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Research Paper

QUATERNARY CODES AND A CLASS OF 2-DESIGNS INVARIANT UNDER THE GROUP A_8

REZA KAHKESHANI*

Abstract. In this paper, we use the Key-Moori Method 1 and construct a quaternary code $\mathcal{C}_8$ from a primitive representation of the group $PSL_2(9)$ of degree 15. We see that $\mathcal{C}_8$ is a self-orthogonal even code with the automorphism group isomorphic to the alternating group A_8. It is shown that by taking the support of any codeword ω of weight l in $\mathcal{C}_8$ or $\mathcal{C}_8^{\perp}$, and orbiting it under A_8, a 2-$(15, l, \lambda)$ design invariant under the group A_8 is obtained, where $\lambda = \binom{l}{2}|\omega^{A_8}|/\binom{15}{2}$. A number of these designs have not been known before up to our best knowledge. The structure of the stabilizers $(A_8)_\omega$ is determined and moreover, primitivity of A_8 on each design is examined.

1. Introduction

In [16, 17], Key and Moori considered the primitive permutation representations of the Janko groups J_1 and J_2, and constructed designs, codes and graphs invariant under J_1, J_2

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*Corresponding author

or $\bar{J}_2$, where $\bar{J}_2 = \mathrm{Aut}(J_2) = J_2{:}2$. Together with Rodrigues [18], they considered the groups $PSp_n(q)$, A_6 and A_9, and showed that it is possible to construct some designs $\mathcal{D}$ from a group G such that $\mathrm{Aut}(\mathcal{D})$ and $\mathrm{Aut}(G)$ have no containment relationship. Moori and Rodrigues [20, 21] applied the same method to the sporadic simple groups M^cL and Co_2. They constructed a self-orthogonal doubly even code $\mathcal{C}$ invariant under $M^cL{:}2$ using a primitive representation of degree 275 of the McLaughlin group M^cL. Using the actions of M^cL and $M^cL{:}2$ on $\mathcal{C}$, they constructed 1-designs $\mathcal{D}_\omega$, and investigated the maximality of the stabilizers and the primitivity of the actions on $\mathcal{D}_\omega$. They also examined some designs and their binary codes constructed from a primitive representation of degree 2300 of the group Co_2 of Conway. Rodrigues [28] considered the primitive representations of the projective symplectic groups $S_4(3)$ and $S_4(4)$, and showed that some of the constructed codes are optimal or near optimal. Recently, Moori and his research group constructed a number of symmetric 1-designs from the maximal subgroups of the small Ree groups ${}^2G_2(q)$ [22, 23]. Furthermore, see [1, 12] for construction of designs and codes from the Mathieu group M_{11} and the Held's simple group He.

In [8, 9, 11, 13, 14], the authors constructed some designs and found their automorphism groups using all the primitive representations of the projective special linear groups $PSL_2(q)$, where $q \leq 65$ is a prime power. Moreover, the authors examined some of their binary codes and automorphism groups [10, 15]. In [7], Darafsheh considered the group $PSL_2(2^n)$ and found three designs $\mathcal{D}_{2^n+1}$, $\mathcal{D}_{2(2^n-1)}$ and $\mathcal{D}_{2^n-1}$ with parameters 1-$\left(\binom{2^n}{2}, 2^n+1, 2^n+1\right)$, 1-$\left(\binom{2^n+1}{2}, 2(2^n-1), 2(2^n-1)\right)$ and 1-$\left(\binom{2^n+1}{2}, 2^n-1, 2^n-1\right)$, respectively, such that $\mathrm{Aut}(\mathcal{D}_{2(2^n-1)}) \cong S_{2^n+1}$. Recently, Moori and Saeidi [24, 25, 26] constructed some designs and their binary codes from the Tits group ${}^2F_4(2)'$, some certain 1-designs invariant under the projective special linear group $PSL_2(2^n)$ and some designs from the Suzuki groups $Sz(2^{2n+1})$. In [10], Darafsheh et al. considered the primitive representations of the groups $PSL_2(8)$ and $PSL_2(9) \cong A_6$, and constructed designs $\mathcal{D}_7$, $\mathcal{D}_{14}$ and $\mathcal{D}_8$ with parameters 1-$(36, 7, 7)$, 1-$(36, 14, 14)$ and 2-$(15, 8, 4)$, respectively, such that $\mathrm{Aut}(\mathcal{D}_7) \cong PSL_2(8)$, $\mathrm{Aut}(\mathcal{D}_{14}) \cong S_9$ and $\mathrm{Aut}(\mathcal{D}_8) \cong A_8$. They showed that the binary codes associated with these designs, i.e., C_7, C_{14} and C_8, have parameters $[36, 28, 3]$, $[36, 8, 8]$ and $[15, 4, 8]$, respectively. The authors showed that $C_8^\perp$ is the Hamming code $\mathcal{H}_4(2)$ and $C_7^\perp = C_{14}$. Also, $\mathrm{Aut}(C_7) = \mathrm{Aut}(C_{14}) \cong S_9$ and $\mathrm{Aut}(C_8) \cong A_8$. They determined the structure of the stabilizers $(S_9)_\omega$ and $(A_8)_\omega$ for any non-zero codeword ω, and showed that the stabilizers $(S_9)_\omega$ and $(A_8)_\omega$ are maximal subgroups of S_9 and A_8, respectively. It was seen that $\mathcal{C}_{14}$ is the smallest non-trivial F_2-module on which S_9 acts irreducibly on it. They showed that if we take $\mathrm{Supp}(\omega)$, for any ω in C_{14} and C_8, and orbit it under the automorphism groups S_9 and A_8, respectively, then 1-$(36, l, k_l)$ designs and a 1-$(15, 8, 8)$ design are obtained, where $l = \mathrm{wt}(\omega)$ and $k_l = l|\omega^G|/36$. It was seen that the groups S_9 and A_8 act primitively on these designs.

