v) + d(v, a) = 1 + k,$$

and

$$k = d(v, a) \leq d(v, u) + d(u, a) = 1 + i.$$

Thus $|i - k| \leq 1$. By the same argument, $|j - \ell| \leq 1$. This shows that $k = i - 1, i$ or $i + 1$ and $\ell = j - 1, j$ or $j + 1$. Whence $\varphi(e)$ is an edge in $\mathcal{P}(x, y, d)$ and so G is a subgraph of $\mathcal{P}(x, y, d)$.

Clearly, $v_{0,d} = a, v_{d,0} = b \in G$. To show that (ii) does also hold, note that if, for example, $N(v_{i,j}) \cap \{v_{i-1,j-1}, v_{i-1,j}, v_{i-1,j+1}\} = \emptyset$, then there is no path with the length i from $v_{i,j}$ to a . \square

3. ENUMERATING OF METRIC TWO-DIMENSIONAL GRAPHS WITH THE BASIC DISTANCE 1

Lemma 2.5 shows that any generalized path $\mathcal{P}(x, y, d)$ can be regarded as a larger path $\mathcal{P}(x', y', d')$. Thus the generalized path mentioned in Theorem 2.6 is not unique. A simple argument based on the property (ii) of Theorem 2.6 implies that if $x = \max\{d(v, a) : v \in G\}, y = \max\{i : (i, i + d) = \langle v|B \rangle, \text{ for some } v \in G\}$ and $d = d(a, b)$, then the boundary $\partial\mathcal{P}(x, y, d)$

$v_{0,d}, v_{1,d-1}, v_{2,d-2}, \dots, v_{d,0}, v_{d+1,1}, v_{d+2,2}, \dots, v_{x,x-d}, v_{1,d+1}, v_{2,d+2}, \dots, v_{y,y+d}$ of $\mathcal{P}(x, y, d)$ are vertices of G . Whence this x, y and d are the least possible values such that G is a subgraph of $\mathcal{P}(x, y, d)$.

Definition 3.1. Let G be a simple metric two-dimensional graph. We say that G is fitted in $\mathcal{P}(x, y, d)$, denoted by $G \sqsubseteq \mathcal{P}(x, y, d)$, if

$$\begin{aligned} x &= \max\{d(v, a) : v \in G\}, \\ y &= \max\{i : (i, i + d) = \langle v|B \rangle, \text{ for some } v \in G\}, \\ d &= d(a, b), \end{aligned}$$

or equivalently G contains the boundary $\partial\mathcal{P}(x, y, d)$

$v_{0,d}, v_{1,d-1}, v_{2,d-2}, \dots, v_{d,0}, v_{d+1,1}, v_{d+2,2}, \dots, v_{x,x-d}, v_{1,d+1}, v_{2,d+2}, \dots, v_{y,y+d}$ of $\mathcal{P}(x, y, d)$. The parameters x and y are called the length and width of G and are denoted by $\ell(G)$ and $w(G)$, respectively.

We now want to enumerate the number of n vertex metric two-dimensional graph with the basic distance 1. Prior to this, we enumerate the number of n vertex metric two-dimensional graph with the length x , width y , and the basic distance 1. We denote the latter number by $\nu(n; x, y)$.

Lemma 3.2. $\nu(n; x, y) \geq 1$ if and only if $x + y + 2 \leq n \leq 2x + y + 1$.

Proof. Suppose that there is an n vertex metric two-dimensional graph G with the length x , width y and the basic distance 1. Using Theorem 2.6, we fit it in $\mathcal{P}(x, y, 1)$. Since the boundary of $\mathcal{P}(x, y, 1)$ has $x + y + 1$ elements, we should have $n \geq x + y + 1$. If $n = x + y + 1$, then $G = \partial\mathcal{P}(x, y, 1)$ which is a path and has metrics dimension 1. Thus $n \geq x + y + 2$. Moreover, $n = |V(G)| \leq |V(\mathcal{P}(x, y, 1))| = 2x + y + 1$.

On the other hand, if $x + y + 2 \leq n \leq 2x + y + 1$, then we can write $n = x + y + 1 + r$, where $1 \leq r \leq x$. Now consider the subgraph of $\mathcal{P}(x, y, 1)$ induced by $\partial\mathcal{P}(x, y, 1) \cup \{v_{1,1}, \dots, v_{r,r}\}$. This is an n vertex subgraph of $\mathcal{P}(x, y, 1)$ satisfying (i) and (ii) of Theorem 2.6. \square

Based on Lemma 3.2, for simplicity, we denote $\nu(n; x, y)$ by $\mu(m; x, y)$. We note that $\mu(m; x, y) \geq 1$ if and only if $1 \leq m \leq x$.

