



Comparison of the stability of the downward continuation of gravity field by using the Tikhonov regularization and improved regularization operators

Ata Eshaghzadeh^{1*} and Alireza Hajian²

1. Graduate student of geophysics, Institute of Geophysics, University of Tehran, Iran

2. Department of Physics, Najafabad Branch, Islamic Azad University, Isfahan, Iran

Received: 1 July 2017; Accepted: 22 November 2017

Corresponding author: eshagh@alumni.ut.ac.ir

Keywords

C-norm
Cutoff wavenumber
Downward continuation
Improved regularization operator
Kohe Namak
Tikhonov regularization

Extended Abstract

summary

Downward continuation of potential field data plays an important role in interpretation of gravity and magnetic data. For its inherent instability, many methods have been presented to downward continue stably and precisely. The Tikhonov regularization approach is one of the most robust. It is based on a low-pass filter derivation in the Fourier spectral domain, by means of a minimization problem solution. In this manuscript, we propose an improved regularization operator for downward continuation of potential field data. First, we simply define a special wavenumber named the cutoff wavenumber to divide the potential field spectrum into the signal part and the noise part based on the radially averaged power spectrum of potential field data. Next, we use the conventional downward continuation operator to downward continue the signal and the Tikhonov regularization operator to suppress the noise. Moreover, the

parameters of the improved operator are defined by the cutoff wavenumber which has an obvious physical significance. For computing the α parameter, it is necessary that the C-norm of the potential field must be calculated. The improved operator can not only eliminate the influence of the high-wavenumber noise but also avoid the attenuation of the signal. Experiments through synthetic gravity and real gravity data from Kohe Namak region, Ghom province, Iran show that the downward continuation precision of the proposed operator is higher than the Tikhonov regularization operator.

Introduction

The analytical continuation of potential fields is recognized as being a powerful tool in the transformation of geophysical potential fields (mainly in gravity and geomagnetic fields). Continuation of the potential field data above the level of measurement is known as upward continuation and in the opposite direction (below the level of measurement but must be above sources), it is known as downward continuation. During the potential field data processing and interpretation, upward continuation is often used to enhance the regional components in the original data by attenuating shallow surface sources manifestation and downward continuation is often used to enhance the detection of shallower sources by extracting the local anomalies and calculate the depth of the important shallowest sources. From the point of view of potential theory, upward continuation is a stable transformation and it can be performed for any reasonable height level in the source free area (in the Fourier spectral domain, it is a low pass filter). A problem occurs in the case of downward continuation, as this is an unstable operation (e.g., Parker, 1977). It has been analytically derived from potential field theory by various authors (e.g., Baranov, 1975; p. 47–49), that we can only continue downwards an interconnected potential field function to the depth of its nearest source (its upper edge).

Methodology and Approaches

The Tikhonov regularization operator has the following form (Abedi et al., 2013; Li et al., 2013; Zeng et al., 2013):

$$R = \frac{1}{1 + \alpha e^{2h\omega_r}} \cdot e^{h\omega_r} \quad (1)$$

where α is the regularization parameter. If we want to design a better regularization low-pass filter, it is desired to all-pass the signal parts while at the same time suppressing the noise parts. As a result, we proposed the following

improved regularization low-pass filter:

$$filter = \begin{cases} 1 \rightarrow \text{if } \omega_r \leq \omega_c \\ \frac{1}{1 + \alpha e^{2h\omega_r}} \rightarrow \text{if } \omega_r > \omega_c \end{cases} \quad (2)$$

where ω_c is the cutoff wavenumber which divides the potential field spectrum into two parts. The signal part of the spectrum is fit separately from the noise. At last, the noise is constrained to have zero slope, and both solutions are constrained to merge at the wavenumber ω_c . Slope and intercept of the signal and mean noise level are fit by least squares to spectral samples partitioned at ω_c . In order to regularize the downward continuation function without overfiltering or underfiltering the data, it is necessary to choose the regularization parameters, α and ω_c , which taper the continuation function beginning near the wavenumber ω_c . The improved regularization low-pass filter in Eq. (2) changes into:

$$filter = \begin{cases} 1 \rightarrow \text{if } \omega_r \leq \omega_c \\ \frac{1}{1 + e^{2h(\omega_r - \omega_c)}} \rightarrow \text{if } \omega_r > \omega_c \end{cases} \quad (3)$$

where ω_c is decided by the radially averaged power spectrum.

Results and Conclusions

The downward continuation of potential field data is an inverse problem that requires the use of regularization theory, especially when the noise is present in the data. In this manuscript, we presented an improved regularization operator and regularization parameter choice method to accomplish the downward continuation of potential field data based on the characteristic of the potential field spectrum. We demonstrate the proposed operator on noise corrupted gravity model and real aeromagnetic anomalies, and the results show that the proposed regularization operator can obtain more precise results compared to the Tikhonov regularization operator.