Moreover, the authors examined self-dual designs and their binary codes obtained from the primitive representations of the group $PSL_2(13)$ using the Key-Moori Method 1 and showed that $PSL_2(13)$ and $PSL_2(13){:}2$ appear as the full automorphism group of the obtained designs and binary codes [15].

In this paper, motivated by the mentioned works, we consider designs from the primitive representations of the group $PSL_2(9)$ and construct their quaternary codes. It is seen that the obtained codes are all trivial except the one constructed from the primitive representation of $PSL_2(9)$ of degree 15. In fact, the natural action of $PSL_2(9)$ on the right cosets of its maximal subgroup S_4 gives us a self-orthogonal even code $\mathcal{C}_8$ with parameters $[15, 4, 8]_4$ invariant under its automorphism group A_8. Its dual is a $[15, 11, 3]_4$ code and $\jmath \in \mathcal{C}_8^{\perp}$. Furthermore, we examine the stabilizers $(A_8)_\omega$, where ω is a codeword in $\mathcal{C}_8$ or $\mathcal{C}_8^{\perp}$, and determine their structures. Also, the maximality of the stabilizers and the primitivity of the actions are investigated. Moreover, for any codeword ω in $\mathcal{C}_8$ and $\mathcal{C}_8^{\perp}$, we take the support of ω and orbit it under A_8 to form the 1-$(15, l, k_l)$ designs $\mathcal{D}_\omega$ and $\mathcal{D}'_\omega$, respectively, where ω is of weight l and $k_l = l|\omega^{A_8}|/15$. It is shown that these 1-designs are in fact designs with parameters 2-$(15, l, \lambda)$ invariant under the group A_8, where $\lambda = \binom{l}{2}|\omega^{A_8}|/\binom{15}{2}$. As we will see, a number of these 2-designs are new.

2. Preliminaries

Let $\mathcal{S} = (\mathcal{P}, \mathcal{B}, \mathcal{I})$ be an incidence structure which consists of two disjoint sets $\mathcal{P}$ and $\mathcal{B}$, and an incidence relation $\mathcal{I} \subseteq \mathcal{P} \times \mathcal{B}$. The sets $\mathcal{P}$, $\mathcal{B}$ and $\mathcal{I}$ are called point, block and flag sets, respectively. We will write $p\mathcal{I}B$ if and only if $(p, B) \in \mathcal{I}$. A block $B \in \mathcal{B}$ is sometimes identified with the set of points p incident with it, and we then write $p \in B$ when $p\mathcal{I}B$. In fact, $\mathcal{I}$ is here the membership relation $\in$. When we replace each block by its complement, the complementary structure $\overline{\mathcal{S}}$ is obtained. The dual of $\mathcal{S} = (\mathcal{P}, \mathcal{B}, \mathcal{I})$ is defined to be the structure $\mathcal{S}^\top = (\mathcal{B}, \mathcal{P}, \mathcal{I}^\top)$, where $B\mathcal{I}^\top p$ if and only if $p\mathcal{I}B$. The information of an incidence structure $\mathcal{S}$ can be represented in a $(0, 1)$-matrix M whose rows and columns are labled by points and blocks of $\mathcal{S}$, respectively. The matrix M, which is called the incidence matrix of $\mathcal{S}$, is of size $|\mathcal{P}| \times |\mathcal{B}|$ such that the entry $(p, B) \in \mathcal{P} \times \mathcal{B}$ is 1 if and only if p is incident with B, and 0 otherwise. For any $B \in \mathcal{B}$, let v^B denote the incidence vector of B, where B is considered as a subset of $\mathcal{P}$. The vector v^B is the column of M which is labeled by B. The incidence matrix of $\mathcal{S}^\top$ is $M^\top$, where the incidence matrix of $\mathcal{S}$ is M. A bijection $\varphi : \mathcal{P} \to \mathcal{P}'$ is called an isomorphism between two structures $\mathcal{S} = (\mathcal{P}, \mathcal{B}, \mathcal{I})$ and $\mathcal{S}' = (\mathcal{P}', \mathcal{B}', \mathcal{I}')$ if $p\mathcal{I}B \leftrightarrow \varphi(p)\mathcal{I}'\varphi(B)$ for any $p \in \mathcal{P}$ and $B \in \mathcal{B}$. The structures $\mathcal{S}$ and $\mathcal{S}'$ are called isomorphic, written $\mathcal{S} \cong \mathcal{S}'$, if there is an isomorphism φ between them. The structure $\mathcal{S}$ is said to be self-dual if $\mathcal{S} \cong \mathcal{S}^\top$. An isomorphism of $\mathcal{S}$ onto itself is an automorphism of $\mathcal{S}$. The automorphism group of $\mathcal{S}$, denoted by $\mathrm{Aut}(\mathcal{S})$, is the set of all the automorphisms of $\mathcal{S}$. The incidence structure $\mathcal{D} = (\mathcal{P}, \mathcal{B}, \mathcal{I})$

is named a t-(v,k,λ) design if $|\mathcal{P}| = v$, $|B| = k$ for any $B \in \mathcal{B}$, and any t distinct points of $\mathcal{P}$ are incident with precisely λ blocks of $\mathcal{B}$. The number of blocks, b, is $\lambda\binom{v}{t}/\binom{k}{t}$. A design $\mathcal{D}$ is called trivial if $\mathcal{B}$ consists of all the k-subsets of $\mathcal{P}$, i.e., $b = \binom{v}{k}$. The design $\mathcal{D}$ is symmetric if $v = b$. The number of blocks incident with s points is denoted by λ_s, where $s \leq t$. It is known that λ_s is independent of the set and $\lambda_s = \lambda\binom{v-s}{t-s}/\binom{k-s}{t-s}$. Thus, any t-(v,k,λ) design is a s-(v,k,λ_s) design. The number of blocks incident with any point is called the replication number r, and we have $r = \lambda_1$. It is known that in any 2-(v,k,λ) design, $bk = vr$ and $r(k-1) = \lambda(v-1)$. If $\mathcal{D}$ is a t-(v,k,λ) design then the complementary design $\overline{\mathcal{D}}$ is a t-$(v,v-k,\overline{\lambda})$ design, where $\overline{\lambda} = \sum_{s=0}^{t}(-1)^s\binom{t}{s}\lambda_s$. Moreover, $\mathcal{D}^\top$ is a design with b points such that the size of any block is r. Note that the complement of a 2-(v,k,λ) design is a design with parameters 2-$(v,v-k,b-2r+\lambda)$. We refer the reader to [5] for known t-(v,k,λ) designs. Since the complement of a t-design is again a t-design, the convention is to mention only t-(v,k,λ) designs with $k \leq v/2$. For more details, see [2, 3, 5].