Lemma 3.3. $\mu(x; x, y) = 4 \times 20^{y-1} \times 10^{x-y}$ for each $x \geq y \geq 1$ and $\mu(x; x, 0) = 2 \times 10^{x-1}$ for each $x \geq 1$.

Proof. Let G be an n vertex metric two-dimensional graph G with the length x , width y and the basic distance 1, where $n = 2x + y + 1$. Thus $G \subseteq \mathcal{P}(x, y, 1)$ and the induced subgraph $\partial\mathcal{P}(x, y, 1)$ of $\mathcal{P}(x, y, 1)$ should be a subgraph of G . For other vertices

$$\{v_{1,1}, \dots, v_{y,y}, v_{y+1,y+1}, \dots, v_{x,x}\},$$

we should put the edges in such a way that (ii) of Theorem 2.6 is satisfied. For $v_{1,1}$ putting edges $v_{1,1}v_{0,1}$ and $v_{1,1}v_{1,0}$ is compulsory, and we have 4 choices for ‘to put’ or ‘not to put’ the edges $v_{1,1}v_{1,2}$ and $v_{1,1}v_{2,1}$.

If $1 < i \leq y$, then for $v_{i,i}$ putting one of the 5 sets of edges,

$$\begin{aligned} & \{v_{i,i}v_{i-1,i-1}\}, \{v_{i,i}v_{i-1,i}, v_{i,i}v_{i,i-1}\}, \{v_{i,i}v_{i-1,i-1}, v_{i,i}v_{i-1,i}\}, \\ & \{v_{i,i}v_{i-1,i-1}, v_{i,i}v_{i,i-1}\}, \{v_{i,i}v_{i-1,i-1}, v_{i,i}v_{i-1,i}, v_{i,i}v_{i,i-1}\} \end{aligned}$$

is compulsory and we have 4 choices for ‘to put’ or ‘not to put’ the edges $v_{i,i}v_{i,i+1}$ and $v_{i,i}v_{i+1,i}$.

If $y + 1 \leq i \leq x$, then the 4 choices decreases into 2 choices, since we do not have $v_{i,i+1}$.

Finally, if $y = 0$, then we have 2 choices for $v_{1,1}$ and 10 choices for $v_{i,i}$ when $1 < i \leq x$. \square

Though we know that $\mu(0; x, y) = 0$, but for the following recursive relation, we need to assume, as a convenient, that $\mu(0; x, y) = 1$.

Furthermore, for $y \geq 1$, we assume that

$$\omega(j) = \begin{cases} 4, & j = 1, \\ 20, & 2 \leq j \leq y, \\ 10, & y + 1 \leq j \leq x, \end{cases}$$

and for $y = 0$ we assume that

$$\omega(j) = \begin{cases} 2, & j = 1, \\ 10, & 2 \leq j \leq x. \end{cases}$$

Theorem 3.4. *Let x be a positive integer, let y be a nonnegative integer, and let $1 \leq m < x$. Then $\mu(m; x, y)$ satisfies the recursive relation*

$$\mu(m; x, y) = \sum_{i=1}^{m+1} \left(\prod_{j=1}^{i-1} \omega(j) \right) \cdot \mu(m - (i - 1); x - i, \max\{y - i, 0\}),$$

with the boundary values

$$\mu(0; x, y) = 1, \quad \mu(x; x, y) = \prod_{j=1}^x \omega(j).$$

Proof. To determine $\mu(m; x, y)$, we in fact need to enumerate the number of $n = x + y + 1 + m$ vertex metric two-dimensional subgraphs G of $\mathcal{P}(x, y, 1)$ with the basic distance 1. Let $m < x$. Then there is a vertex $v_{i,i} \in \mathcal{P}(x, y, 1) \setminus G$. Let i be the first index such that $v_{i,i} \in \mathcal{P}(x, y, 1) \setminus G$. Then $1 \leq i \leq m + 1$. Since $v_{1,1}, \dots, v_{i-1,i-1} \in G$, we have $\prod_{j=1}^{i-1} \omega(j)$ choices for selecting appropriate edges. Then we have $\mu(m - (i - 1); x - i, \max\{y - i, 0\})$ choices for selecting other edges for other vertices of G . \square

Corollary 3.5. *Let x be a positive integer and let $1 \leq m < x$. Then*

$$\mu(m; x, 0) = \mu(m; x - 1, 0) + \sum_{i=2}^{m+1} 2 \times 10^{i-2} \cdot \mu(m - (i - 1); x - i, 0).$$

Example 3.6. We evaluate $\mu(m; x, 0)$ for $m = 1, 2, 3$ and $x > m$.

