JOURNAL OF RESEARCH ON APPLIED GEOPHYSICS

(JRAG)

2018, VOL 4, NO 1

(DOI): 10.22044/JRAG.2016.744

Shahrood University of Technology

Iranian Geophysical Society - 1975

Seismic travel time inversion for velocity model estimation using combination of Tikhonov and total variation regularizations

Mohammad Alrajawi[1], Hamid Reza Siahkoohi[2*] and Ali Gholami[3]

1- Ph.D. Candidate, Institute of Geophysics, University of Tehran, Tehran, Iran
2- Professor, Institute of Geophysics, University of Tehran, Tehran, Iran
3- Associate Professor, Institute of Geophysics, University of Tehran, Tehran, Iran

Received: 7 May 2016; Accepted: 15 September 2016

Corresponding author: hamid@ut.ac.ir

Keywords
Traveltime Inversion
Inverse Problems
Tikhonov Regularization
TV Regularization
Vertical Seismic Profile (VSP)
Regularization Parameters

Extended Abstract

Summary

A variety of methods has been presented to invert arrival time of seismic waves for velocity distribution over the survey area. In real world, the velocity distribution model consists of blocky structures as well as smooth varying parts. In such cases, implementation of Tikhonov regularization will recover the smooth varying portion of the velocity model; while the Total Variation (TV) regularization is capable of recovering the blocky varying parts of the velocity model. In this research, a technique for solving inverse problems based on a combination of second order Tikhonov and TV regularizations is proposed. The methods are tested on both synthetic and real arrival times and the results are presented.

Introduction

Inverse problems are applied in different disciplines. These methods in geophysical applications become a bit complicated due to special conditions of geophysical data. The conditions arise from the fact that geophysical data are collected only once due to the high acquisition cost; or it is due to the effects of discretization of the problem. In general, the issues that we encounter in geophysical inverse problems are classified as ill-posed inverse problems. Inverse problems are divided into three categories: Over-determined, Under-determined, and Even-determined. Over-determined systems often have no solutions; it is possible for an over-determined system of equations to have either many solutions or exactly one solution. For even-determined problems, it is possible to have a unique solution. A system of equations with fewer equations than variables is under-determined. In many cases under-determined systems of equations have infinitely many solutions. Under-determined systems need the regularization. In such cases regularization is required to choose a suitable model from those fitting the data. Two conventional regularization methods are Tikhonov regularization and TV regularization.

Methodology and Approaches

For a linear inverse problem $d = Gm + e$ where $d \in \Re^m$ is data, $m \in \Re^n$ is model parameters, $e \in \Re^m$ is noise, and $G \in \Re^{m \times n}$ is forward operator, the Tikhonov regularization is expressed as: $J = \|d - Gm\|_2^2 + \lambda \|L_i m\|_2^2$, where $\lambda \rangle 0$ is the regularization parameter controlling the conditioning of the problem and matrix L is a regularization operator. The Tikhonov regularization brings the advantage of linearity of the problem so that m can be determined analytically by solving normal equations $\hat{m} = (G^T G + \lambda L_i^T L_i)^{-1} G^T d$.

The TV regularization restricts the domain of possible candidate solutions to those having sparse gradient, and can be expressed as the following minimization equation: $J = \|d - Gm\|_2^2 + \lambda \|L_1 m\|_1$ with a solution *m* which can be determined by solving normal equations: $(G^T G + \frac{\lambda}{2} L_1^T \vartheta^{k+1} L_1) m^{k+1} = G^T d$, where $\vartheta^{k+1} = diag_{i=1...M-1}(\frac{1}{\sqrt{[L_1 m^k]_i^2 + \varepsilon}})$ is a diagonal matrix and ε is a small positive number.

A technique for solving inverse problems based on a combination of second order Tikhonov and TV regularizations is proposed. Assume signal x is a combination of two signals $x = x_1 + x_2$, where x_1 is a blocky component and x_2 is a smooth component of x. The proposed method can be approximated by using a new minimization problem of the form $\arg\min_{x1,x2}\left\langle \|y - G(x_1 + x_2)\|_2^2 + \mu(\|L_2x_2\|_2^2 + \xi\|L_1x_1\|_1^1)\right\rangle$, where μ, ξ are regularization parameters, and L_1 and L_2 are first-order and second-order derivative operators. An iteratively re-weighted least squares (IRLS) technique is used as a fast and an efficient algorithm for minimization of the cost function. The solution x can be determined iteratively by solving normal weighted equations:

$\begin{cases}(G^TG + \frac{\mu\xi}{2}L_1{}^T\vartheta(x_1)L_1)x_1 + G^TGx_2 = G^Ty \\ G^TGx_1 + (G^TG + \mu L_2{}^TL_2)x_2 = G^Ty\end{cases}$, where $\vartheta(x_1) = diag_{i=1\ldots M-1}(\frac{1}{\sqrt{[L_1x_1]_i^2 + \varepsilon}})$ is a diagonal matrix and ε is a small positive number. In this study, a method also is presented for determination of the regularization parameters similar to the L-curve method.

Results and Conclusions

In this paper, a technique for solving inverse problems based on a combination of second order Tikhonov and TV regularizations is proposed. The method eliminates their individual weaknesses and recovers both blocky and smooth portions of the model.