Any subspace of the vector space F_q^n is called a q-ary linear code of length n. In fact, a linear code $\mathcal{C}$ over the finite field F_q is an $[n,k,d]_q$ code if $\mathcal{C}$ is a code of length n, dimension k and minimum distance d. When the minimum distance d of $\mathcal{C}$ is unknown, $\mathcal{C}$ is referred to as an $[n,k]_q$ code. Binary, ternary and quaternary codes are codes over F_2, F_3 and F_4, respectively. The code $\mathcal{C} \leq F_q^n$ is called trivial if it is the zero vector $\{0\}$, the whole space F_q^n, a subspace $\langle v\rangle$ of dimension 1 or the subspace $\langle v\rangle^\perp$ of dimension $n-1$. An $[n,k,d]_q$ code is called optimal if d is the largest possible minimum distance between all the $[n,k]_q$ codes. Elements of $\mathcal{C}$ are called codewords and $\mathrm{Supp}(c)$, support of c, is the set of non-zero coordinate positions of $c \in \mathcal{C}$. The weight of c, denoted by $\mathrm{wt}(c)$, is defined to be the number of non-zero coordinates of c. For any $x, y \in F_q^n$, $d(x,y) = \mathrm{wt}(x-y)$, where d is the Hamming distance between x and y. The diameter of $\mathcal{C}$ is $\mathrm{diam}(\mathcal{C}) = \max\{d(x,y) \mid x, y \in \mathcal{C}\}$. The all-one vector $\jmath$ is a vector all of whose coordinates are one. The orthogonal subspace of $\mathcal{C} \leq F_q^n$ is the dual code $\mathcal{C}^\perp$ and the hull of $\mathcal{C}$ is $\mathcal{C} \cap \mathcal{C}^\perp$. The code $\mathcal{C}$ is self-orthogonal if $\mathcal{C} \subseteq \mathcal{C}^\perp$, and self-dual if $\mathcal{C} = \mathcal{C}^\perp$. The code $\mathcal{C}$ is called weakly self-dual if $\mathcal{J} \in \mathcal{C} \subseteq \mathcal{C}^\perp$, where $\mathcal{J}$ is the all-one vector. The homogeneous polynomial $W_{\mathcal{C}}(x,y) = \sum_{c\in\mathcal{C}} x^{n-\mathrm{wt}(c)}y^{\mathrm{wt}(c)}$ is the weight enumerator of $\mathcal{C}$. The MacWilliams identity $W_{\mathcal{C}^\perp}(x,y) = \frac{1}{|\mathcal{C}|}W_{\mathcal{C}}(x+(q-1)y, x-y)$ holds for any q-ary code $\mathcal{C}$. A binary code $\mathcal{C}$ is called to be even (doubly even) if the weight of any codewords is divisible by 2 (4). Two q-ary linear codes $\mathcal{C}$ and $\mathcal{C}'$ of the same length are called equivalent if $\mathcal{C}$ can be obtained from $\mathcal{C}'$ by permuting the coordinate positions and possibly multiplying each coordinate position by a non-zero element of F_q. Two codes $\mathcal{C}$ and $\mathcal{C}'$ are called isomorphic and written $\mathcal{C} \cong \mathcal{C}'$ if $\mathcal{C}'$ can be obtained from $\mathcal{C}$ by permuting the coordinate positions. An automorphism φ of $\mathcal{C}$ is a permutation φ of the coordinate positions that maps $\mathcal{C}$ to $\mathcal{C}$. The automorphism group of $\mathcal{C}$, denoted by $\mathrm{Aut}(\mathcal{C})$, is the set of all the automorphisms of $\mathcal{C}$. The weight class $\mathcal{C}_{(l)}$ of $\mathcal{C}$

consists of all the codewords of weight l. An automorphism thus preserves each weight class of the code. The space spanned by the incidence vectors of the blocks of $\mathcal{S} = (\mathcal{P}, \mathcal{B}, \mathcal{I})$ over F_q is a linear code $\mathcal{C}_q \leq F_q^{\mathcal{P}}$, where $F_q^{\mathcal{P}}$ is the vector space of functions from $\mathcal{P}$ to F_q. Hence, $\mathcal{C}_q = \langle v^B \mid B \in \mathcal{B} \rangle$. When p is prime, the p-rank of $\mathcal{S}$ is the dimension of $\mathcal{C}_p$. The code $\mathcal{C}_q$ obtained from a t-design $\mathcal{D}$ is called a design's code. For more details, see [2, 19].