A simple verification shows that $\mu(1; x, 0) = 2x$. For $m = 2 < x$ we have

$$\begin{aligned} \mu(2; x, 0) &= \mu(2; x - 1, 0) + 2\mu(1; x - 2, 0) + 20\mu(0; x - 3, 0) \\ &= \mu(2; x - 1, 0) + 2 \cdot 2(x - 2) + 20 \\ &= \mu(2; x - 1, 0) + 4(x + 3). \end{aligned}$$

Iterating the above equation, we have

$$\begin{aligned}
 \mu(2; x, 0) &= \mu(2; x - 1, 0) + 4(x + 3) \\
 &= \mu(2; x - 2, 0) + 4(x + 2) + 4(x + 3) \\
 &= \mu(2; x - 3, 0) + 4(x + 1) + 4(x + 2) + 4(x + 3) \\
 &= \dots \\
 &= \mu(2; 2, 0) + 4(2 + 4) + \dots + 4(x + 3) \\
 &= 20 + 4 \left(\frac{(x + 3)(x + 4)}{2} - 15 \right) \\
 &= 2(x - 1)(x + 8).
 \end{aligned}$$

Finally, for $m = 3 < x$, we have

$$\begin{aligned}
 \mu(3; x, 0) &= \mu(3; x - 1, 0) + 2\mu(2; x - 2, 0) + 20\mu(1; x - 3, 0) \\
 &\quad + 200\mu(0; x - 4, 0) \\
 &= \mu(3; x - 1, 0) + 2 \cdot 2(x - 3)(x + 6) + 20 \cdot 2(x - 3) + 200 \\
 &= \mu(3; x - 1, 0) + 4(x^2 + 13x + 2).
 \end{aligned}$$

A similar method gives

$$\mu(3; x, 0) = \frac{4}{3}x^3 + 28x^2 + \frac{104}{3}x - 192.$$

Corollary 3.7. *Let x be a positive integer and let $1 \leq m < x$. Then $\mu(m; x, 0)$ is a polynomial of x of degree m .*

Proof. Using induction on $m + x$, we can assume that the right hand side of Corollary 3.5 is a polynomial of x of degree m . Whence the left hand side is also a polynomial of x of degree m . \square

We now can simply evaluate $\nu(n)$; the number of all n vertex labeled metric two-dimensional graph with the basis $B = \{a, b\}$ and the basic distance 1.

Theorem 3.8. *The number of all n vertex labeled metric two-dimensional graph G with the basis $B = \{a, b\}$ and the basic distance 1, is*

$$\nu(n) = \sum_{y=0}^{\lfloor \frac{n-1}{3} \rfloor} \sum_{x=\lceil \frac{n-y-1}{2} \rceil}^{n-y-2} \mu(n - x - y - 1; x, y).$$

Proof. Each G can be fitted in a $\mathcal{P}(x, y, 1)$ where, by Lemma 3.2, we should have $x + y + 2 \leq n \leq 2x + y + 1$. Thus the valid values of x and y are $0 \leq y \leq \lfloor \frac{n-1}{3} \rfloor$ and $\lceil \frac{n-y-1}{2} \rceil \leq x \leq n - y - 2$. We know that the number of metric two-dimensional subgraph of $\mathcal{P}(x, y, 1)$ is $\mu(n - x - y - 1; x, y)$. \square

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مشخص سازی گراف های با بعد متریک دو و شمارش آنها

مصطفی محقق نژاد، فریدون رهبرنیا، مجید میرزاووزیری و رضا قنبری
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بعد متریک گراف G عبارت است از حداقل تعداد راس های لازم در زیرمجموعه B از رئوس گراف، به طوری که تمام راس های دیگر به واسطه فاصله ی آنها تا راس های B ، به طور منحصر به فرد تعیین شوند. در این حالت B را پایه متریک گراف G می نامیم. فاصله پایه یک گراف دو بعدی G ، به صورت فاصله بین دو عنصر B تعریف می شود. در این مقاله، ابتدا گراف های با بعد متریک دو مشخص سازی می شود و سپس تعداد رئوس گراف های با بعد متریک دو، که فاصله پایه آنها یک باشد را خواهیم شمرد.

کلمات کلیدی: بعد متریک گراف ها، مجموعه کاشف، پایه متریک گراف، فاصله پایه ای، شمارش گراف ها.