We follow the $\mathbb{ATLAS}$ [6] notation for the structure of groups. The groups $G.H$, $G{:}H$ and $G \cdot H$ denote a general extension, a split extension and a non-split extension, respectively. When p is prime, p^n denotes the elementary abelian group of order p^n. The general linear group of degree n, $GL_n(q)$, consists of all the invertible $n \times n$ matrices over F_q. The subset of $GL_n(q)$ consisting of matrices with determinent 1 forms the special linear group $SL_n(q)$. The map $\varphi : GL_n(q) \to F_q^*$, $A \mapsto \det A$, is an epimorphism with $\ker(\varphi) = SL_n(q)$. The group $GL_n(q)$ acts on the points of the $(n-1)$-dimensional projective space $PG(n-1, q)$ with the kernel $Z = \{\lambda I \mid \lambda \in F_q^*\}$. The projective general linear group $PGL_n(q)$ is defined to be $GL_n(q)/Z$ and the projective special linear group $PSL_n(q)$ is $SL_n(q)/(SL_n(q) \cap Z)$. It is possible to identify the projective line $PG(1, q)$ by $\{[\lambda : 1] \mid \lambda \in F_q\} \cup \{\infty = [1 : 0]\}$. Hence, the matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in PGL_2(q)$ maps $[\lambda : 1]$ to $[\frac{\lambda a + c}{\lambda b + d} : 1]$ if $0 \neq \lambda b + d$, and to ∞ otherwise. Also, this matrix maps ∞ to itself if $b = 0$, and to $[ab^{-1} : 1]$ otherwise. It is known that $PGL_2(q)$ and $LF_2(q) = \left\{ x \overset{f}{\mapsto} \frac{ax+c}{bx+d} \mid ad - bc \neq 0 \right\}$ are permutation isomorphic. The group $LF_2(q)$ is called the linear fractional group. Moreover, $L_2(q) = \left\{ x \overset{f}{\mapsto} \frac{ax+c}{bx+d} \mid 0 \neq ad - bc \text{ is square} \right\}$ is a subgroup of $LF_2(q)$ which is isomorphic to $PSL_2(q)$. For further properties, we refer the reader to [3, 29].

3. The method

We construct designs from the group $PSL_2(9)$ by the following method.

Theorem 3.1. [16, 17]*(Key-Moori Method 1) Let G be a finite primitive permutation group acting on the set Ω of size n. Let $\Delta \neq \{\omega\}$ be an orbit of the stabilizer G_ω, where $\omega \in \Omega$, and set $\mathcal{B} := \Delta^G = \{\Delta^g \mid g \in G\}$. Then, the incidence structure $\mathcal{D} = (\Omega, \mathcal{B}, \in)$ is a symmetric 1-$(n, |\Delta|, |\Delta|)$ design. Moreover, if Δ is a self-paired orbit then $\mathcal{D}$ is self-dual and G acts primitivly as an automorphism group on the points and blocks of $\mathcal{D}$.*

Note that there is only one orbit of length 1 since the point stabilizer is a maximal subgroup and G is simple. Moreover, If Δ is any union of orbits of G_ω, including the singleton orbit $\{\omega\}$, then $\mathcal{D}$ is still a symmetric 1-design with the group operating [16]. By Theorem 3.2, the Key-Moori Method 1 gives all possible designs $\mathcal{D} = (\mathcal{P}, \mathcal{B})$ on which G acts primitively on $\mathcal{P}$ and $\mathcal{B}$:

Max.	Deg.	#	Len.	$\lvert\mathrm{Aut}(\mathcal{D})\rvert$	Code	Dual	$\lvert\mathrm{Aut}(\mathcal{C})\rvert$
S_4	15	3	6(1)	720	—	—	—
			8(1)	$8!/2$	$[15,4,8]$	$[15,11,3]$	$8!/2$

Table 1. Quaternary codes from the group $PSL_2(9)$.

wt	$(A_8)_\omega$	Max.	$\mathcal{D}_\omega$	$\overline{\mathcal{D}}_\omega$	# blocks
(8)	$2^3{:}L_3(2)$	Yes	2-(15,8,4)	2-(15,7,3)	15
(12)	$2^4{:}(S_3 \times S_3)$	Yes	2-(15,12,22)	2-(15,3,1)	35

Table 2. The stabilizers $(A_8)_\omega$ and 1-designs $\mathcal{D}_\omega$ from $\mathcal{C}_8$

Theorem 3.2. [16] *If a group G acts primitively on the points and the blocks of a symmetric 1-design $\mathcal{D} = (\mathcal{P}, \mathcal{B})$ then $\mathcal{B} = \Delta^G$, where Δ is a union of orbits of a point-stabilizer G_ω, $\omega \in \mathcal{P}$.*

Theorem 3.3. [16] *If $\mathcal{D}$ is a symmetric 1-design constructed by applying the Key-Moori Method 1 on the group G then $G \leq \mathrm{Aut}(\mathcal{D})$. Moreover, if $\mathcal{C}$ is a design's code of $\mathcal{D}$ over a finite field then $\mathrm{Aut}(\mathcal{D}) \leq \mathrm{Aut}(\mathcal{C})$.*

As the above theorems show, we can construct designs and their q-ary codes for any given permutation group using the Key-Moori Method 1 and a computer program in Magma [4]. Moreover, the following theorem shows that we can obtain designs using the actions of the automorphism groups on the supports of the codewords:

Theorem 3.4. [20] *Let $\mathcal{C}$ be a linear code and suppose that ω is a codeword of weight l in $\mathcal{C}$. Then, $\mathrm{Supp}(\omega)^{\mathrm{Aut}(\mathcal{C})}$ is a 1-(n, l, k_l) design $\mathcal{D}_\omega$, where n is the length of $\mathcal{C}$ and $k_l = l|\omega^{\mathrm{Aut}(\mathcal{C})}|/n$.*

As it is seen, we can construct designs for any given code by Theorem 3.4 and a computer program in Magma.

4. The quaternary codes from $PSL_2(9)$

We construct quaternary codes from all the primitive representations of the group $PSL_2(9)$ using the Key-Moori Method 1. By Theorem 3.2, it is sufficient to use the action of $PSL_2(9)$ on the set of the right cosets of each maximal subgroup. We examine these design's codes for all primitive representations of $PSL_2(9)$ by Magma.

The projective special linear group $PSL_2(9)$ is a simple group of order 360 with five maximal subgroups up to conjugacy of orders 24, 24, 36, 60 and 60 isomorphic to S_4, S_4, $3^2{:}4$, A_5 and A_5, respectively. For the last three subgroups, the stabilizers $PSL_2(9)_\omega$ are transitive on the

wt	$(A_8)_\omega$	Max.	$\mathcal{D}'_\omega$	$\overline{\mathcal{D}'}_\omega$	# blocks
(3)	$2^4{:}(S_3 \times S_3)$	Yes	2-(15,3,1)	2-(15,12,22)	35
(4)	$2^4{:}D_{12}$	No	2-(15,4,6)	2-(15,11,55)	105
$(5)_1$	$2^3{:}D_8$	No	2-(15,5,30)	2-(15,10,135)	315
$(5)_2$	S_5	No	2-(15,5,16)	2-(15,10,72)	168
$(6)_1$	D_{12}	No	2-(15,6,240)	2-(15,9,576)	1680
$(6)_2$	$S_4 \times 2$	No	2-(15,6,60)	2-(15,9,144)	420
$(6)_3$	$3^2{:}D_8$	No	2-(15,6,40)	2-(15,9,96)	280
$(6)_4$	$2^3{:}S_4$	No	2-(15,6,15)	2-(15,9,36)	105
$(7)_1$	D_8	No	2-(15,7,504)	2-(15,8,672)	2520
$(7)_2$	2^3	No	2-(15,7,504)	2-(15,8,672)	2520
$(7)_3$	$S_4 \times 2$	No	2-(15,7,84)	2-(15,8,112)	420
$(7)_4$	$2^3{:}L_3(2)$	Yes	2-(15,7,3)	2-(15,8,4)	15
$(8)_1$	2^3	No	2-(15,8,672)	2-(15,7,504)	2520
$(8)_2$	D_8	No	2-(15,8,672)	2-(15,7,504)	2520
$(8)_3$	S_4	No	2-(15,8,224)	2-(15,7,168)	840
$(8)_4$	$S_4 \times 2$	No	2-(15,8,112)	2-(15,7,84)	420
$(8)_5$	$2^3{:}L_3(2)$	Yes	2-(15,8,4)	2-(15,7,3)	15
$(9)_1$	D_8	No	2-(15,9,864)	2-(15,6,360)	2520
$(9)_2$	D_{12}	No	2-(15,9,576)	2-(15,6,240)	1680
$(9)_3$	$3^2{:}D_8$	No	2-(15,9,96)	2-(15,6,40)	280
$(9)_4$	$S_4 \times 2$	No	2-(15,9,144)	2-(15,6,60)	420
$(9)_5$	$2^3{:}S_4$	No	2-(15,9,36)	2-(15,6,15)	105
$(10)_1$	D_{12}	No	2-(15,10,720)	2-(15,5,160)	1680
$(10)_2$	S_4	No	2-(15,10,360)	2-(15,5,80)	840
$(10)_3$	$2^4{:}2^2$	No	2-(15,10,135)	2-(15,5,30)	315
$(10)_4$	S_5	No	2-(15,10,72)	2-(15,5,16)	168
$(11)_1$	$2^4{:}(S_3 \times 2)$	No	2-(15,11,55)	2-(15,4,6)	105
$(11)_2$	$S_4 \times 2$	No	2-(15,11,220)	2-(15,4,24)	420
$(11)_3$	S_4	No	2-(15,11,440)	2-(15,4,48)	840
$(12)_1$	$S_4 \times 2$	No	2-(15,12,264)	2-(15,3,12)	420
$(12)_2$	$2^4{:}(S_3 \times S_3)$	Yes	2-(15,12,22)	2-(15,3,1)	35

Table 3. The stabilizers $(A_8)_\omega$ and 1-designs $\mathcal{D}'_\omega$ from $\mathcal{C}_8^\perp$

set $\Omega \setminus \{\omega\}$ and the constructed designs are trivial. Hence, we don't consider them. The primitive representations of the first two subgroups are also permutation isomorphic. Thus, we consider the action of $PSL_2(9)$ on the set of right cosets of its maximal subgroup S_4, which gives us a primitive representation of $PSL_2(9)$ of degree 15. Then, the point stabilizer $PSL_2(9)_\omega$ has three orbits of lengths 1, 6 and 8. In fact, we obtain two self-dual designs $\mathcal{D}_6$ and $\mathcal{D}_8$ with parameters 1-$(15, 6, 6)$ and 2-$(15, 8, 4)$, respectively [10]. All of the information about quaternary codes is given in Table 1. In this table, the shape of the maximal subgroup and its index are under the headings 'Max.' and 'Deg.', respectively. The number of orbits and their lengths are shown under the symbols '#' and 'Len.', respectively. The entry line $m(n)$ indicates that there exist n orbits of length m. The order of the automorphism group of each obtained design is written under the heading '$|\operatorname{Aut}(\mathcal{D})|$'. The symbols 'Code' and 'Dual' denote the parameters of the quaternary codes and their duals, respectively. The order of the automorphism group of each quaternary code is under the heading '$|\operatorname{Aut}(\mathcal{C})|$'.

Let $\mathcal{C}_6$ and $\mathcal{C}_8$ be two quaternary codes constructed from the designs $\mathcal{D}_6$ and $\mathcal{D}_8$, respectively. Magma shows that $\mathcal{C}_6$ is a trivial quaternary code and thus, will not be considered. Moreover, $\mathcal{C}_8$ is a quaternary code of length 15, dimension 4, and minimum distance 8. Its dual code $\mathcal{C}_8^\perp$ is a quaternary code with minimum distance 3 and hence, $\mathcal{C}_8^\perp$ is a $[15, 11, 3]$ quaternary code. By [27], $\mathcal{C}_8$ and $\mathcal{C}_8^\perp$ have minimum distances two and one less than the optimal codes, respectively. The weight enumerators of $\mathcal{C}_8$ and $\mathcal{C}_8^\perp$ are

$$
\begin{aligned}
W_{\mathcal{C}_8}(x,y) =& x^{15} + 45x^7y^8 + 210x^3y^{12},\\
W_{\mathcal{C}_8^\perp}(x,y) =& x^{15} + 105x^{12}y^3 + 315x^{11}y^4 + 2394x^{10}y^5 + 15750x^9y^6 + 54855x^8y^7 + 160695x^7y^8\\
&+ 391020x^6y^9 + 688212x^5y^{10} + 949095x^4y^{11} + 937965x^3y^{12} + 659610x^2y^{13}\\
&+ 277830xy^{14} + 56457y^{15}.
\end{aligned}
$$

Hence, $\mathcal{C}_8$ and $\mathcal{C}_8^\perp$ have the diameters 12 and 15, respectively. Moreover, $\mathcal{C}_8$ is even and $\jmath \in \mathcal{C}_8^\perp$. Magma shows that $\dim(\operatorname{hull}(\mathcal{C}_8)) = 4$ and hence, $\mathcal{C}_8$ is self-orthogonal. Computations with Magma implies that $|\operatorname{Aut}(\mathcal{C}_8)| = 8!/2$. On the other hand, $\operatorname{Aut}(\mathcal{D}_8) \cong A_8$ [10]. Therefore, $\operatorname{Aut}(\mathcal{C}_8) \cong A_8$ by Theorem 3.3. These results give us the following theorem:

Theorem 4.1. *Consider the primitive permutation representation of $PSL_2(9)$ of degree 15 and let Δ be the orbit of the point stabilizer of length 8. Then, the constructed design $\mathcal{D}_8$ is a symmetric 2-$(15, 8, 4)$ design invariant under its automorphism group A_8. Let $\mathcal{C}_8$ be the quaternary code constructed by $\mathcal{D}_8$. Then, $\mathcal{C}_8$ is a self-orthogonal even code with parameters $[15, 4, 8]_4$ and 45 codewords of minimum weight. Its dual, $\mathcal{C}_8^\perp$, is a $[15, 11, 3]_4$ code with 105 codewords of minimum weight. Moreover, $\operatorname{Aut}(\mathcal{C}_8) \cong A_8$.*

5. The Designs $\mathcal{D}_\omega$ and $\mathcal{D}'_\omega$

Consider the quaternary code $\mathcal{C}_8$ and its automorphism group A_8. Hence, A_8 naturally acts on two codes $\mathcal{C}_8$ and $\mathcal{C}_8^\perp$. It is deduced from Theorem 3.4 that if ω be a codeword of weight l in $\mathcal{C}_8$ or $\mathcal{C}_8^\perp$ then $\mathrm{Supp}(\omega)^{A_8}$ forms a 1-$(15, l, k_l)$ design, where $k_l = l|\omega^{A_8}|/15$.

All of the information about the actions of A_8 on the codes $\mathcal{C}_8$ and $\mathcal{C}_8^\perp$ are given in Tables 2 and 3, respectively. In Table 2, weight of a codeword is under the heading 'wt' and the entry line '$(l)_i$' denotes the ith orbit of weight l, i.e., $\mathcal{C}_{8(l)_i}$. If the action of A_8 on $\mathcal{C}_{8(l)}$ is transitive then there is only one orbit of weight l and the entry line is denoted by '(l)'. The heading '$(A_8)_\omega$' denotes the structure of the stabilizer and the maximality of stabilizers are given in the third column. The parameters of the constructed designs and their complements are given under '$\mathcal{D}_\omega$' and '$\overline{\mathcal{D}}_\omega$', respectively. The number of blocks is under the symbol '# blocks'. The Tables 2 and 3 have the same structure, except that designs in Table 3 are under the symbols '$\mathcal{D}'_\omega$' and '$\overline{\mathcal{D}'}_\omega$'. The designs obtained from the codewords of weight 13, 14, and 15 in $\mathcal{C}_8^\perp$ are trivial and thus, we will not consider them. In the following theorems, typical codewords in the weight classes $\mathcal{C}_{(l)_i}$ and $\mathcal{C}_{(l)}$ are denoted by $\omega_{(l)_i}$ and $\omega_{(l)}$, respectively.

Theorem 5.1. *Consider the action of the automorphism group A_8 on the quaternary code $\mathcal{C}_8$. If $\omega \in \mathcal{C}_8$ is a non-zero codeword then the weight class $(\mathcal{C}_8)_\omega$ is an orbits such that $(A_8)_\omega$ is a maximal subgroup of A_8. Moreover, A_8 acts primitively on the designs $\mathcal{D}_\omega$.*

Proof. We know that the non-zero codewords of $\mathcal{C}_8$ are of weights 8 or 12. By Magma, we form $\mathrm{Supp}(\omega)^{A_8}$ for any non-zero codeword ω. Our computations with Magma show that $\mathrm{Supp}(\omega_{(8)})^{A_8} = \mathcal{C}_{(8)}$ and $\mathrm{Supp}(\omega_{(12)})^{A_8} = \mathcal{C}_{(12)}$. Hence, A_8 is transitive on each of the weight classes $\mathcal{C}_{(8)}$ and $\mathcal{C}_{(12)}$. We use Magma to construct $(A_8)_{\omega_{(8)}}$ and $(A_8)_{\omega_{(12)}}$ as permutation groups inside A_8. It is seen that $(A_8)_{\omega_{(8)}}$ and $(A_8)_{\omega_{(12)}}$ are maximal subgroups of A_8 of orders 1344 and 576, respectively. By [6], $(A_8)_{\omega_{(8)}} \cong 2^3{:}L_3(2)$ and $(A_8)_{\omega_{(12)}} \cong 2^4{:}(S_3 \times S_3)$. Since A_8 is transitive on $\mathcal{C}_{(8)}$ and $\mathcal{C}_{(12)}$, the designs $\mathcal{D}_{(8)}$ and $\mathcal{D}_{(12)}$ with parameters 1-(15,8,8) and 1-(15,12,28) are obtained, respectively. Magma gives us $\mathcal{D}_{(8)}$ and $\mathcal{D}_{(12)}$ as 2-designs and thus, $\mathcal{D}_{(8)}$ and $\mathcal{D}_{(12)}$ are designs with parameters 2-(15,8,4) and 2-(15,12,22), respectively. The transitivity of A_8 on $\mathcal{C}_{(8)}$ and $\mathcal{C}_{(12)}$, and the maximality of $(A_8)_{\omega_{(8)}}$ and $(A_8)_{\omega_{(12)}}$ imply that A_8 acts primitively on the designs $\mathcal{D}_{(8)}$ and $\mathcal{D}_{(12)}$. See Table 2. □

Similarly, the following theorem is deduced:

Theorem 5.2. *Suppose that A_8 acts on $\mathcal{C}_8^\perp$ as the full automorphism group. Under this action, 35 orbits $\mathcal{C}_{(0)}$, $\mathcal{C}_{(3)}$, $\mathcal{C}_{(4)}$, $\mathcal{C}_{(5)_1}$, $\mathcal{C}_{(5)_2}$, $\mathcal{C}_{(6)_1}$, $\mathcal{C}_{(6)_2}$, $\mathcal{C}_{(6)_3}$, $\mathcal{C}_{(6)_4}$, $\mathcal{C}_{(7)_1}$, $\mathcal{C}_{(7)_2}$, $\mathcal{C}_{(7)_3}$, $\mathcal{C}_{(7)_4}$, $\mathcal{C}_{(8)_1}$, $\mathcal{C}_{(8)_2}$, $\mathcal{C}_{(8)_3}$, $\mathcal{C}_{(8)_4}$, $\mathcal{C}_{(8)_5}$, $\mathcal{C}_{(9)_1}$, $\mathcal{C}_{(9)_2}$, $\mathcal{C}_{(9)_3}$, $\mathcal{C}_{(9)_4}$, $\mathcal{C}_{(9)_5}$, $\mathcal{C}_{(10)_1}$, $\mathcal{C}_{(10)_2}$, $\mathcal{C}_{(10)_3}$, $\mathcal{C}_{(10)_4}$, $\mathcal{C}_{(11)_1}$, $\mathcal{C}_{(11)_2}$, $\mathcal{C}_{(11)_3}$, $\mathcal{C}_{(12)_1}$, $\mathcal{C}_{(12)_2}$, $\mathcal{C}_{(13)}$, $\mathcal{C}_{(14)}$ and $\mathcal{C}_{(15)}$ are obtained. If ω be a codeword in $\mathcal{C}_{(3)}$, $\mathcal{C}_{(7)_4}$,*

$\mathcal{C}_{(8)_5}$ or $\mathcal{C}_{(12)_2}$ then $(A_8)_\omega$ is a maximal subgroup of A_8 and moreover, A_8 acts primitively on the related design $\mathcal{D}'_\omega$.

Proof. By Magma, we form $\mathrm{Supp}(\omega)^{A_8}$ for any codeword $\omega \in \mathcal{C}_8^\perp$. Clearly, $\mathrm{Supp}(\omega)^{A_8}$ is an orbit and A_8 is transitive on it. Our computations with Magma imply that there are 35 orbits $\mathcal{C}_{(0)}$, $\mathcal{C}_{(3)}$, $\mathcal{C}_{(4)}$, $\mathcal{C}_{(5)_1}$, $\mathcal{C}_{(5)_2}$, $\mathcal{C}_{(6)_1}$, $\mathcal{C}_{(6)_2}$, $\mathcal{C}_{(6)_3}$, $\mathcal{C}_{(6)_4}$, $\mathcal{C}_{(7)_1}$, $\mathcal{C}_{(7)_2}$, $\mathcal{C}_{(7)_3}$, $\mathcal{C}_{(7)_4}$, $\mathcal{C}_{(8)_1}$, $\mathcal{C}_{(8)_2}$, $\mathcal{C}_{(8)_3}$, $\mathcal{C}_{(8)_4}$, $\mathcal{C}_{(8)_5}$, $\mathcal{C}_{(9)_1}$, $\mathcal{C}_{(9)_2}$, $\mathcal{C}_{(9)_3}$, $\mathcal{C}_{(9)_4}$, $\mathcal{C}_{(9)_5}$, $\mathcal{C}_{(10)_1}$, $\mathcal{C}_{(10)_2}$, $\mathcal{C}_{(10)_3}$, $\mathcal{C}_{(10)_4}$, $\mathcal{C}_{(11)_1}$, $\mathcal{C}_{(11)_2}$, $\mathcal{C}_{(11)_3}$, $\mathcal{C}_{(12)_1}$, $\mathcal{C}_{(12)_2}$, $\mathcal{C}_{(13)}$, $\mathcal{C}_{(14)}$ and $\mathcal{C}_{(15)}$. Since A_8 is transitive on each orbit, these orbits form 1-designs $\mathcal{D}'_{(l)_i}$. Furthermore, we construct $(A_8)_\omega$ as a permutation group inside A_8. Magma shows that $(A_8)_{\omega_{(3)}}$, $(A_8)_{\omega_{(7)_4}}$, $(A_8)_{\omega_{(8)_5}}$ and $(A_8)_{\omega_{(12)_2}}$ are maximal subgroups of A_8 of orders 576, 1344, 1344 and 576, respectively. By [6], it is deduced that $(A_8)_{\omega_{(3)}} \cong (A_8)_{\omega_{(12)_2}} \cong 2^4{:}(S_3 \times S_3)$ and $(A_8)_{\omega_{(7)_4}} \cong (A_8)_{\omega_{(8)_5}} \cong 2^3{:}L_3(2)$. Note that the orbits $\mathcal{C}_{(3)}$, $\mathcal{C}_{(7)_4}$, $\mathcal{C}_{(8)_5}$ and $\mathcal{C}_{(12)_2}$ form the designs $\mathcal{D}'_{(3)}$, $\mathcal{D}'_{(7)_4}$, $\mathcal{D}'_{(8)_5}$ and $\mathcal{D}'_{(12)_2}$ with parameters 1-(15,3,7), 1-(15,7,7), 1-(15,8,8) and 1-(15,12,28), respectively. Magma gives us these designs as 2-designs. Thus, they are designs with parameters 2-(15,3,1), 2-(15,7,3), 2-(15,8,4) and 2-(15,12,22), respectively. The transitivity of A_8 on these weight subclasses and the maximality of the related stabilizers $(A_8)_{\omega_{(l)_i}}$ imply that A_8 acts primitively on the designs $\mathcal{D}'_{(3)}$, $\mathcal{D}'_{(7)_4}$, $\mathcal{D}'_{(8)_5}$ and $\mathcal{D}'_{(12)_2}$. See Table 3. Note that the designs $\mathcal{D}'_{(13)}$, $\mathcal{D}'_{(14)}$ and $\mathcal{D}'_{(15)}$ are trivial and will not be considered. □

By an exhaustive search, we see that all the obtained 1-designs are 2-designs. Therefore, for any codeword ω of weight l in $\mathcal{C}_8$ or $\mathcal{C}_8^\perp$, the 1-$(15, l, k_l)$ designs are in fact 2-$\left(15, l, \binom{l}{2}|\omega^{A_8}|/\binom{15}{2}\right)$ designs. Note that the number of blocks in each of these designs is $|\omega^{A_8}|$. Now, the following theorem is implied:

Theorem 5.3. *Consider the quaternary codes $\mathcal{C}_8$ and $\mathcal{C}_8^\perp$, and their full automorphism group A_8. If ω is a codeword of weight l in $\mathcal{C}_8$ or its dual then* $\mathrm{Supp}(\omega)^{A_8}$ *forms a 2-$(15, l, \lambda)$ design, where $\lambda = \binom{l}{2}|\omega^{A_8}|/\binom{15}{2}$. Moreover,* $\mathrm{Aut}(\mathrm{Supp}(\omega)^{A_8}) \cong A_8$.

As it is seen in Tables 2 and 3, we have 33 non-trivial 2-designs. Magma shows that $\mathcal{D}_{\omega_{(8)}} = \overline{\mathcal{D}'}_{\omega_{(7)_4}} = \mathcal{D}'_{\omega_{(8)_5}}$, $\mathcal{D}_{\omega_{(12)}} = \overline{\mathcal{D}'}_{\omega_{(3)}} = \mathcal{D}'_{\omega_{(12)_2}}$, $\mathcal{D}'_{\omega_{(4)}} = \overline{\mathcal{D}'}_{\omega_{(11)_1}}$, $\mathcal{D}'_{\omega_{(5)_1}} = \overline{\mathcal{D}'}_{\omega_{(10)_3}}$, $\mathcal{D}'_{\omega_{(5)_2}} = \overline{\mathcal{D}'}_{\omega_{(10)_4}}$, $\mathcal{D}'_{\omega_{(6)_1}} = \overline{\mathcal{D}'}_{\omega_{(9)_2}}$, $\mathcal{D}'_{\omega_{(6)_2}} = \overline{\mathcal{D}'}_{\omega_{(9)_4}}$, $\mathcal{D}'_{\omega_{(6)_3}} = \overline{\mathcal{D}'}_{\omega_{(9)_3}}$, $\mathcal{D}'_{\omega_{(6)_4}} = \overline{\mathcal{D}'}_{\omega_{(9)_5}}$, $\mathcal{D}'_{\omega_{(7)_1}} = \overline{\mathcal{D}'}_{\omega_{(8)_2}}$, $\mathcal{D}'_{\omega_{(7)_2}} = \overline{\mathcal{D}'}_{\omega_{(8)_1}}$, and $\mathcal{D}'_{\omega_{(7)_3}} = \overline{\mathcal{D}'}_{\omega_{(8)_4}}$. Moreover, $\mathcal{D}'_{\omega_{(7)_1}}$ and $\mathcal{D}'_{\omega_{(7)_2}}$ are non-isomorphic designs with the same parameters. Therefore, 19 different designs $\mathcal{D}'_{\omega_{(3)}}$, $\mathcal{D}'_{\omega_{(4)}}$, $\mathcal{D}'_{\omega_{(5)_1}}$, $\mathcal{D}'_{\omega_{(5)_2}}$, $\mathcal{D}'_{\omega_{(6)_1}}$, $\mathcal{D}'_{\omega_{(6)_2}}$, $\mathcal{D}'_{\omega_{(6)_3}}$, $\mathcal{D}'_{\omega_{(6)_4}}$, $\mathcal{D}'_{\omega_{(7)_1}}$, $\mathcal{D}'_{\omega_{(7)_2}}$, $\mathcal{D}'_{\omega_{(7)_3}}$, $\mathcal{D}'_{\omega_{(7)_4}}$, $\overline{\mathcal{D}'}_{\omega_{(8)_3}}$, $\overline{\mathcal{D}'}_{\omega_{(9)_1}}$, $\overline{\mathcal{D}'}_{\omega_{(10)_1}}$, $\overline{\mathcal{D}'}_{\omega_{(10)_2}}$, $\overline{\mathcal{D}'}_{\omega_{(11)_2}}$, $\overline{\mathcal{D}'}_{\omega_{(11)_3}}$ and $\overline{\mathcal{D}'}_{\omega_{(12)_1}}$ are obtained. Magma shows that all of these designs are invariant under the full automorphism group A_8. According to [5], there are 80 non-isomorphic 2-(15,3,1) designs, five non-isomorphic symmetric 2-(15,7,3) designs and at least 31300 2-(15,4,6) designs. So, up to our best knowlege, 16 new 2-$(15, l, \lambda)$ designs are obtained. See Table 3.

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Reza Kahkeshani

Department of Pure mathematics

Faculty of Mathematical Sciences

University of Kashan

Kashan, Iran.

kahkeshanireza@kashanu.ac.ir